HYDROLIGHT TECHNICAL NOTE 6

RUNNING HYDROLIGHT WITH INTERNAL SOURCES OR INELASTIC SCATTERING AND INFINITELY DEEP WATER

There is a “well known” (well known, that is, to those who have looked closely at their output) problem with HydroLight version 4 when run with inelastic scatter or internal sources and an infinitely deep bottom. These notes explain the problem, how to work around it in HydroLight version 4, and how the fix will be automated in HydroLight-EcoLight version 5.

When a run includes internal sources or inelastic scatter, radiances and other quantities can appear wrong (and sometimes actually are wrong) near the greatest depth requested in the output, which is the depth where the bottom boundary condition is applied in order to solve the radiative transfer equation. This situation can occur in any run that uses an infinitely deep bottom and includes fluorescence, Raman scatter, or bioluminescence.

Let us suppose that the greatest depth you’re interested in is z_{max}, and let z_b be the depth at which the bottom boundary is applied. In most runs, $z_{max} = z_b$ = the last depth at which you requested output when submitting the HydroLight run. The origin of the problem is lies in the bottom boundary condition that describes the infinitely deep water column below the greatest depth of the requested output, z_b. Suppose you do a run with no inelastic scatter or internal sources (i.e., no fluorescence, Raman scatter, or bioluminescence); I’ll just call this a source-free run. You tell your run to go to z_b = 25 m depth, say, with an infinitely deep water column below 25 m. HydroLight then computes (using *Light and Water* Eq. 9.76) the radiance reflectance (essentially the BRDF) of the infinitely deep layer of **source-free** water below 25 m. That BRDF is then placed at z_b = 25 m and serves as the bottom boundary condition for solving the RTE. The BRDF fully describes the effect of the infinitely deep layer of water below 25 m on the upwelling radiance at 25 m, and the computed radiances from the sea surface down to 25 m are correct. Life is good.

Now suppose you resubmit this run, but include inelastic scatter or bioluminescence; I’ll call this a run with an internal source. **The computed bottom boundary BRDF applied at z_b = 25 m is the exactly same as for the source-free run.** You then have a water column with an internal source from the surface down to 25 m, and no internal source below 25 m. That’s a perfectly fine radiative transfer problem, but it’s probably not the one you wanted so solve, which presumably should include a source all the way down to infinity.

HydroLight could apply a proper lower boundary condition for the "bottom" at z_b, which would correctly describe the effects of both the source function and elastic scatter processes in the infinitely deep layer below the "bottom" at z_b, but **only** if the contribution of the source in the infinitely deep layer to the upwelling radiance at z_b is known *a priori*. This is analogous to how the boundary condition at the sea surface is applied at depth 0. You give HydroLight the downwelling sky radiance, which HydroLight then uses to construct the boundary condition at the sea surface. The needed incident sky radiance is obtained either from ancillary approximate sky radiance models built into HydroLight, from the user's measurements, or from solving the RTE for the atmosphere to obtain the incident sky radiance. In any case, specifying the sky radiance is not a part of solving the RTE within the water body itself, which is what HydroLight does.

In the same fashion, to input the effects of the source in the infinitely deep water below your last output depth z_b, you need to give HydroLight the upwelling radiance at z_b or, equivalently, the BRDF that describes the effects of both the water column elastic scattering and internal sources below the depth z_b where the BRDF is applied.

So how can you compute the needed BRDF that includes source effects? Just as for the incident sky radiance, you have to model it, measure it, or solve the RTE from z_b down to infinity to compute the source contribution at z_b. But, of course, you're trying to avoid solving the RTE down to infinity by applying the BRDF at your requested maximum depth of interest. In the case of source-free, homogeneous water, the solution of the RTE from z_b down to infinity depends only on the depth-independent IOPs, which are known in advance. The integration of the RTE therefore can be converted to a matrix eigenvalue problem and solved analytically (*Light and Water*, Section 9.5. Eq. 9.76 is essentially the BRDF of the layer of infinitely deep, homogeneous, source-free water). However, the internal source at a given wavelength generally depends on depth and all shorter wavelengths because the elastically scattered light at shorter wavelengths (which determine the source at a longer wavelength via inelastic scattering) depends on depth and wavelength in a manner that cannot be known in advance of solving the RTE. *I therefore can't figured out how to include internal sources into that matrix calculation.* Until I do, which may never happen, **the only practical way to get the correct solution at the depth of greatest interest z_{max} is to let HydroLight solve the RTE to some still greater depth z_b, such that the effects of the bottom BRDF for source-free water at z_b no longer affect the solution at z_{max}, where your interest in the water column stops.**

And how much deeper than z_{max} should you run HydroLight when sources are present, so that you still get a good result at z_{max}? How far that is in meters depends on the water column optical properties in the simulation being done and also on which radiometric variables or AOPs are of interest, as seen in the following example simulations.

Example Simulation 1

I did a few HydroLight 4.3 simulations to see how much effect the source-free bottom boundary has on the water-column radiances above the boundary. The first set of runs was done with the "abconst" IOP model. I set $a = 0.2$ and $b = 0.8$ m^{-1}, so that c = 1.0 m^{-1}. Thus the geometric depth in meters is numerically equal to the optical depth $\zeta = cz$. A Petzold average-particle phase function was used. The sun was placed at a 50 deg zenith angle in a clear sky with typical marine atmospheric conditions. The sea surface was level. The maximum depth of the requested output, $z_{\rm max}$, was 60 m (60 optical depths). This is the depth $z_{\rm b}$ at which the bottom boundary condition was applied. The water was infinitely deep, homogeneous, and source-free below depth 60. The RTE was then solved from depth 0 down to depth 60. Figure 1 shows the upwelling (nadir-viewing, L_u), downwelling (zenith-viewing, L_d) radiances, and the horizontal radiance looking toward (L_{h_180}), at right angles to (L_{h_90}), and away from (L_{h_0}) the sun's azimuthal direction.

The runs in Fig. 1 had either no internal source, or an isotropically emitting internal source $S_{\rm o}$ (modeled here as bioluminescence) that was constant with depth. The top pair of plots in Fig. 1 shows the radiances for no internal source, $S_{\rm o} = 0$. As best seen in the right panel, it takes roughly 20 optical depths for the surface boundary effects on all radiances to damp out. Below depth 20, the radiances then decay exponentially with depth, as best seen in the left panel.

The second row of plots in Fig. 1 shows the results when a weak, isotropically emitting internal source with strength $S_{\rm o} = 0.0001$ W m^{-2} nm^{-1} is included in the run. The radiance emitted by this source is negligible compared to the solar-generated radiances near the sea surface. However, at depths of 20 to 40, the internal source begins to dominate the radiance distribution, and below depth 40, the solar radiance is negligible compared to the source radiance. The radiance then becomes isotropic with magnitude $L = S_{\rm o}/(4\pi a) = 3.98{\rm x}10^{-5}$ W m^{-2} sr^{-1} nm^{-1}. (The asymptotic radiance in homogeneous water with an isotropically emitting source depends only on the source strength and the absorption coefficient a. This is one of the rare exact analytic solutions of the RTE.) However, below a depth of about 50, the radiances begin to depart from the constant value seen between depths 40 and 50. The radiance distribution is "smelling the bottom boundary" and responding to the discontinuity in the source function at depth 60. The effect is most noticeable on L_u, because there is no upwelling source radiance arriving from below depth 60.

The third row of plots in Fig. 1 shows the results for a strong internal source, $S_{\rm o} = 0.1$ W m^{-2} nm^{-1}. The radiances due to the internal source are now comparable in magnitude to the near-surface solar radiances. The last row of figures shows the results for a very strong source, $S_{\rm o} = 1.0$ W m^{-2} nm^{-1}, for which the source radiance is much larger than even the near-surface solar radiances. In all cases, we see that the radiance distribution is being affected by the bottom boundary condition for a distance of 10-20 optical depths from the bottom. This is comparable to depth below the sea surface for which the upper boundary condition affects the radiances.

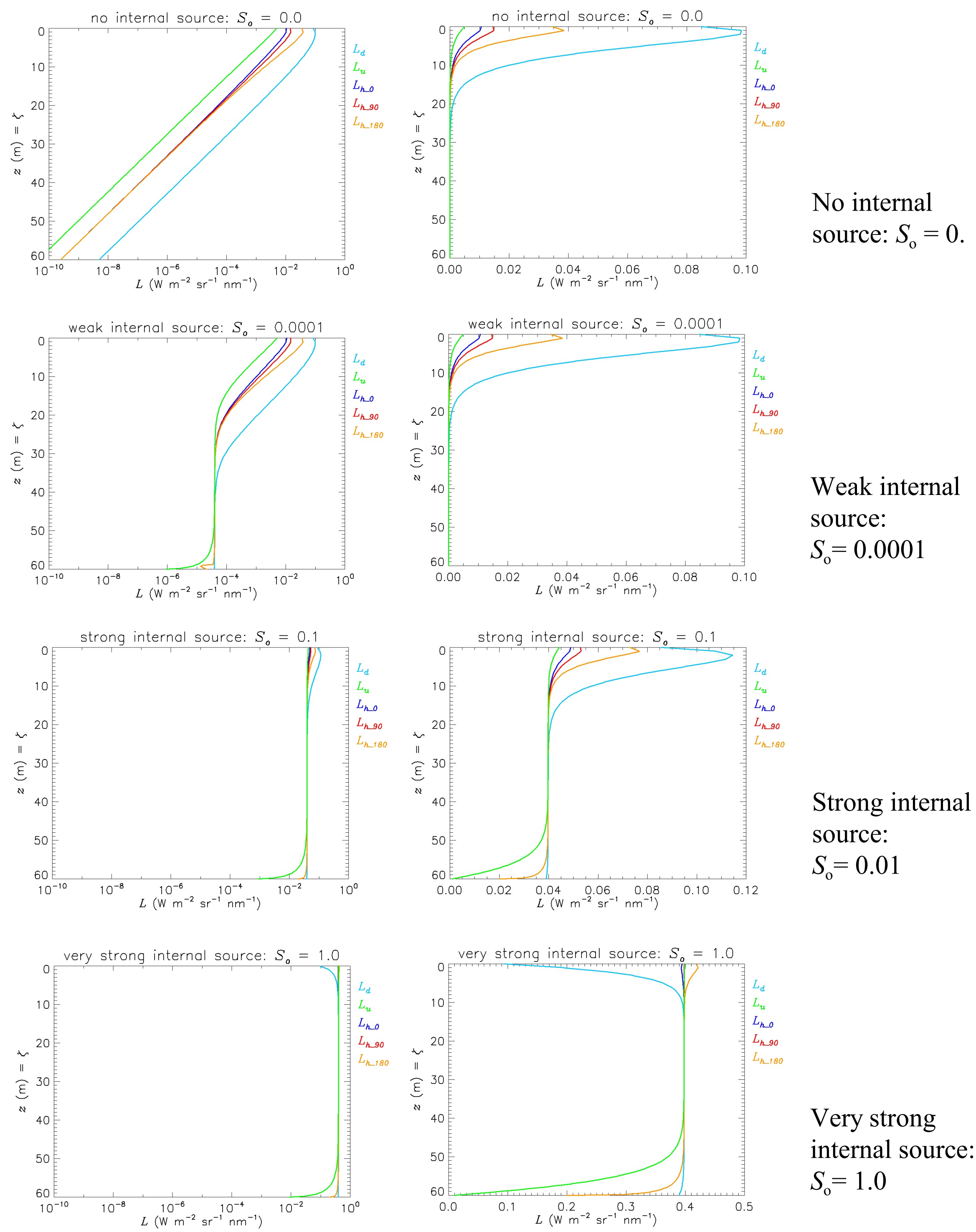

Fig. 1. Radiances for various strengths of internal sources S_o. The left panels have a logarithmic abscissa for the radiance, the right panels are the same data with a linear abscissa.

These simulations show that the surface and boundary conditions can affect the internal radiance distribution to a distance of 10-20 optical depths from the boundary, at least for this particular simulation. If we were interested in the radiances down to 40 optical depths, say, then running HydroLight down to 60 optical depths would give a solution at 40 that is only slightly affected by the incorrect boundary condition at depth 60.

Example Simulation 2

Let us now consider how the bottom boundary location effects a typical simulation of ocean waters. The Case 1 IOP model was used with a chlorophyll concentration of 1 mg m^{-3}. The particles had a Petzold average-particle phase function. The sun was placed at a 50 deg zenith angle in a clear sky with typical marine atmospheric conditions. The sea surface was level. The baseline run included no inelastic scatter, and the source runs included Raman scatter, chlorophyll fluorescence, and CDOM fluorescence. The runs were made from 350 to 750 nm with 10 nm resolution.

Figure 2 compares the upwelling (nadir-viewing) radiance L_u at $z_{\text{max}} = 25$ m depth when the bottom z_b was placed at 50 m in the HydroLight run, for the baseline run (elastic only) vs. the corresponding run with inelastic effects. We see that for wavelengths below about 600 nm, the contribution of fluorescence and Raman scatter to the total (elastic plus inelastic) upwelling radiance at 25 m is small. However, for red and near-IR wavelengths, most of the upwelling radiance comes from the solar radiance at shorter wavelengths, which can penetrate to 25 m, being inelastically scattered to longer wavelengths, for which the high absorption by water prevented solar radiation from reaching 25 m. It is clearly important to include inelastic scattering if accurate radiances are to be obtained at long wavelengths and deep depths.

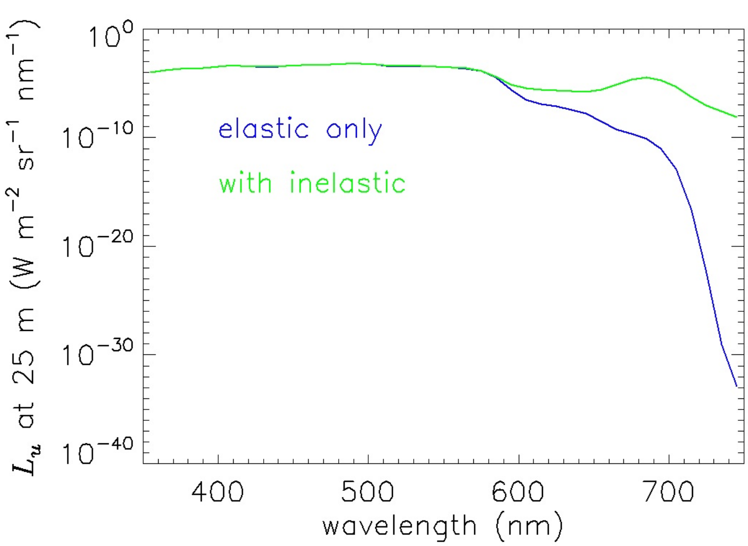

Fig. 2. Upwelling radiance at 25 m depth for elastic (blue) vs. inelastic (green) scattering.

Figure 3 compares the upwelling radiances L_u at z_{max} = 25 m when the bottom was placed at z_b = 25, 30, or 50 m. Each run included inelastic scatter. We see that there is almost no difference at blue and green wavelengths. This is because the inelastic contribution to L_u is small compared to the elastically scattered solar radiation at 25 m at these wavelengths, thus the source-free bottom boundary condition when applied at z_b = 25 m makes little difference in the radiance. However, at red and IR wavelengths, the difference between applying the bottom boundary condition at 25 vs. 30 or 50 m makes almost a two-order-of-magnitude difference in L_u at 25 m. For the red and IR wavelengths, there is only a small difference at 25 m for the bottom placed at 30 vs. 50 m, which is barely visible on this plot.

Figure 4 shows the depth profile of L_u at 485 nm, for the three different values of z_b. There is only a small difference (less than 2%) in the L_u values at 25 m for z_b = 25 vs 50 m. Figure 5 shows the L_u depth profile at 685 nm. Now the effect of the source-free bottom boundary condition is quite evident. Placing the bottom at 25 m is clearly not acceptable if it is important to obtain accurate upwelling radiances at red or IR wavelengths. However, it is seen that the bottom boundary effects extend through only about 5 m of water (about 4 optical depths at 685 nm for these IOPs) at this wavelength. Thus placing the bottom at 30 m, which increases the run time by only 20%, is almost as good as placing it at 50 m, which doubles the run time.

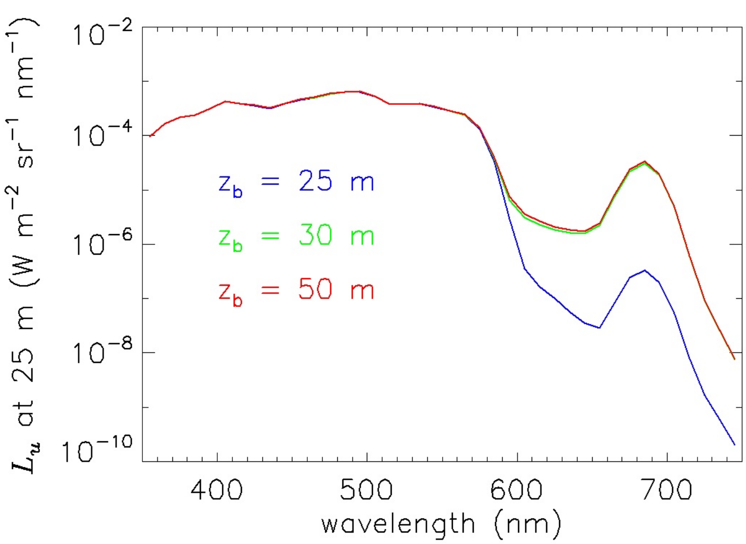

Fig. 3. Upwelling radiance at z_{max} = 25 m depth for inelastic scattering when the bottom boundary is placed at z_b = 25 (blue), 30 (green), or 50 (red) m.

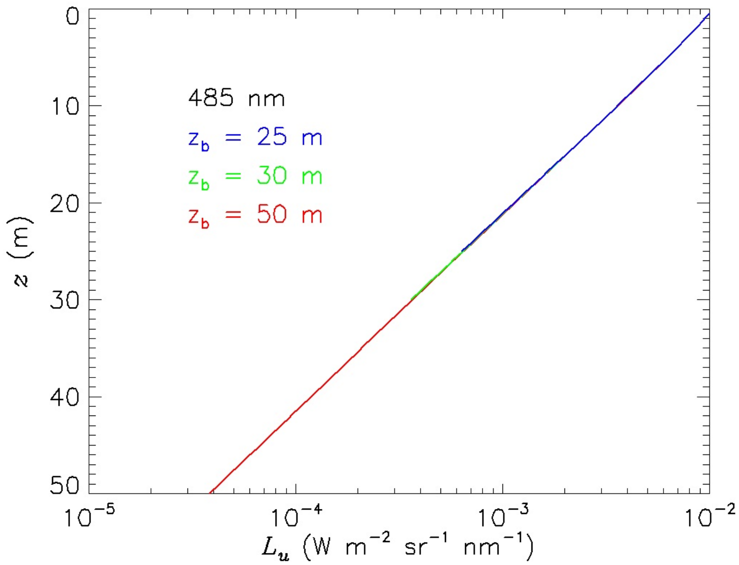

Fig. 4. $L_u(z)$at 485 nm for three different bottom depth in the HydroLight run. There is very little effect of the source-free bottom boundary condition.

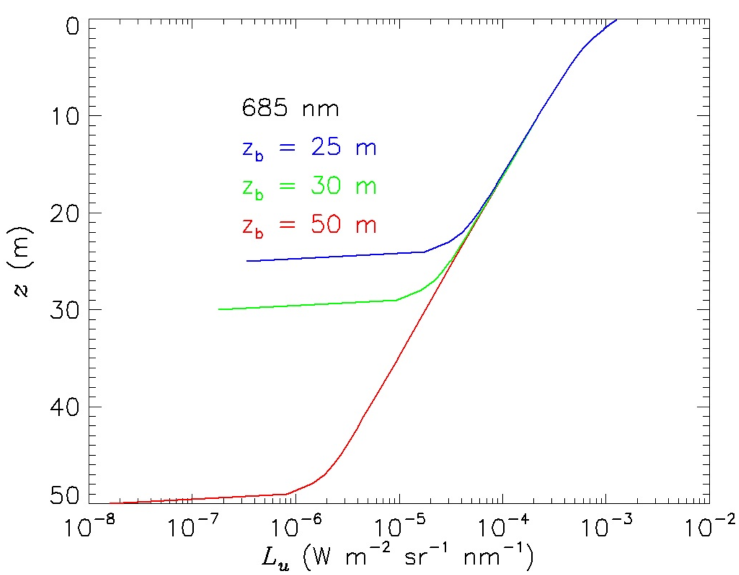

Fig. 5. $L_u(z)$at 685 nm for three different bottom depths in the HydroLight run. The source-free bottom boundary conditions is now quite important.

A final bit of understanding can be obtained by comparing the effect of the infinitely deep, source-free boundary condition with a Lambertian bottom boundary condition, as is used for finite-depth water. For simplicity, I used a black bottom (zero reflectance). The IOPs and surface conditions are the same as for Figs. 3-5. Figure 6 compares six runs. Four have the bottom at 25 m and two have the bottom at 35 m. The runs compare source-free vs. source (chlorophyll and CDOM fluorescence and Raman scatter), and infinite bottom vs. black bottom.

In Fig. 6a, which is for 485 nm, we see that the upwelling radiance L_u at 25 m is almost the same whether or not inelastic scatter is included, when the bottom is at 25 m. The reason is that the radiance distribution is dominated by elastically scattered solar radiation at 485 nm, which easily penetrates to 25 m for these water conditions. However, there is a large difference between the infinitely deep and black bottoms within 10-15 m of the bottom, when placed at 25 m. When the bottom is at 35 m (green curves), the black bottom gives L_u about 16% too small at 25 m, and the infinite bottom at 35 m gives L_u(25 m) that is almost identical to L_u(25 m) when the infinite bottom is placed at 25 m.

At 635 nm, Fig. 6b, both the elastically scattered solar radiation at 635 nm and inelastically scattered radiance from shorter wavelengths contribute to the upwelling radiance. We see that both the presence or absence of inelastic scattering and the type of bottom boundary condition affect L_u near the bottom at 25 m. However, placing the bottom at 35 m gives an accurate L_u at 25 m, regardless of whether the bottom at 35 m is black or infinitely deep.

At 685 nm, Fig. 6c, very little elastically scattered solar radiation penetrates to 25 m, and the radiance is determined primarily by inelastically scattered light from blue and green wavelengths. At this wavelength the type of bottom boundary makes very little difference for the source and no-source pairs of runs. However, the use of either bottom condition at 25 m gives an incorrect L_u near 25 m, compared to what is obtained by running HydroLight to a depth of 35 m. When the bottom is placed at 35 m, a good solution is obtained at 25 m regardless of whether the bottom is black or infinite.

In summary, for these run conditions, we see that we can get an accurate L_u at 25 m at all wavelengths, when inelastic scattering is included in the run, by placing an infinite bottom at 35 m. We then simply ignore the L_u values below our depth of greatest interest, 25 m. Note that the L_u values below 25 m are not actually “bad,” they are simply the solution to a different radiative transfer problem than the one we wanted to solve.

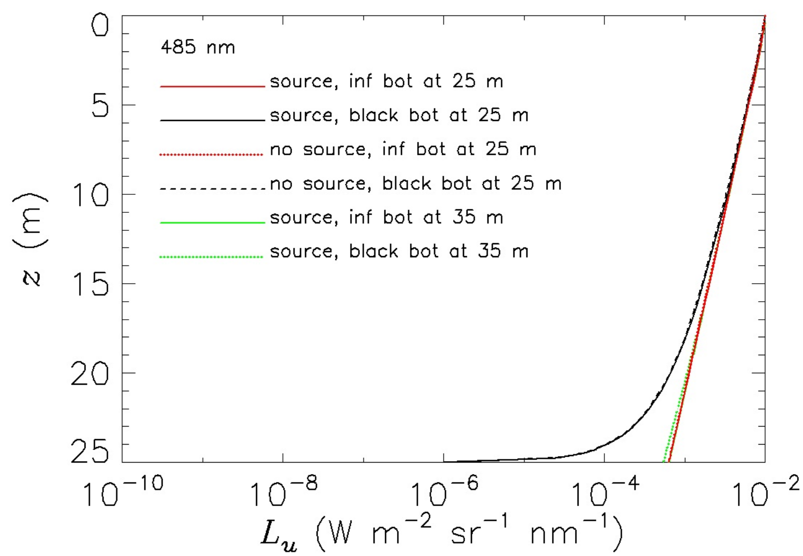

Fig. 6a. Source vs. no source, black bottom vs. infinite bottom, for 485 nm. L_u depends primarily on the bottom boundary condition and is almost independent of the inelastic scattering.

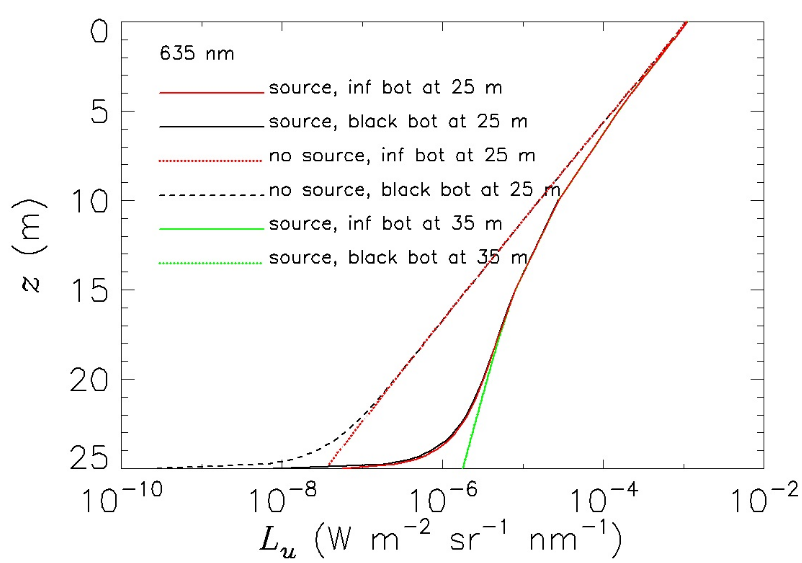

Fig. 6b. Source vs. no source, black bottom vs. infinite bottom, for 635 nm. L_u now depends both on the bottom boundary condition and on the inelastic scattering. The green curves are indistinguishable.

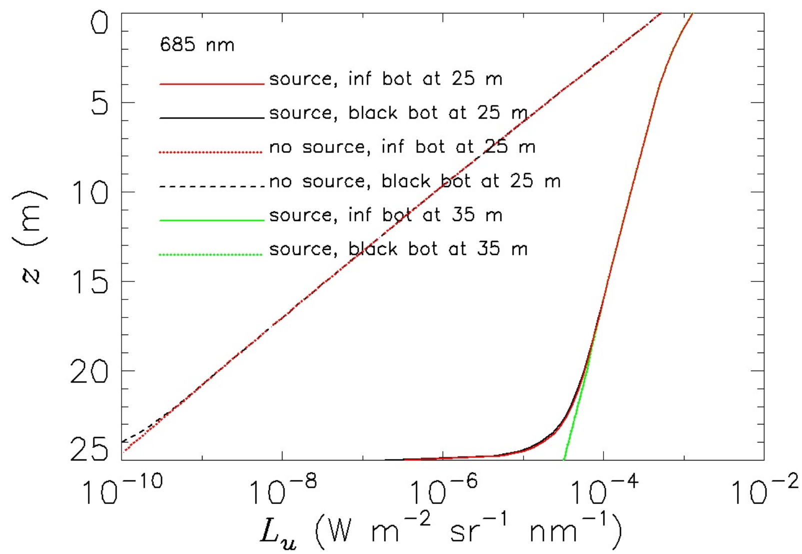

Fig. 6c. Source vs. no source, black bottom vs. infinite bottom, for 685 nm. L_u now depends strongly on the inelastic scattering and only very weakly on the bottom boundary condition, when applied at a given depth. The green curves are indistinguishable.

Dynamic Bottom Boundary Depths in HydroLight-EcoLight Version 5.0

Although the situation described above has not been a problem for most users, it causes occasional confusion as to why HydroLight output behaves the way it does near the bottom boundary. To avoid this confusion in the future, HydroLight-EcoLight version 5 (HE5) will implement a dynamic bottom boundary condition, which automatically computes the depth to which a run needs to solve the RTE, so that the radiances and other quantities are accurate at the last depth of interest to the user. This is done as follows.

Runs for a wide range of water conditions from very clear to very turbid, and for wavelengths from 300 to 1000 nm, show that applying the bottom boundary condition at 20 optical depths below the greatest depth of interest is sufficient to give good radiances at the greatest depth of interest. HE5 therefore does the following:

At the start of a run it computes the wavelength-dependent depth

$$z_b(\lambda) = z_{max}(\text{requested}) + 20/c(z_{max}, \lambda),$$

where $c(z_{max}, \lambda)$ is the beam-attenuation coefficient at the maximum output depth requested by the user. It then finds the maximum of $z_b(\lambda)$, $z_b(\lambda_{max})$, which occurs at some wavelength λ_{max}.

The infinitely deep, source-free bottom boundary condition is then applied at depth

$$z_b(\lambda) = z_b(\lambda_{max}) \text{ if } \lambda \le \lambda_{max}$$

$$z_b(\lambda) = z_{max}(\text{requested}) + 20/c(z_{max}, \lambda) \text{ if } \lambda > \lambda_{max}$$

These bottom depths guarantee that the RTE is solved as deeply as needed at all wavelengths $\lambda < \lambda_{max}$ in order to compute the inelastic scattering source terms needed at wavelength λ_{max}, where the water is clearest, and that it is solved no deeper than needed at longer wavelengths (especially beyond 700 nm) where water absorption is high.

Use of the dynamic bottom boundary depth is the default in HE5 whenever bioluminescence or inelastic scatter are included and the water column is infinitely deep. If no source terms are included or the bottom is placed at a finite depth, then the bottom boundary condition is applied at z_{max}(requested), just as in previous versions of HydroLight. If desired, the dynamic bottom boundary option can be turned off in the "Change Defaults" form of the HE5 User Interface. In order to avoid a new type of confusion, output at depths below the user-requested maximum depth z_{max}(requested) is not included in the various output files. Ignorance is bliss.

Limited experience (while developing HE5) shows that the dynamic bottom boundary condition typically adds about 20% to the run time, compared to the use of a fixed bottom depth. That is a reasonable computational price to pay in order to avoid spurious results in simulations where the effects of sources are of interest at great depths. The penalty for not going deep enough can be a "wrong" answer for your problem.

Curt Mobley **13 May 2008**