Limnol. Oceanogr., 29(2), 1984, 350–356

Dependence of relationship between inherent and apparent optical properties of water on solar altitude

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Abstract

A computer simulation study, using the Monte Carlo calculation procedure, has been carried out to determine in what way the relationships between the apparent and the inherent optical properties of natural waters are affected by the angle of incidence of the photons on the surface. With the particular normalized volume scattering function used in these simulations, the vertical attenuation coefficient for downward irradiance at the midpoint of the euphotic zone, $K_d(z_m)$, can be expressed as a function of the absorption (a) and scattering (b) coefficients, and the cosine of the incident photons just below the surface (μ_o), in accordance with

$$K_d(z_m) = \frac{1}{\mu_o}[a^2 + (0.473\mu_o - 0.218)ab]^{1/2}.$$

Thus, as the direction of the incident light departs increasingly from the vertical, $K_d(z_m)$ increases but becomes progressively less responsive to increases in scattering at constant absorption. The irradiance reflectance just below the surface [$R(0)$] at all angles of incidence increases linearly with the ratio of the backscattering coefficient (b_b) to the absorption coefficient, and at any value of b_b/a increases as the angle of the light departs further from the vertical, in approximate conformity with

$$R(0) = (0.975 - 0.629\mu_o)b_b/a.$$

The optical properties of natural waters most commonly measured by aquatic scientists are functions of the underwater irradiance, such as the vertical attenuation coefficient for downward irradiance (K_d) or the irradiance reflectance ($R = E_u/E_d$, where E_u and E_d are upward and downward irradiances at a given depth). Although strictly speaking these are not properties of the water itself but of the light field established within it and vary to some extent both with depth and with solar altitude, their values are nevertheless largely a function of the composition of the water, and it is useful to be able to treat them as properties of the water. Preisendorfer (1961) refers to properties such as these, whose value at any given point in the medium is dependent on the radiance distribution at that point, as *apparent optical properties* to distinguish them from the *inherent optical properties* such as the absorption coefficient (a), the scattering coefficient (b), and the beam attenuation coefficient (c)—intrinsic properties of the aquatic medium itself whose value is not affected by the prevailing distribution of radiance.

Since the apparent properties are largely determined by the inherent properties, a knowledge of the relationship between them is of value. It makes it possible to estimate the inherent properties from the more easily measured apparent properties and also to predict the way in which the biologically more relevant apparent properties will change as the result of changes in the inherent optical properties associated with anticipated changes in the absorbing and scattering components of the water.

The complex behavior of photons in water, resulting from the combined effects of scattering and absorption and the shape of the volume scattering function, makes the establishment of explicit theoretical relationships between the apparent and the inherent optical properties of water difficult, although it is possible if the underwater radiance distribution is known (Preisendorfer 1976). On the other hand, using appropriate computer simulation techniques we can calculate, for any given set of values of the inherent optical properties, the values of the apparent optical properties or any characteristics of the underwater light field (Beardsley and Zaneveld 1969; Plass and Kattawar 1969; Gordon et al. 1975; Kirk 1981*a*). Such computer modeling could in principle give rise to results of general utility if, as the result of systematic study of the dependence of the apparent on the inherent optical properties, widely applicable empirical relationships could be found.

Gordon et al. (1975), by least-squares curve fitting to results obtained by the Monte Carlo calculation procedure, arrived at a power series which related K_d to a combination of inherent optical properties. Kirk (1981*a*), also using the Monte Carlo procedure, was able to fit the results for vertically incident light satisfactorily to the simple relationship

$$K_d(z_m) = (a^2 + 0.256ab)^{1/2}, \qquad (1)$$

z_m being the depth at which irradiance is 10% of that just beneath the surface, i.e. the midpoint of the euphotic zone. In the case of irradiance reflectance, Gordon et al. fitted their data to another power series which can be simplified (Jerlov 1976) to

$$R(0) = \text{constant}\frac{b_b}{a + b_b} \qquad (2)$$

where $R(0)$ is the irradiance reflectance just below the surface, b_b is the backscattering coefficient, and the constant has the value 0.32 for zenith sun and 0.37 for an overcast sky. Kirk (1981*a*) found the relationship

$$R(0) = 0.328b_b/a \qquad (3)$$

to hold satisfactorily for zenith sun, in good agreement with the relationship

$$R(0) = 0.33b_b/a \qquad (4)$$

obtained by Prieur (1976) using a different calculation procedure.

These various relationships have been found useful in a number of situations. Morel and Prieur (1977) used Eq. 4 in their analysis of the reasons for variation in ocean color. Bukata et al. (1979) used the power series of Gordon et al. (1975), in conjunction with their own measurements of K_d, $R(0)$, and c to determine a and b in Lake Ontario. Kirk (1981*b*) used relationships arising out of Monte Carlo simulations to calculate the values of b in certain Australian lakes solely from the measured underwater irradiance values. Phillips and Kirk (in prep.) have used Monte Carlo-derived empirical relationships to determine the spectral variation of a and b in the waters of Jervis Bay, N.S.W., on the basis of their measured values of K_d and c as a function of wavelength.

Since the apparent optical properties are affected by the angular structure of the light field within the water, as well as by the inherent optical properties, we would expect the relationship between the inherent and apparent optical properties to depend in part on the angular structure of the light flux incident on the water surface, i.e. to vary with solar altitude and with the proportion of diffuse and direct solar radiation. The nature of this dependence has not hitherto been systematically explored, although Gordon et al. (1975) and Kirk (1981*a*) did carry out simulations for more than one solar altitude.

The work described here was carried out to determine, by Monte Carlo calculation, in what way the relationships between apparent and inherent optical properties are affected by variation in the angle of the solar beam on the surface.

Calculation procedure

The calculation of the nature of the light field established within water bodies having various values of the ratio of scattering to absorption coefficient and illuminated from above with a stream of parallel photons at various angles to the surface was carried out by the Monte Carlo procedure as described previously (Kirk 1981*a,c*). As before, the water was assumed to have a normalized volume scattering function, $\tilde{\beta}(\theta)$, identical to that measured by Petzold (1972) in the water of San Diego harbor. Each simulation run, with a given value of a, b, and angle of incident light, was carried out with a total of 10^6 photons. The water surface was assumed to be flat.

The zenith angle of the incident light flux was varied from 0° to 89° (corresponding to a range of solar altitude from 90° down to 1°). At each angle of incidence, the absorption coefficient was given the value $1.0 \cdot m^{-1}$, and simulation was carried out for a series of values of the scattering coefficient ranging from 1.0 to $18.0 \cdot m^{-1}$ or higher. For each value of b the values of downward irradiance just above and below z_m were used to calculate $K_d(z_m)$, and the values of upward and downward irradiance just below the surface were used to obtain $R(0)$.

To simulate overcast conditions I assumed a Standard Overcast Sky, with a cardioidal radiance distribution [radiance at any zenith angle, θ', proportional to $(1 + 2 \cos \theta')$]. The cumulative angular distribution of photons incident on unit area from such a sky is given by

$$F(\theta') = (3 \cos^2\theta' + 4 \cos^3\theta')/7.$$

By setting a random number, R, equal to $F(\theta')$ and solving the resultant cubic equation, the computer chooses an appropriate angle of incidence for each photon. The Fresnel equation in conjunction with another random number is used to determine whether each photon is reflected at the surface or passes through into the water.

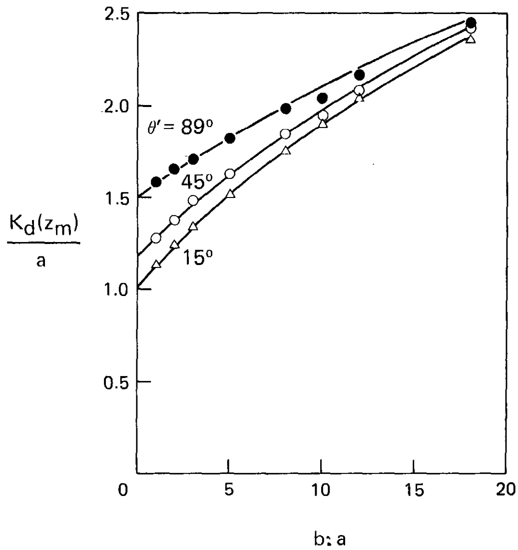

Fig. 1. Ratio of vertical attenuation coefficient to absorption coefficient as a function of b:a at different angles of incidence. The zenith angle of the incident flux above the surface is indicated next to each curve. The points are the values obtained in Monte Carlo simulations; the curves were calculated using Eq. 10.

Results

Dependence of K_d *on* a, b, *and* θ'—A stream of photons incident on the surface at zenith angle, θ', undergoes refraction at the air/water boundary in accordance with Snell's Law to give rise to a stream of photons within the water which, just under the surface, is at an angle, θ, to the nadir. The cosine of this angle we shall indicate by μ_o.

Kirk (1981*a*) determined the value of K_d at the midpoint of the euphotic zone (z_m, where the downward irradiance is reduced to 10% of the subsurface value) for a series of values of b:a in water illuminated with vertically incident light. The starting point for the series of values of $K_d(z_m)$ obtained was the value when there is no scattering ($b = 0$), and K_d for vertically incident light in nonscattering water (in which K_d does not vary with depth) is equal to the absorption coefficient, a. For a nonvertical (but parallel) incident light stream (since for every meter of depth the photons traverse a pathlength of $1/\mu_o$), the value of K_d in the absence of scattering is a/μ_o. This effect of the initial direction of the light stream will also be present in scattering waters regardless of what additional consequences may arise from the occurrence of scattering. Thus we may plausibly anticipate that in such waters the dependence of $K_d(z_m)$ on a, b, and μ_o will take the general form

$$K_d(z_m) = \frac{1}{\mu_o} f(a, b, \mu_o). \qquad (5)$$

In the previous study it was found that for vertically incident light the dependence of $K_d(z_m)$ on a and b takes the form

$$K_d(z_m) = (a^2 + gab)^{1/2} \qquad (6)$$

where g is a constant whose value, with the particular normalized volume scattering function used in the simulations, is 0.256. It seemed possible that for nonvertical light a relationship might exist, in accordance with Eq. 5 but similar in form to Eq. 6, in which g is a function of μ_o.

At all seven angles of incidence studied the $K_d(z_m)$ values determined by Monte Carlo calculation in fact conformed satisfactorily to an equation of the general form

$$K_d(z_m) = \frac{1}{\mu_o}[a^2 + \text{constant} \times ab]^{1/2}. \qquad (7)$$

For a given value of μ_o the value of the constant at each value of b:a was calculated from the $K_d(z_m)$ value using a rearranged form of Eq. 7:

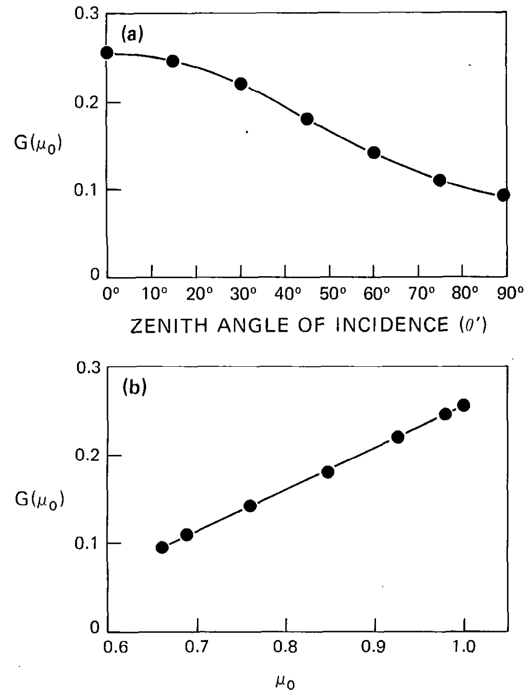

Fig. 2. Variation of $G(\mu_o)$ with (a) zenith angle of incidence and (b) cosine of the incident photons after refraction at the surface.

$$\text{constant} = \frac{a}{b}\left\{\left[\frac{K_d(z_m)\mu_o}{a}\right]^2 - 1\right\} \quad (8)$$

and the average value over the range b:a = 1.0 to 18.0 was determined; this is given the symbol $G(\mu_o)$, and we can now write

$$K_d(z_m) = \frac{1}{\mu_o}[a^2 + G(\mu_o)ab]^{1/2}, \quad (9)$$

or

$$\frac{K_d(z_m)}{a} = \frac{1}{\mu_o}\left[1 + G(\mu_o)\frac{b}{a}\right]^{1/2}. \quad (10)$$

In Fig. 1 the individual values of $K_d(z_m/a$ arising out of the Monte Carlo calculation for series of values of b:a for light incident at θ' = 15°, 45°, and 89° have been plotted together with the curves obtained using Eq. 10 with the appropriate values of $G(\mu_o)$ (i.e. the average value of the constant for b:a = 1 to 18, at a given μ_o). As can be seen, the values of $K_d(z_m)$ obtained from Monte Carlo simulation lie close to the corresponding curves, indicating that Eq. 9 and 10 are reasonable average representations of the relationship between $K_d(z_m)$, b, a, and μ_o. Agreement at the greatest zenith angle (θ' = 89°) is somewhat less good than at the other angles.

As the zenith angle of incidence increased (i.e. as solar altitude decreased), the value of the factor, $G(\mu_o)$, decreased (Fig. 2a). When $G(\mu_o)$ was plotted against μ_o (the cosine of the angle of the light just after passing through the surface), a linear relationship was observed (Fig. 2b), and in fact $G(\mu_o)$ can be accurately expressed as a function of μ_o by the equation

$$G(\mu_o) = 0.473\mu_o - 0.218. \quad (11a)$$

Thus we can now express $K_d(z_m)$ as an explicit function of a, b, and μ_o:

$$K_d(z_m) = \frac{1}{\mu_o}[a^2 + (0.473\mu_o - 0.218)ab]^{1/2}, \quad (12)$$

or

$$\frac{K_d(z_m)}{a} = \frac{1}{\mu_o}\left[1 + (0.473\mu_o - 0.218)\frac{b}{a}\right]^{1/2}. \quad (13)$$

For some purposes it is more useful to have expressions for K_d(av), the average value of K_d throughout the euphotic zone (the layer of water within which downward irradiance is reduced to 1% of the subsurface value). Kirk (1981*a*) showed that K_d(av) is close in value to $K_d(z_m)$ and varies with b:a in a very similar manner. In the present work I found that expressions for K_d(av) and K_d(av)/a, identical in form to Eq. 9 and 10, can be written in which the value of $G(\mu_o)$ is given by

$$G(\mu_o) = 0.425\mu_o - 0.190. \quad (11b)$$

Thus we obtain expressions, analogous to Eq. 12 and 13, for K_d(av) as an explicit function of a, b, and μ_o.

Dependence of R*(0) on* a, b_b, *and* θ'—As outlined earlier, it is already known from

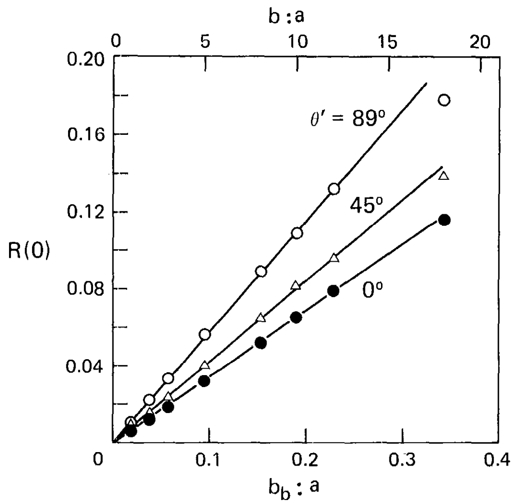

Fig. 3. Linear relationship between irradiance reflectance and the ratio of the backscattering (b_b) to the absorption (a) coefficient at three different angles of incidence.

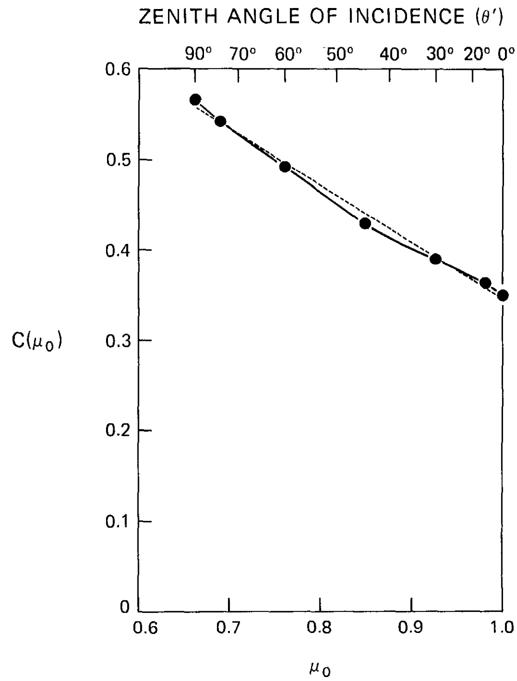

Fig. 4. Variation of $C(\mu_o)$ with μ_o. The continuous curve is drawn to join all the points. The broken line is the best straight line that can be drawn through the points.

previous modeling studies that the irradiance reflectance just below the surface is proportional to the backscattering coefficient and inversely proportional to the absorption coefficient in accordance with

$$R(0) = C(\mu_o)b_b/a \qquad (14)$$

where $C(\mu_o)$ is a function of the angular distribution of the incident light flux and so is a constant for a given angular distribution. Other calculations had shown that reflectance increases as solar altitude decreases (Gordon et al. 1975; Kirk 1981*a*), indicating that $C(\mu_o)$ is a function of μ_o. The manner in which $C(\mu_o)$ varies with μ_o has been investigated.

For values of $b{:}a$ ranging from 1 to 12 a close linear relationship ($r > 0.999$) between $R(0)$ and $b_b{:}a$ was found for all seven angles of incidence ($\theta' = 0°$ to $89°$) studied. The data for three angles of incidence are plotted in Fig. 3: at higher zenith angles some departure from linearity is evident at $b{:}a$ values > 12. The value of $C(\mu_o)$ at each angle of incidence was determined by linear regression for $b{:}a = 1$ to 12 and the values are plotted against μ_o in Fig. 4. It can be seen that $C(\mu_o)$ increases continuously as zenith angle of incidence increases (solar altitude decreases), rising from 0.35 for vertically incident light to 0.57 for light at grazing incidence.

The dependence of $C(\mu_o)$ on μ_o is approximately linear and linear regression analysis of the data yields the relationship

$$C(\mu_o) = -0.629\mu_o + 0.975 \qquad (15)$$

from which, with Eq. 14, we obtain

$$R(0) = (0.975 - 0.629\mu_o)b_b/a. \qquad (16)$$

Dependence of K_d *and* R*(0) on* a *and* b *under overcast conditions*—For light coming from a Standard Overcast Sky (equivalent to a cardioidal radiance distribution), after allowing for angle-dependent reflection at the (presumed flat) surface, the average value of μ_o for the incident photons just below the surface is 0.856. Under these conditions the relationship between $K_d(z_m)$, a, and b is satisfactorily described by Eq. 9 and 10, with $\mu_o = 0.856$ and $G(\mu_o) = 0.168$. In the corresponding equation for K_d(av), $G(\mu_o) = 0.162$.

An average downward cosine of 0.856 for the incident photons just after penetrating the surface would also be obtained with a direct solar beam at a solar zenith angle of 43.6°. From Eq. 11a this gives rise to a $G(\mu_o)$ value of 0.187, significantly, if not grossly, different from the value of 0.168 for a Standard Overcast Sky.

Under overcast conditions, as for parallel solar beams, for values of $b{:}a$ ranging from 1 to 12 there is a close linear relationship ($r > 0.999$) between $R(0)$ and $b_b{:}a$, in accordance with Eq. 14, $C(\mu_o)$ having the value of 0.427. From Eq. 15 it can be calculated that for direct sunlight at a solar zenith angle of 43.6°, $C(\mu_o) = 0.437$.

Discussion

This study has shown that the relationships between the inherent and the apparent optical properties of natural waters are significantly dependent on the angle of the light flux incident on the water surface and has made it possible to give this dependence quantitative expression. The data in Fig. 1 show that the extent to which the vertical attenuation coefficient, $K_d(z_m)$, for downward irradiance at the midpoint of the euphotic zone is increased by a given amount of scattering (at constant absorption) diminishes the more the direction of the incident flux departs from the vertical. Scattering intensifies vertical attenuation mainly by making the photons travel more obliquely and less vertically (decreasing their average downward cosine, $\bar{\mu}_d$), thus increasing their average pathlength per meter of depth, and so in turn increasing the probability of their being absorbed or scattered upward per meter of depth. As the ratio of scattering to absorption increases to very high values, the angular structure of the light field approaches the limiting situation of an isotropic radiance distribution in which the downwelling photons have an average downward cosine equal to 0.5. It is therefore not surprising that the nearer the value of μ_o for the incident photons is to this value, the less scope there is for increasing $K_d(z_m)$ by scattering.

It has been suggested that the vertical attenuation coefficient for downward irradiance, K_d, can usefully be treated as a "quasi-inherent" property of the aquatic medium on the grounds that its value is relatively insensitive to solar angle (Baker and Smith 1979). By means of the relationships established in this paper the range of usefulness of this proposition can readily be explored. Using Eq. 13 I found, for example, that when the solar zenith angle increases from 0° to 45° and 89°, the value of $K_d(z_m)/a$ increases by 15% and 41% when $b{:}a = 1$, by 8% and 22% when $b{:}a = 5$, and by 5% and 12% when $b{:}a = 10$. Thus K_d is indeed rather insensitive to solar altitude in highly scattering waters, but in clear oceanic waters, with low values of $b{:}a$, the value of K_d is likely to vary significantly with the angle of the sun.

For all natural waters the shape of the volume scattering function is such that there is much more scattering in a forward than in a backward direction. As the incident beam moves away from the vertical, so an increasing proportion of the more intense forward scattering becomes upward rather than downward scattering, and this is why irradiance reflectance increases with decreasing solar altitude (increasing zenith angle) in the manner shown in Figs. 3 and 4 and Eq. 16.

Although it is now clear that the angle of the incident photons on the surface materially influences the relationship between the apparent and inherent properties of natural waters, the dependence on angle of incidence fortunately takes a comparatively simple form, and the expressions given here now make it possible to quantitatively relate the apparent to the inherent optical properties for any solar altitude. These expressions could also be used to explore the relationships between inherent and apparent optical properties for any specific nonparallel incident light field. The incident light can be apportioned into a set of angular intervals and the equations applied to each angular flux in turn.

References

BAKER, K. S., AND R. C. SMITH. 1979. Quasi-inherent characteristics of the diffuse attenuation coefficient for irradiance. Proc. Soc. Photo. Instr. Eng. **208**: 60–63.

BEARDSLEY, G. F., AND J. R. ZANEVELD. 1969. Theoretical dependence of the near-asymptotic apparent optical properties on the inherent optical properties of sea water. J. Opt. Soc. Am. **59:** 373–377.

BUKATA, R. P., J. H. JEROME, J. E. BRUTON, AND S. C. JAIN. 1979. Determination of inherent optical properties of Lake Ontario coastal waters. Appl. Opt. **18:** 3926–3932.

GORDON, H. R., O. B. BROWN, AND M. M. JACOBS. 1975. Computed relationships between the inherent and apparent optical properties of a flat, homogeneous ocean. Appl. Opt. **14:** 417–427.

JERLOV, N. G. 1976. Marine optics. Elsevier.

KIRK, J. T. 1981*a*. Monte Carlo study of the nature of the underwater light field in, and the relationships between optical properties of, turbid yellow waters. Aust. J. Mar. Freshwater Res. **32:** 517–532.

———. 1981*b*. Estimation of the scattering coefficient of natural waters using underwater irradiance measurements. Aust. J. Mar. Freshwater Res. **32:** 533–539.

———. 1981*c*. Monte Carlo procedure for simulating the penetration of light into natural waters. CSIRO Div. Plant Ind. Tech. Pap. 36. 16 p.

MOREL, A., AND L. PRIEUR. 1977. Analysis of variations in ocean color. Limnol. Oceanogr. **22:** 709–722.

PETZOLD, T. J. 1972. Volume scattering functions for selected ocean waters. Scripps Inst. Oceanogr. Publ. 72-78. 79 p.

PLASS, G. N., AND G. W. KATTAWAR. 1969. Radiative transfer in an atmosphere-ocean system. Appl. Opt. **8:** 455–460.

PREISENDORFER, R. W. 1961. Application of radiative transfer theory to light measurements in the sea. IUGG Monogr. 10, p. 11–29.

———. 1976. Hydrologic optics, v. 5. NOAA.

PRIEUR, L. 1976. Transfer radiatif dans les eaux de mer. Application à la détermination de paramètres optiques caractérisant leur teneur en substances dissoutes et leur contenu en particules. D.S. thesis, Univ. Pierre et Marie Curie.

Submitted: 7 April 1983
Accepted: 2 October 1983