HYDROLIGHT TECHNICAL NOTE 1

HOW WELL DOES HYDROLIGHT SIMULATE WIND-BLOWN SEA SURFACES?

I occasionally get comments from people saying that Hydrolight cannot be doing a good job of simulating wind-blown sea surfaces because it uses the Cox-Munk wave slope statistics, rather than a full gravity-capillary wave spectrum, to generate realizations of a random sea surface. Hydrolight does indeed use the Cox-Munk slope statistics, but *these statistics include both gravity and capillary wave slope effects*, just as does a full gravity-capillary wave spectrum. This note briefly explains these matters to make clear exactly what sea surface features Hydrolight simulates well and what it does not simulate well.

Cox and Munk (1954a, 1954b) used photographs of the sea surface made from an airplane to deduce the slope statistics for a given wind speed. *The slope statistics they derived thus include the effects of whatever gravity and capillary waves were on the sea surface when the photographs were made.* Duntley (1954) measured the slope statistics using elevations determined from two closely spaced vertical wires suspended from a pier and reached conclusions consistent with those of Cox and Munk.

The confusion arises because over the years some people came to think of the Cox-Munk statistics as referring only to capillary waves. I was completely guilty of this myself in *Light and Water* §4.3 (youthful ignorance is my only excuse). In some situations this viewpoint is not terribly wrong, though, because much of the variance in the slope of the sea surface is indeed due to the smallest capillary waves. The larger gravity waves are responsible for most of the variance of the sea surface elevation, but they contribute less to the variance of the surface slope.

Hydrolight uses Monte Carlo ray tracing of millions of rays (or photon packets) through tens of thousands of randomly generated sea surface realizations to compute the radiance reflectance and transmittance functions that describe the optical effects of the sea surface (see *L&W* §4.7). When doing Monte Carlo simulations of rays interacting with the sea surface, it is the slope of the surface at the point where a ray intersects it that determines the directions of the reflected and transmitted rays, and the associated Fresnel reflectance and transmittance. Thus if the slope statistics are correct, which they are in Hydrolight, the optical effects of the surface will be accurately modeled in most situations. The exception may occur if the sun or viewing direction is near the horizon. Then effects such as wave shadowing by large gravity waves may become important (see *L&W* Fig. 4.32). Modeling wave shadowing by gravity waves requires that the sea surface elevation statistics also be correct and that the larger gravity waves be properly modeled, which is not the case in Hydrolight (or in any other optical oceanography radiative transfer model of which I'm aware). Note, however, that Hydrolight does include multiple scattering between wave facets.

Elfouhaily et al. (1997; called ECKV below) give a comprehensive directional wave spectrum valid for the full range of gravity and capillary waves. In their notation, the mean square slope (mss) of the sea surface is given by (ECKV Eq. A6)

$$mss = \int_0^\infty S(k)\, k^2\, dk\,, \qquad (1)$$

where $S(k)$ is the omnidirectional (1D) elevation spectrum and k is the wave number. $S(k)k^2$ is then the omnidirectional slope spectrum. The Cox-Munk mss is given by (ECKV Eq. 26)

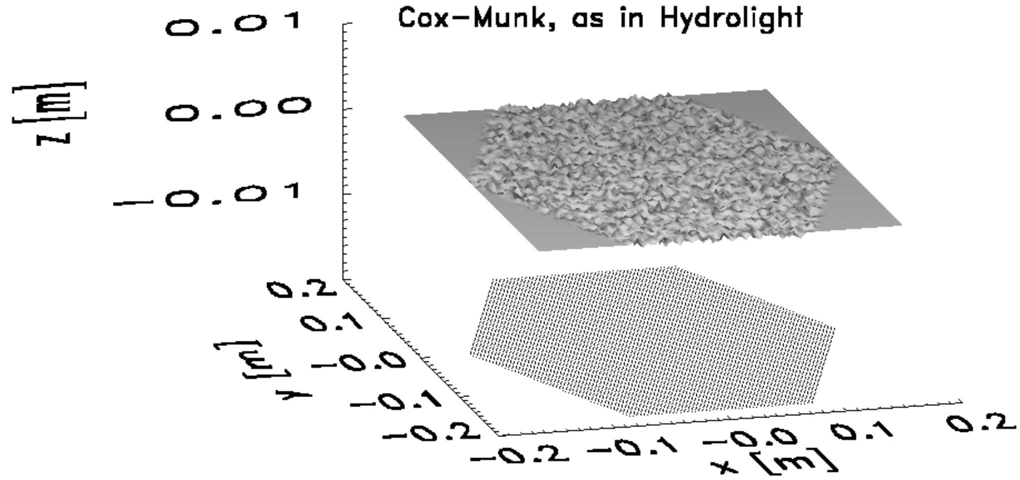
$$mss = 10^{-3}(3 + 5.12\, U_{10}) \pm 0.004\,, \qquad (2)$$

where U_{10} is the wind speed in meters per second at 10 m elevation. Equation (1) can be numerically integrated for a given wind speed, using the elevation spectrum given in ECKV. When this is done for $U_{10} = 6$ m s^{-1}, for example, Eq. (1) gives $mss = 0.036$. The corresponding Cox-Munk value obtained from Eq. (2) is $mss = 0.034 \pm 0.004$. This good agreement between the Cox-Munk *mss* and *mss* computed from the full gravity-capillary wave spectrum shows that Cox-Munk (hence Hydrolight) does indeed properly describe the slope statistics of a fully developed wind-blown surface.

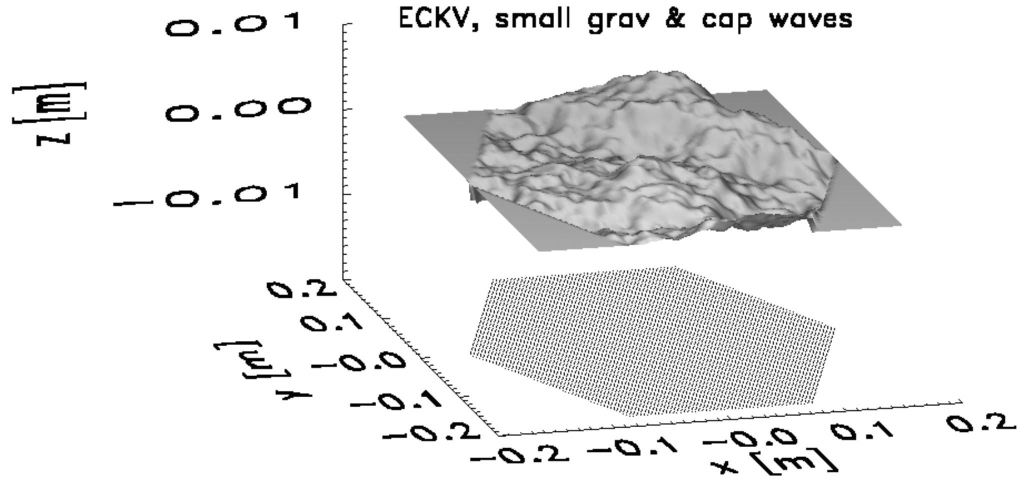
However, there are differences in how Hydrolight models the sea surface compared to a surface generated by resolving all wavelengths of the ECKV spectrum. Hydrolight uses the Cox-Munk wave slope statistics to model a hexagonal patch of sea surface as a number of triangular wave facets, as described in *L&W* §4.3. The vertex elevations of these triangular facets are spatially uncorrelated, because the Cox-Munk equations do not contain spatial correlation information. Figure 1(a) shows a patch of sea surface approximately 0.35 m on a side, covered with numerous small wave facets as generated by Hydrolight for $U_{10} = 6$ m s^{-1}. The resulting sea surface has a crinkly and somewhat unphysical appearance, even though the slopes of the triangular facets are statistically correct. [In Fig. 1, there are $n_{hex} = 50$ "rings" of triangular wave facets around the center of the hexagonal patch of sea surface, compared to $n_{hex} = 2$ in *L&W* Fig 4.2. The 7,651 points where the surface elevation is computed are shown by the dots making up the hexagon below the surface elevation plot.]

The ECKV directional (2D) spectrum can be used to generate sea surface realizations that do have spatial correlation from one triangle vertex to the next, as described in *L&W* §4.9 (after replacing the outdated SWOP spectrum with ECKV). Figure 1(b) shows an example realization of the resulting surface for the same hexagonal patch of water as seen in Fig. 1(a). The surface now resolves the smaller gravity waves and capillary waves. The surface elevation is spatially correlated from each point to the neighboring points, and the surface appears more realistic than that of Fig. 1(a). If we back off from the surface, we see the effects of the largest gravity waves. Figure 1(c) shows a patch of sea surface approximately 70 m on a side. This figure resolves only the largest gravity waves of the ECKV directional spectrum. Figure 1(b) can be thought of as a small patch of sea surface riding on the Fig. 1(c) surface at the middle of the large hexagonal surface patch.

(a)



(b)



(c)

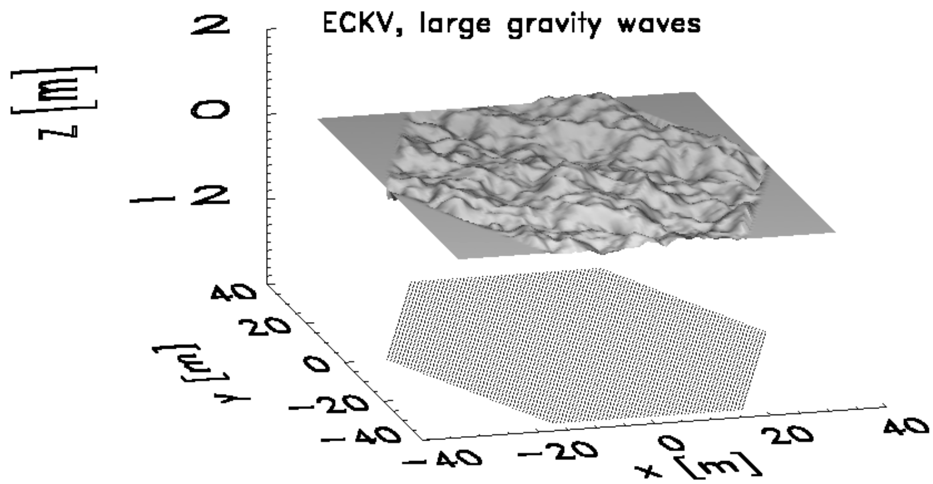


Fig. 1. Realizations of a random sea surface for U_{10} = 6 m s^{-1} and n_{hex} = 50. *x* is the alongwind direction, *y* is crosswind, and *z* is surface elevation.

Computing the variance of the alongwind slope, mss_x, for the row of surface elevation points at $y = 0$ in Fig. 1(a) gives $mss_x = 0.01866$. This compares well with the theoretical value of $mss_x = 0.01896$ (*L&W* Eq. 4.32). This (and many other similar checks) shows that Hydrolight is simulating sea surfaces consistent with the Cox-Munk slope statistics.

The classic Cox-Munk slope statistics as expressed by Eq. (2) agree with the sophisticated ECKV spectrum predictions of Eq. (1), which supports the use of the Cox-Munk equations without modification. Shifrin (2001), however, has synthesized recent work on slope statistics and concluded that the slope variance given by Cox and Munk for a given wind speed U_{10} is too small by a factor of 1.45 for a neutrally stabile marine atmospheric boundary layer. Shifrin's conclusions imply (his Eq. 21) that U_{10} in Eq. (2) should be replaced by $1.45U_{10} + 0.27$. To make a Hydrolight run consistent with Shifrin's recommendation, a classic 6 m s^{-1} wind speed, for example, should be entered into Hydrolight as a 9 m s^{-1} wind speed ($= 1.45 \times 6 + 0.27$). Shifrin also shows how to incorporate atmospheric stability effects on sea surface roughness into the Cox-Munk equations.

The way Hydrolight models wind-blown sea surfaces can be summarized as follows:

- Hydrolight properly simulates the slope statistics of a fully developed gravity-capillary sea surface for a given wind speed. It therefore properly simulates radiative transfer across wind-blown surfaces for solar zenith angles and viewing directions that are not very near the horizon.

- Hydrolight does include multiple scattering between the triangular wave facets used to generate surface realizations.

- Hydrolight does not simulate the elevation statistics of a wind-blown surface. Therefore, it does not account for effects such as wave shadowing of the "back" sides of large gravity waves when the sun is near the horizon. Such effects may be important in some situations (e.g., near sun rise or sun set), but quantitative studies have not been made.

- Hydrolight does not simulate the spatial correlation statistics of wind-blown sea surfaces, so any effects that depend on spatial correlations (e.g., of correlated capillary wave trains riding on the faces of larger gravity waves) are not accounted for. Such effects are unlikely to be important at visible wavelengths.

- Hydrolight does not include effects of atmospheric stability on surface roughness, but such effects can be incorporated into a Hydrolight run by entering a modified wind speed.

- Hydrolight does not include effects of foam generated by whitecaps at high wind speeds.

One question remains: why don't I just start using surfaces generated by the ECKV directional spectrum and properly model all surface statistics. The answer is simply insufficient computer power. Generating Fig. 1(b) or 1(c) takes ~2,500 times longer than generating Fig. 1(a). Moreover, one should in principle resolve all relevant wave scales from the largest gravity waves [as in Fig 1(c)] to the smallest capillary waves [as in Fig. 1(b)]; that would increase the computer time by another enormous factor. Given that I need to generate tens of thousands of surfaces to build up good statistical estimates of the surface radiative transfer properties, such computations are, for the moment, beyond reach (although they would be great fun to do!).

Sincerely,
Curt Mobley
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References

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Elfouhaily, T., B. Chapron, K. Katsaros, and D. Vandemark, 1997. A unified directional spectrum for long and short wind-driven waves, *J. Geophys. Res* **102** (C7), 15781-15796. Note that their Eq. (41) is missing a factor of $L_{PM}J_p$ before the exponential; see their Eq. (32) for comparison.

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