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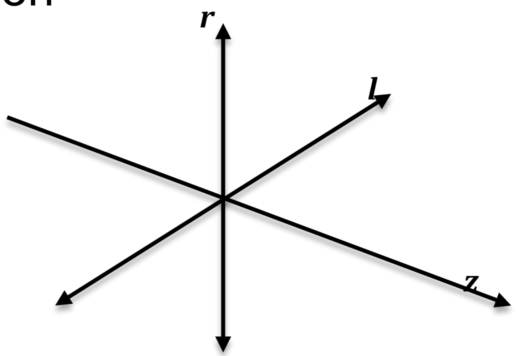
Polarization in Sky and water

At least an hours worth of it

Ken Voss, Ocean Optics Summer class, 2013

Need to describe Stokes vectors and Mueller matrices.

Can describe plane wave propagating in z direction as:



$$E_l(z,t) = a_l \cos(\omega t - kz + \delta_l)$$

$$E_r(z,t) = a_r \cos(\omega t - kz + \delta_r)$$

Let: $\delta = \delta_r - \delta_l$

Some simple cases can be seen:
$\delta=0$,(or a_l or $a_r=0$) light is linearly polarized
$\delta=\pi/2$, $a_l = a_r$ light is circularly polarized
Everything else is called elliptically polarized

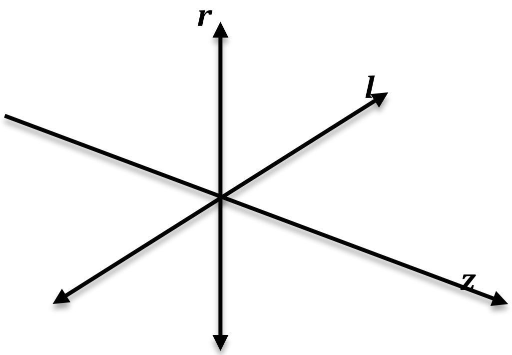
Now in general, ω, for visible light is on the order of 6 x 10^{14}Hz, we can't really measure these oscillations directly. Also the natural light field is not a single plane wave, but a superposition of many waves. So we need a more conv. description….the Stokes vector.

$$I = \langle a_l^2 \rangle + \langle a_r^2 \rangle = \langle E_l E_l^* \rangle + \langle E_r E_r^* \rangle = I_l + I_r$$

$$Q = \langle a_l^2 \rangle - \langle a_r^2 \rangle = \langle E_l E_l^* \rangle - \langle E_r E_r^* \rangle = I_l - I_r$$

$$U = \langle 2 a_l a_r \cos\delta \rangle = \langle E_l E_r^* \rangle + \langle E_r E_l^* \rangle = I_{45} - I_{-45}$$

$$V = \langle 2 a_l a_r \sin\delta \rangle = i\left(\langle E_l E_r^* \rangle - \langle E_r E_l^* \rangle \right) = I_{RCP} - I_{LCP}$$



Some Simple Stokes Vectors:

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Unpolarized light

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ I \\ 0 \\ 0 \end{bmatrix}$$

Polarized along *l*

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ -I \\ 0 \\ 0 \end{bmatrix}$$

Polarized along *r*

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ 0 \\ I \end{bmatrix}$$

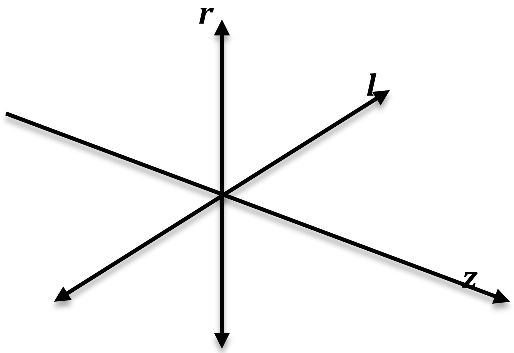
Right Circularly
polarized light

$$I = I_l + I_r$$

$$Q = I_l - I_r$$

$$U = I_{45} - I_{-45}$$

$$V = I_{RCP} - I_{LCP}$$



Can transform these Stokes vectors in a linear process, using Mueller Matrices:

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = \begin{bmatrix} M11 & M12 & M13 & M14 \\ M21 & M22 & M23 & M24 \\ M31 & M32 & M33 & M34 \\ M41 & M42 & M43 & M44 \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

Modify incoming Stokes Vector to outgoing Stokes Vector, example case linear polarizer oriented along *I* axis:

$$1/2 \begin{bmatrix} I+Q \\ I+Q \\ 0 \\ 0 \end{bmatrix} = 1/2 \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

To determine the Stokes vectors, need at least 4 carefully determined measurements. To determine Mueller matrix, need 16 measurements:

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = \begin{bmatrix} M11 & M12 & M13 & M14 \\ M21 & M22 & M23 & M24 \\ M31 & M32 & M33 & M34 \\ M41 & M42 & M43 & M44 \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

Typically, normalize by M11 (which is normal light scattering volume scattering function).

Instruments to measure the Stokes Vector are variations on radiance instruments.

Two types, either single direction at a time (such as 4 co-bore sighted Gershun tubes or 3, since the circular polarization is very small or zero for the most part) or some sort of imaging device, such as fisheye camera with polarizers.

Example Sky instrument

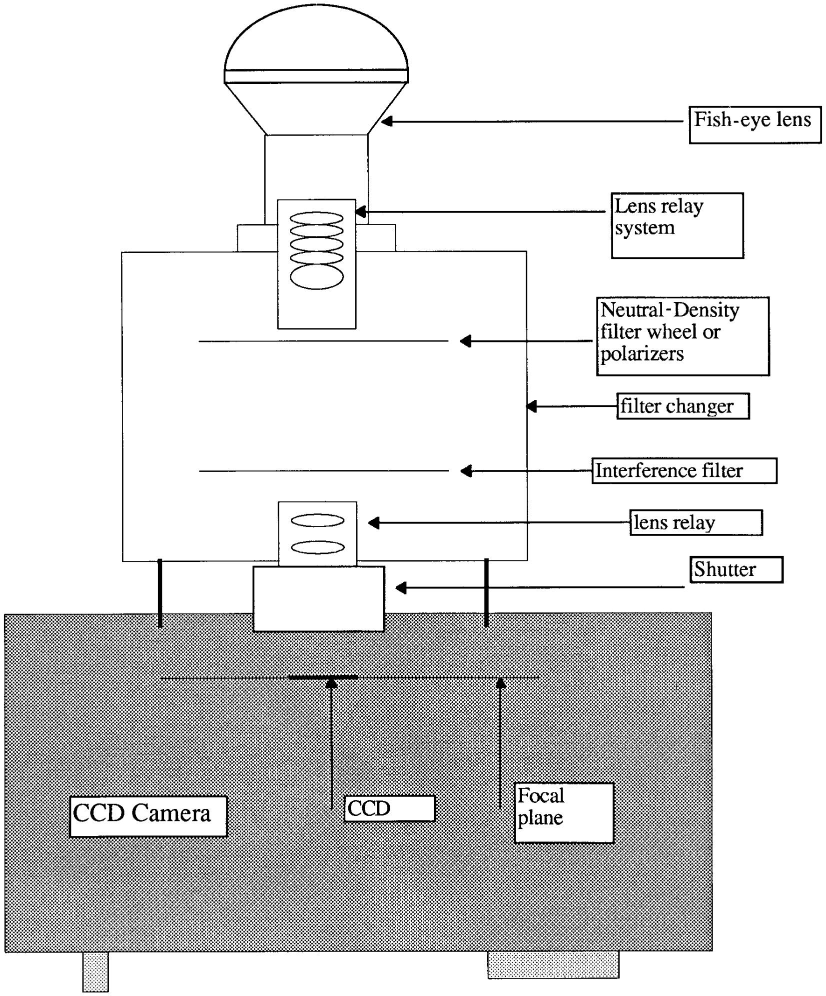
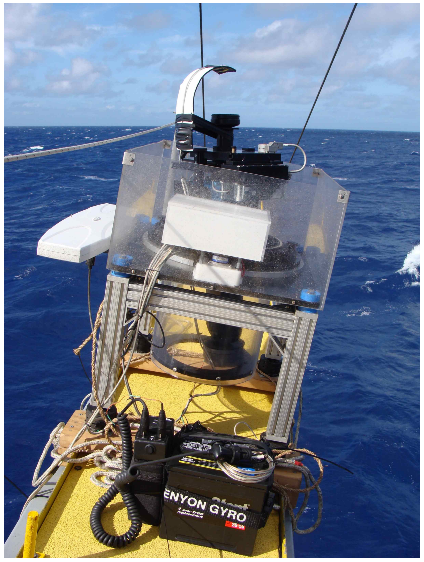
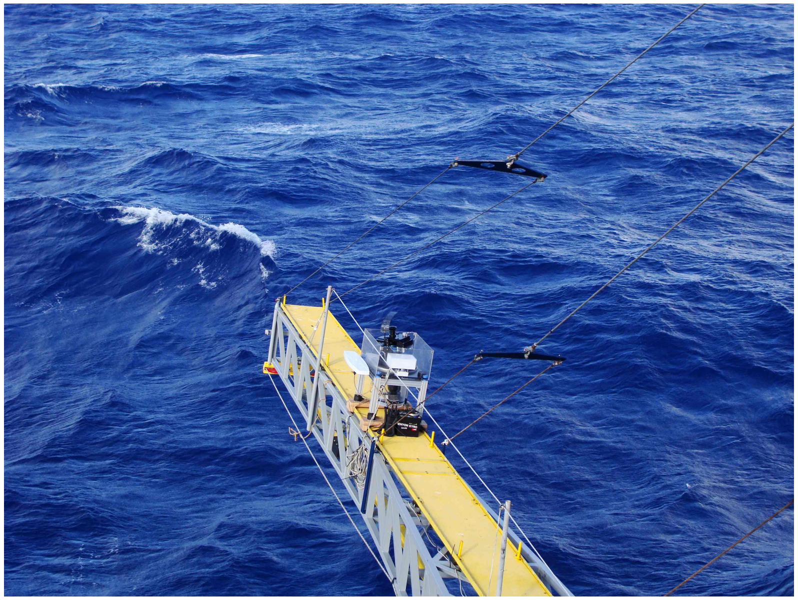


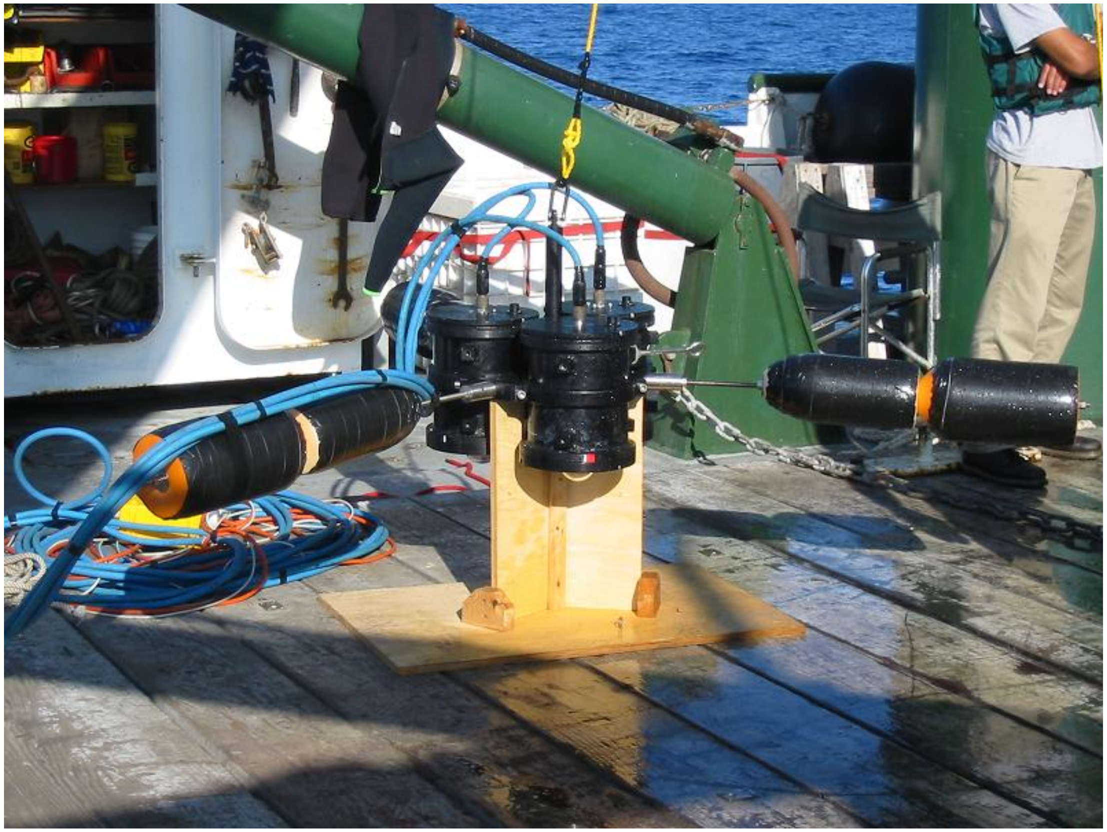
Fig. 1. Block diagram of the RADS-IIP instrument.

- Sky scene relatively stable, hence time sequence can be used
  - Design follows our earlier work (NASA supported): K. J. Voss and Y. Liu, “Polarized radiance distribution measurements of skylight: I. system description and characterization”, 1997, Applied Optics, **36** : 6083-6094.
- Must have stabilization to use on ships/moving platforms: Adapt system used by filming crews



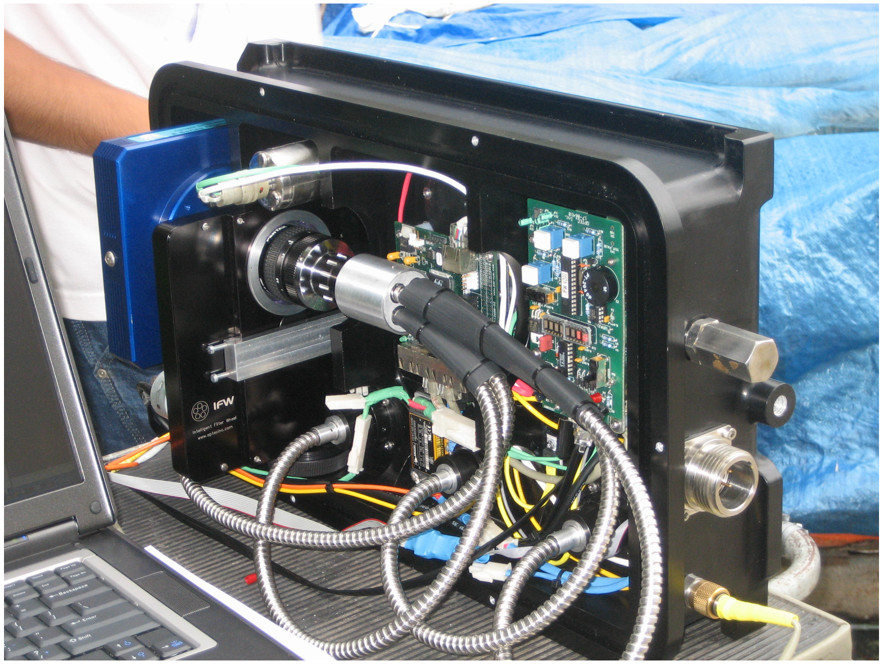


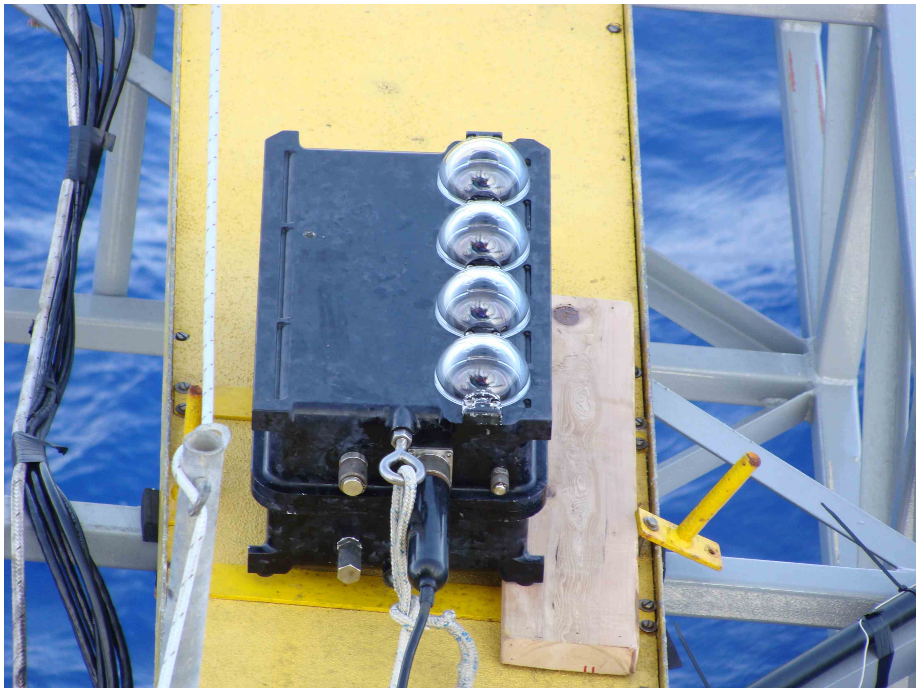
In ocean things light field changes much faster so sequential images won’t work. Built PolRADS for upwelling light field:



For downwelling light built the DPOL instrument:

- New System has 4 lenses, polarizer's in each
- Fiber Optic bundle collects light from each image into one super image
- Super image focused onto camera through spectral filter changer

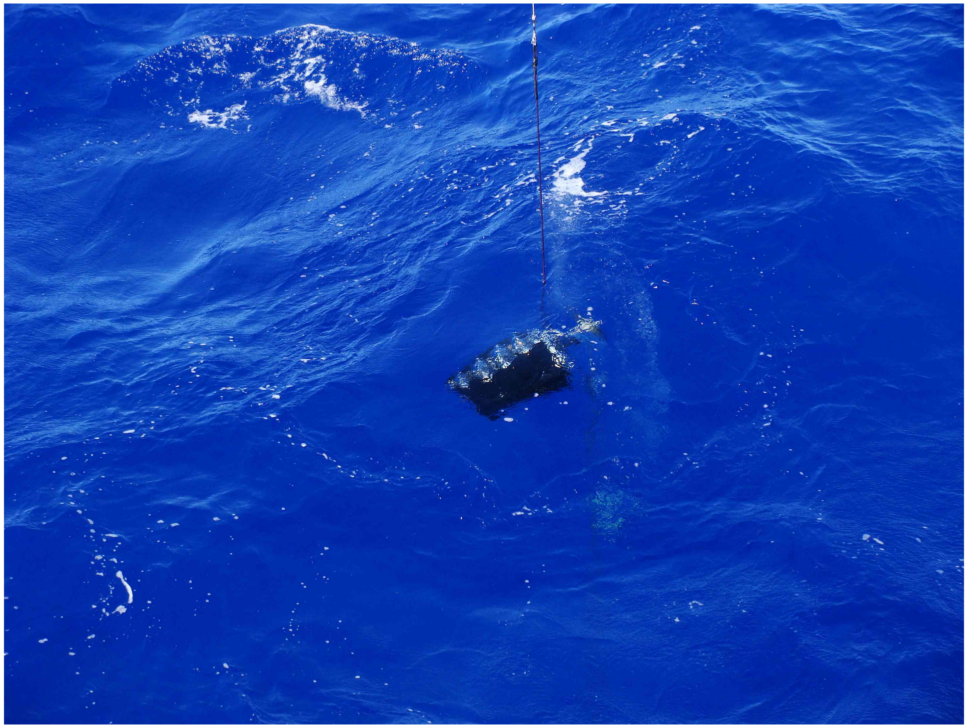


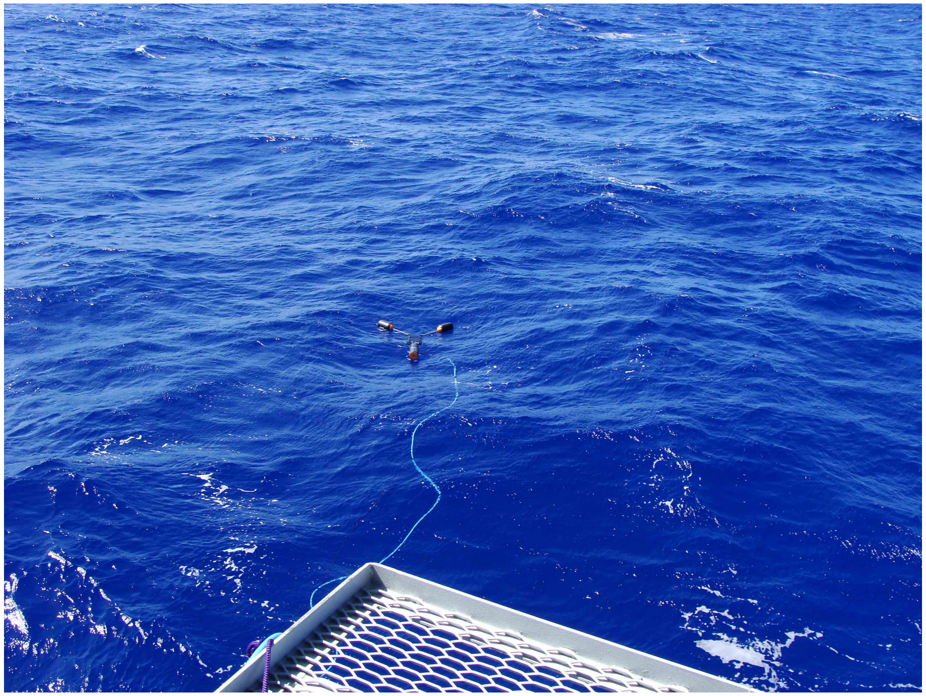


Deploying instrument can be done in one of 3 ways

Traditional, from a wire for downwelling

From floats for upwelling





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3rd option is from a small ROV to avoid ship shadow.

To measure the Mueller matrix need 16 measurements (set of 4 different input Stokes vectors permutated with 4 polarization analyzers). Either do this serially or another method was:

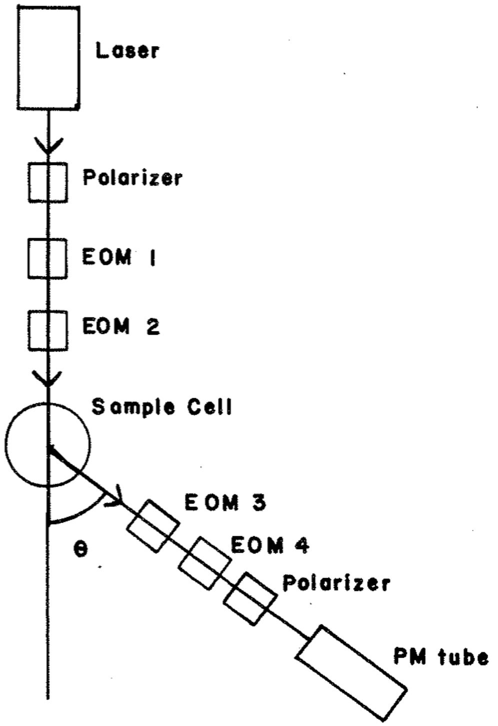
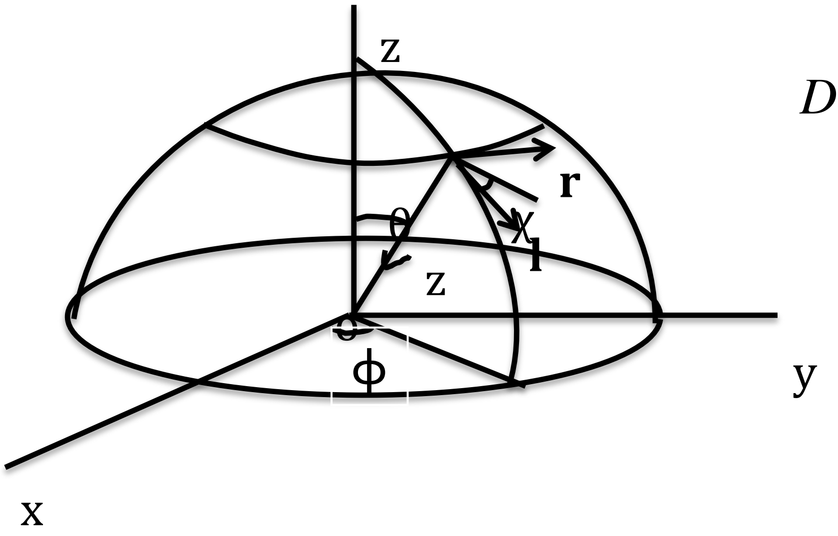


Figure 11. General Instrument Layout

One, last out of position, introduction slide



Degree of polarization

$$DoP = \frac{(Q^2+U^2+V^2)^{1/2}}{I}, \quad 0 \leq DoP \leq 1$$

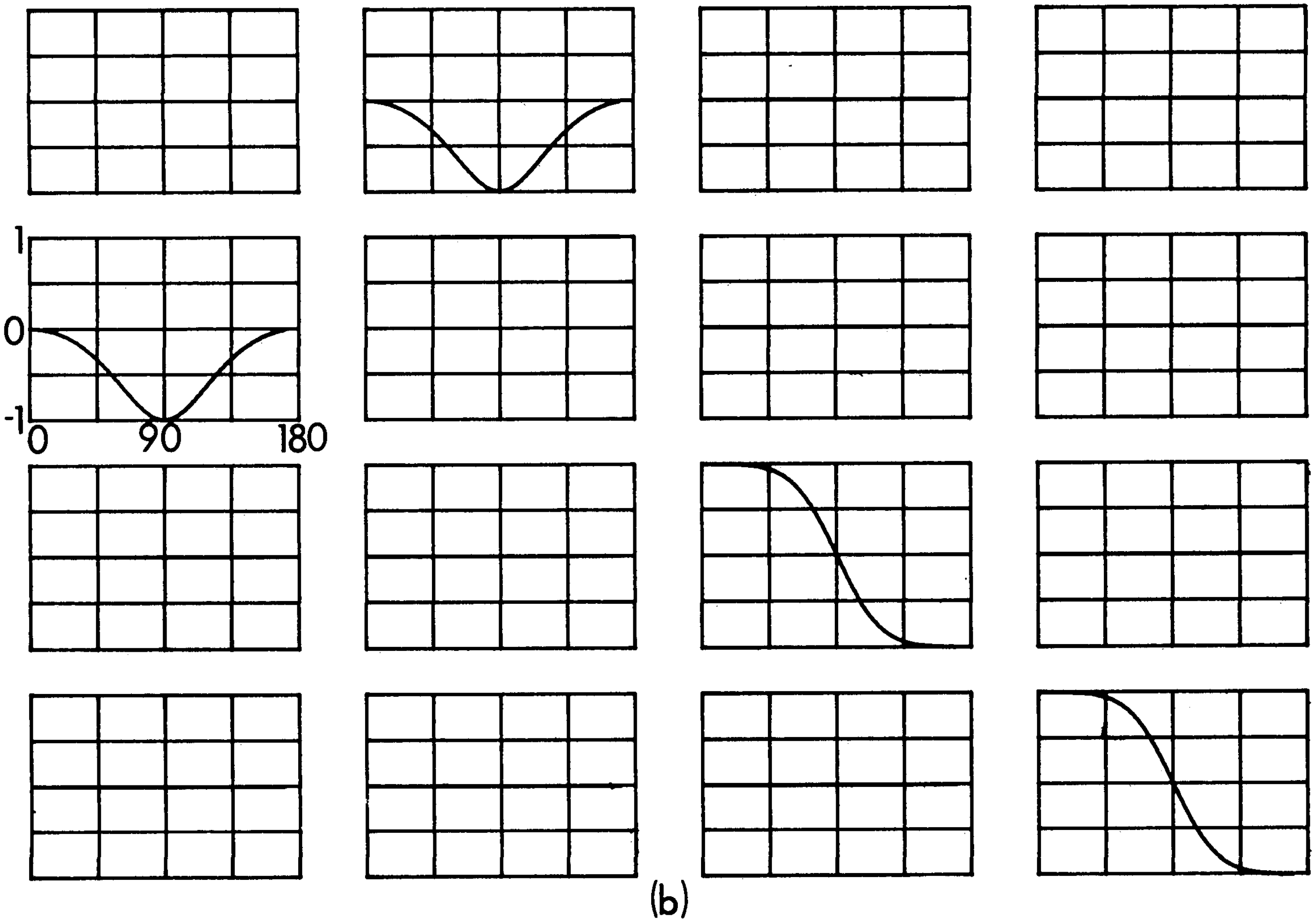
Plane of polarization

$$\tan 2\chi = \frac{U}{Q}, \quad -90^0 \leq \chi \leq 90^0$$

For completeness, ellipticity

$$\sin 2\beta = \frac{2a_l a_r \sin\delta}{a_l^2 + a_r^2} = \frac{V}{(Q^2+U^2+V^2)^{1/2}}$$

Mueller matrix for Rayleigh scattering.



Mueller matrix for Rayleigh scattering. Lets look at some specific angles:

$$M(0°) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

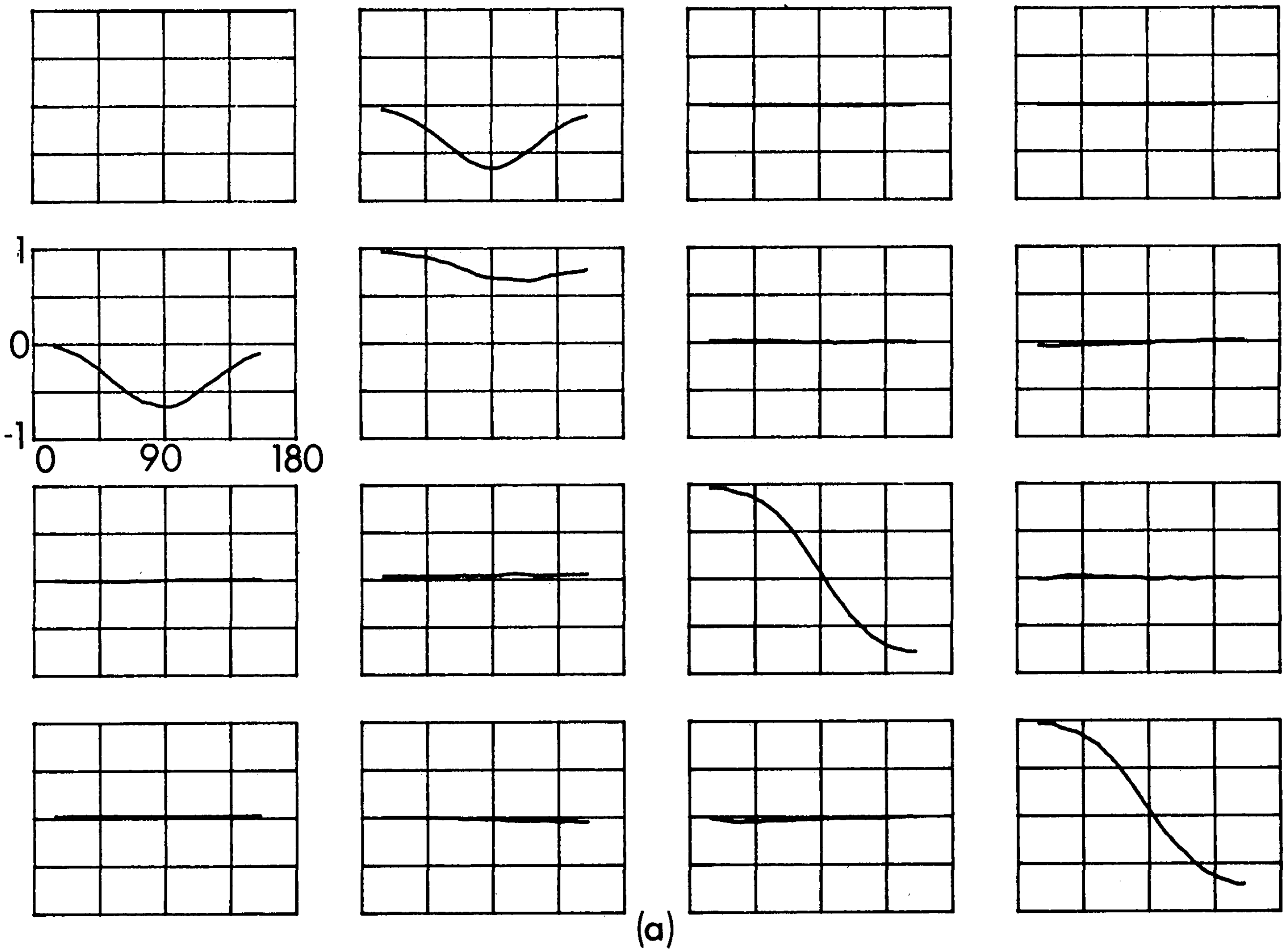
$$M(90°) = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M(180°) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

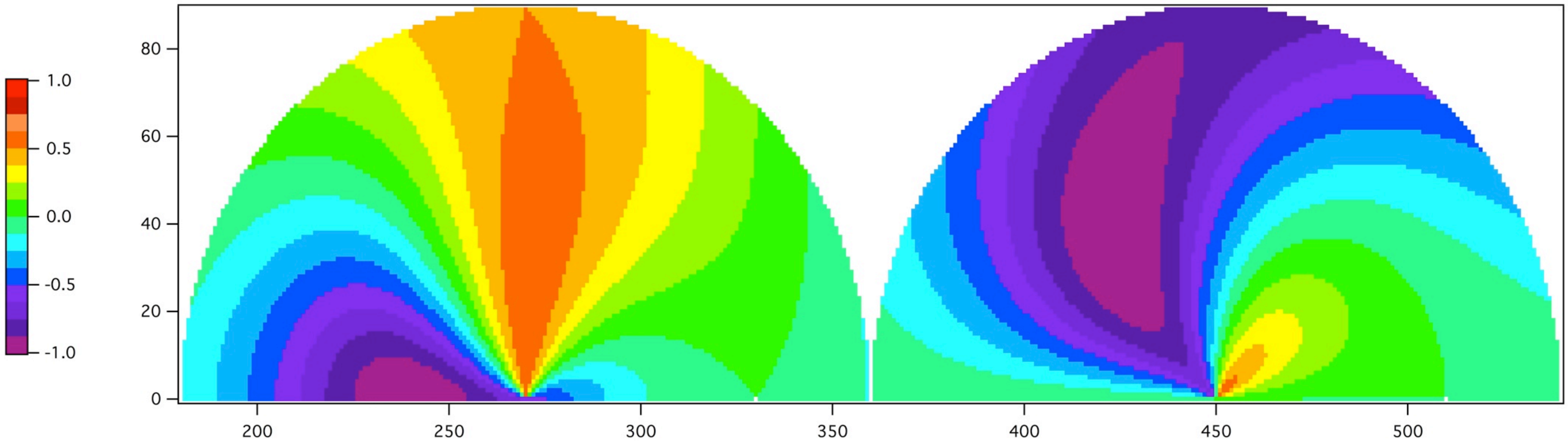
What do these do?

Mueller matrix for Ocean water.

What are the differences?



Case for Rayleigh single scattering in Sky.



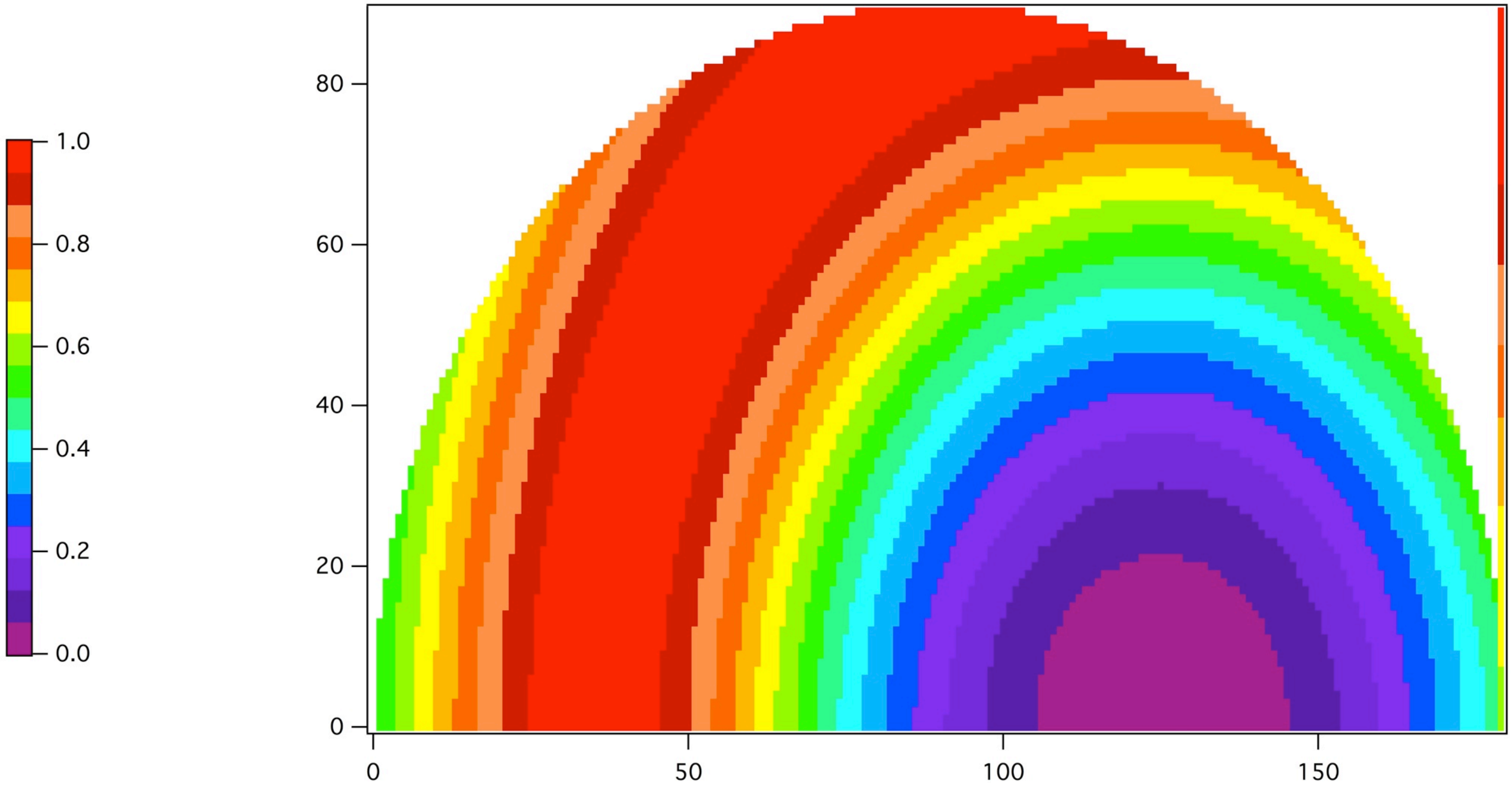
Q/I

U/I

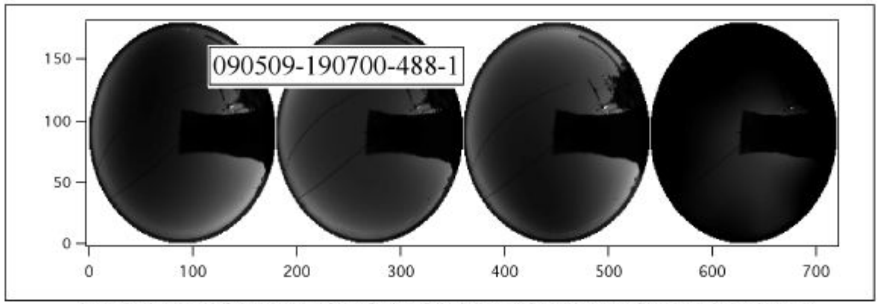
V/I = 0

60 degree zenith angle

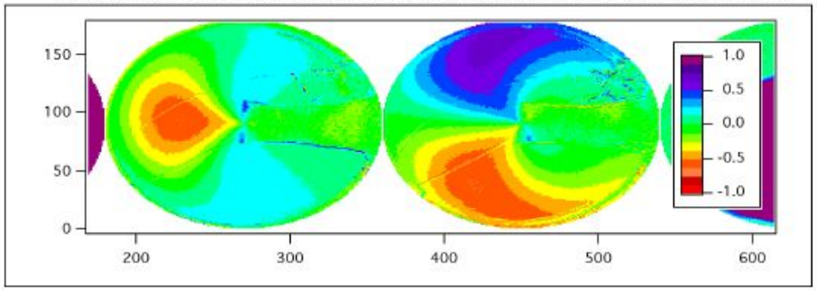
Case for Rayleigh single scattering in Sky.



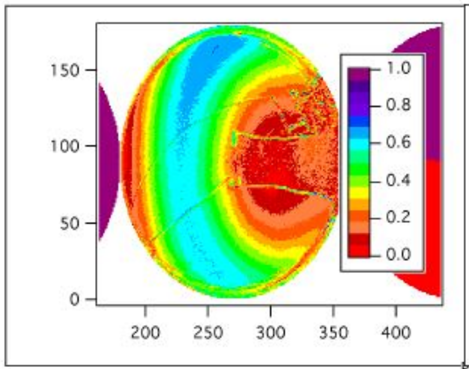
Sky light radiance distribution.



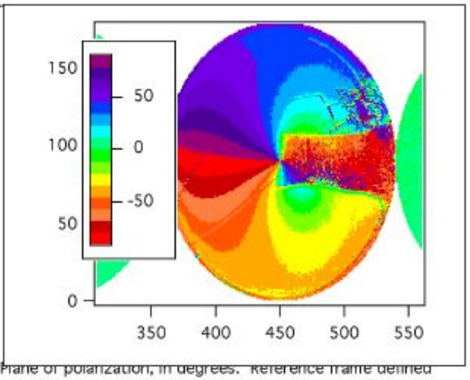
Intensity images, left is intensity from Stokes Calculation, right is intensity from unpolarized image



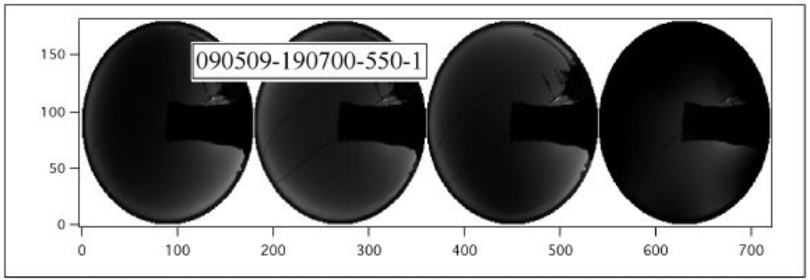
Normalized Stokes vectors Q (on left) and U (on right).



Degree of linear polarization



Plane of polarization, in degrees. Reference frame defined relative to plane with zenith and view direction. 0 deg is in the reference plane, positive angles rotate clockwise (looking along direction of propogation) towards the perpendicular to this plane.



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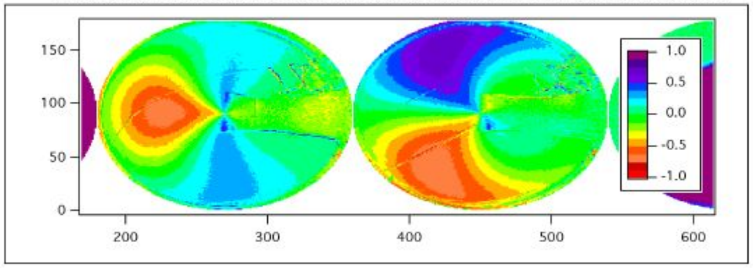

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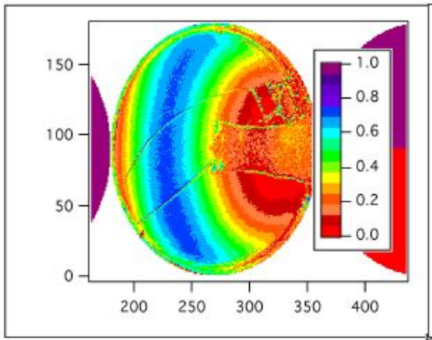
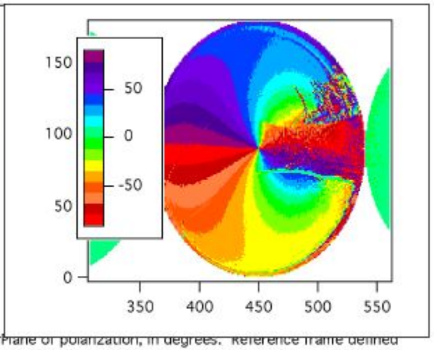
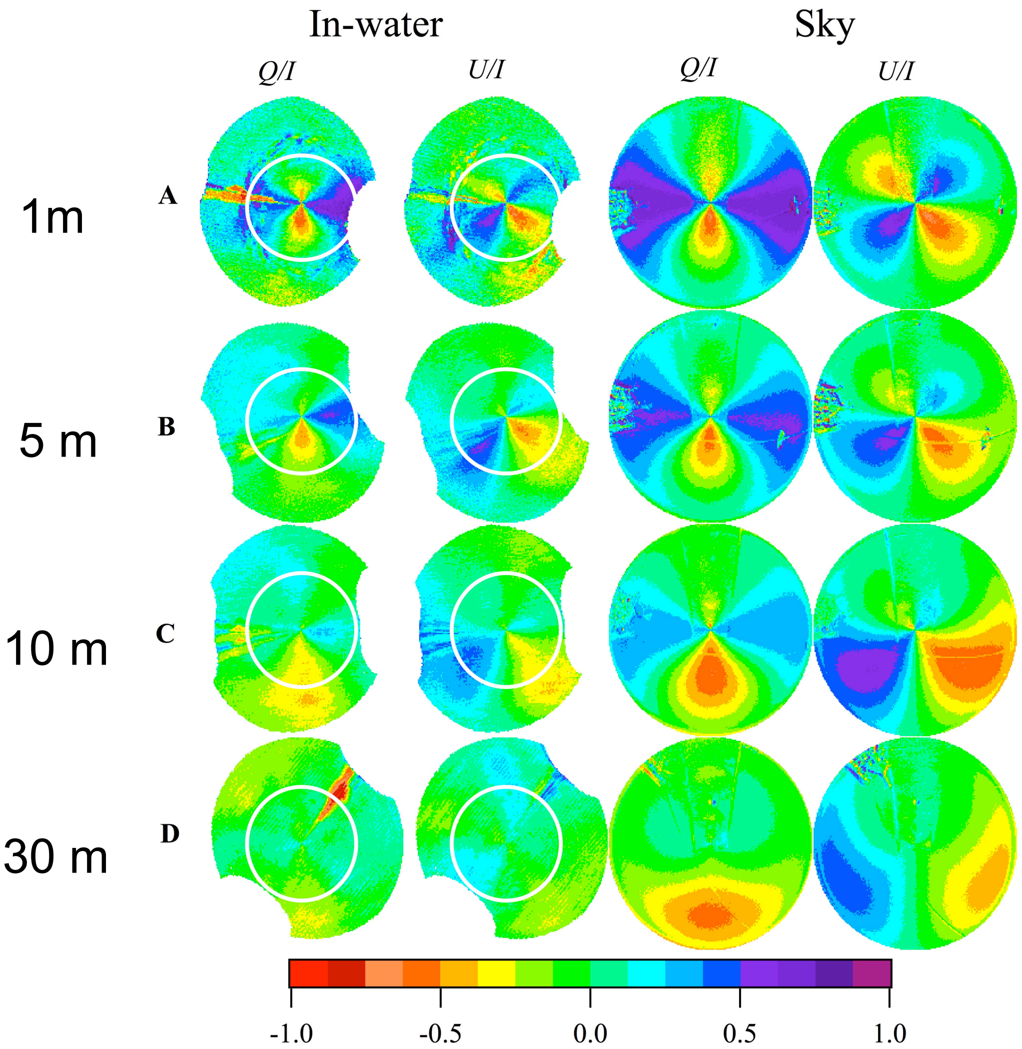


Degree of linear polarization



Plane of polarization, in degrees. Reference frame defined relative to plane with zenith and view direction. 0 deg is in the reference plane, positive angles rotate clockwise (looking along direction of propogation) towards the perpendicular to this plane.

Downwelling radiance distribution.



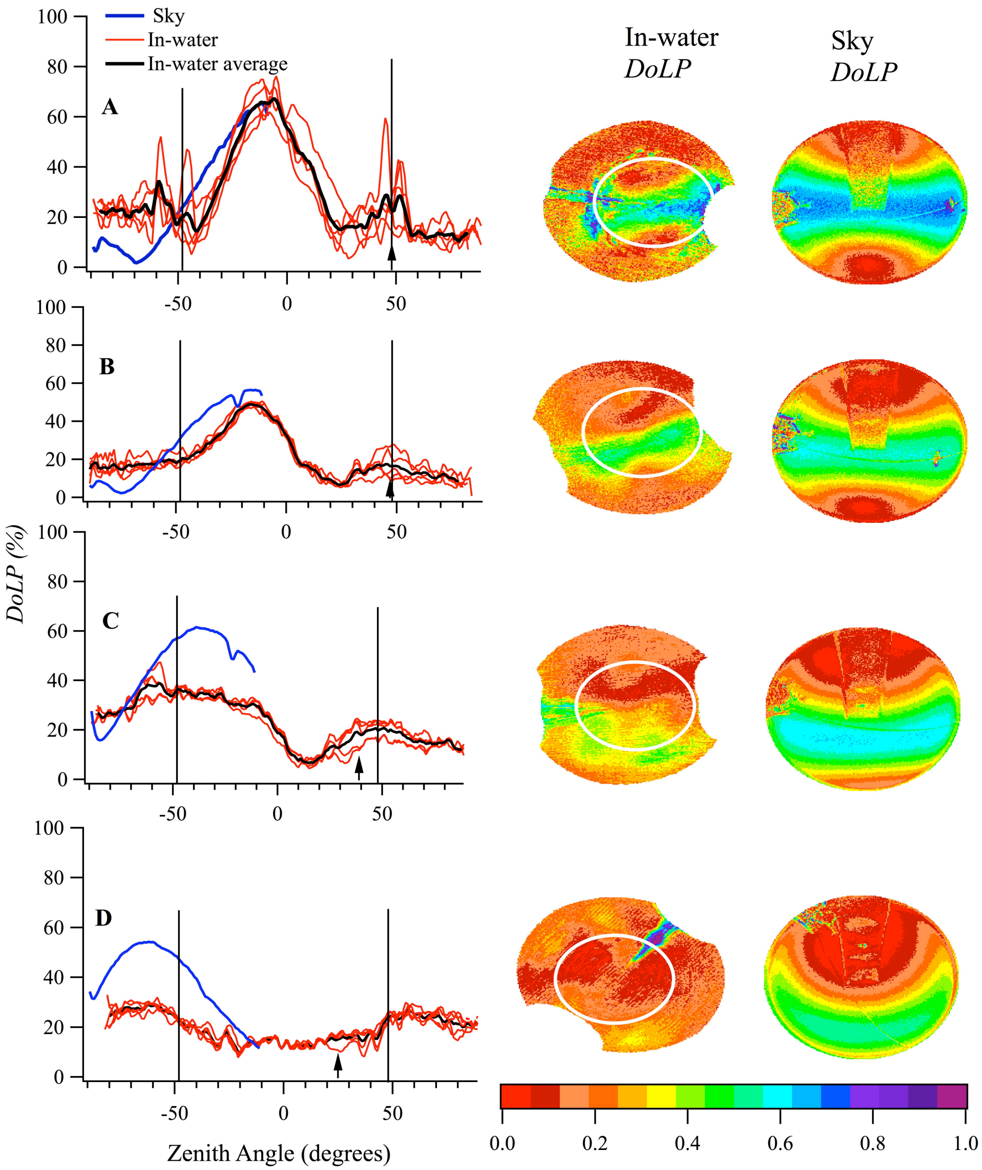
Downwelling radiance distribution.

1m

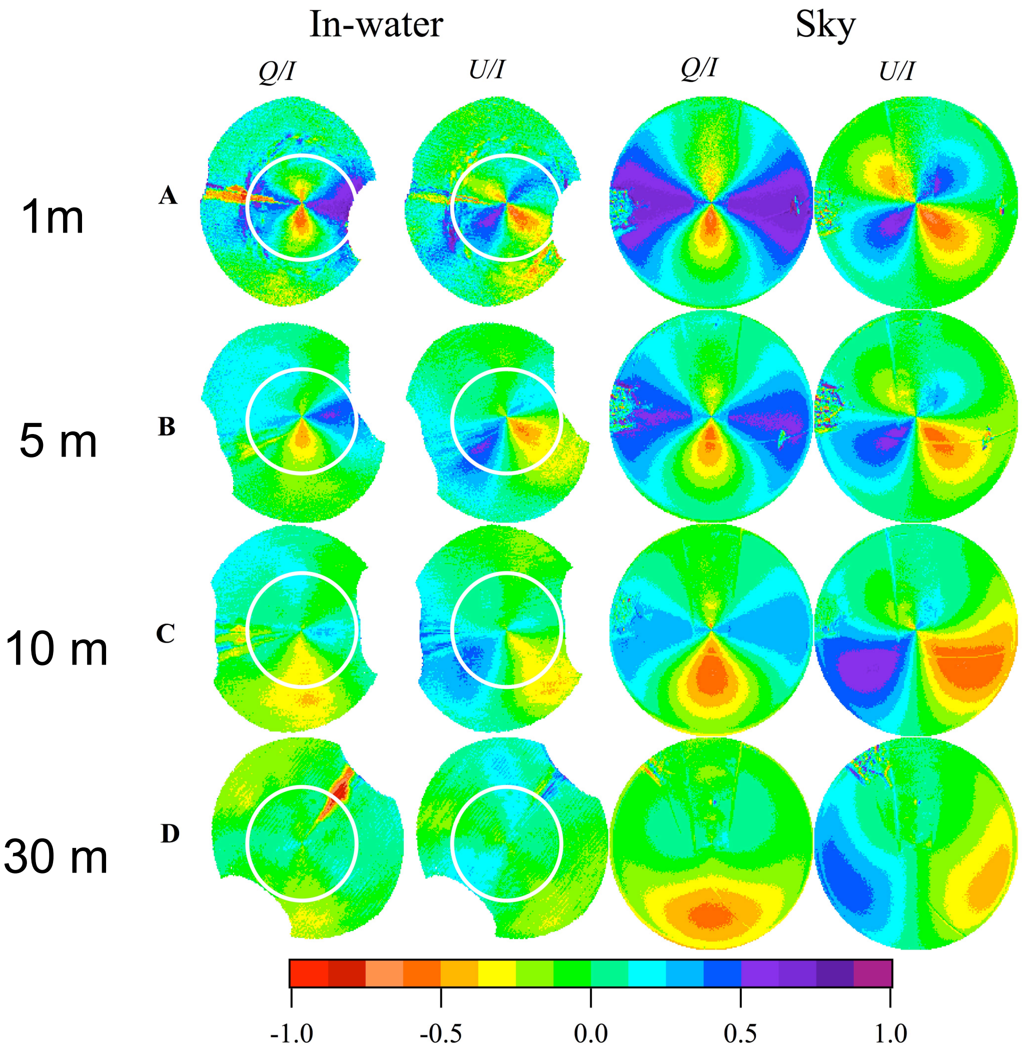
5 m

10 m

30 m



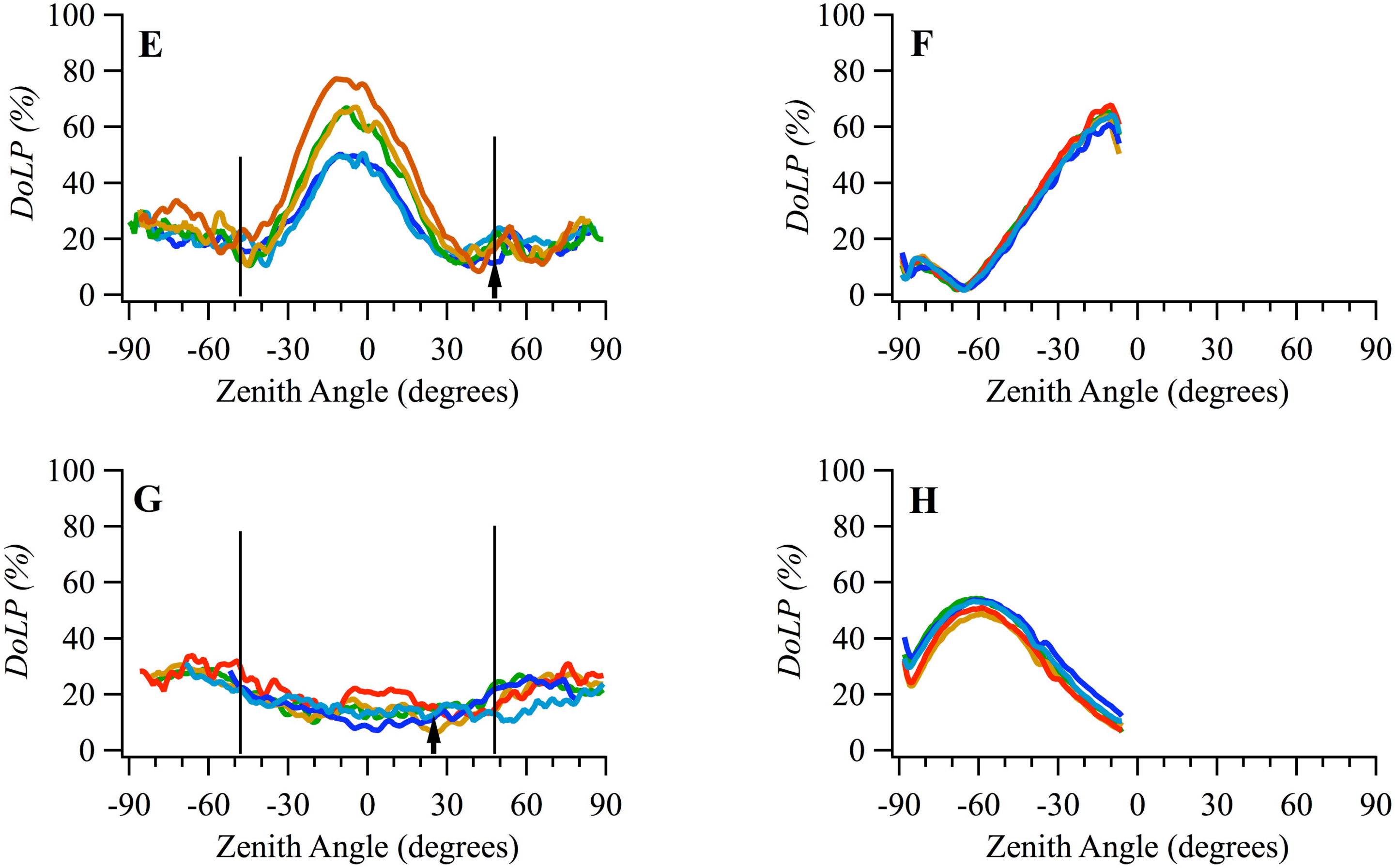
Downwelling radiance distribution.



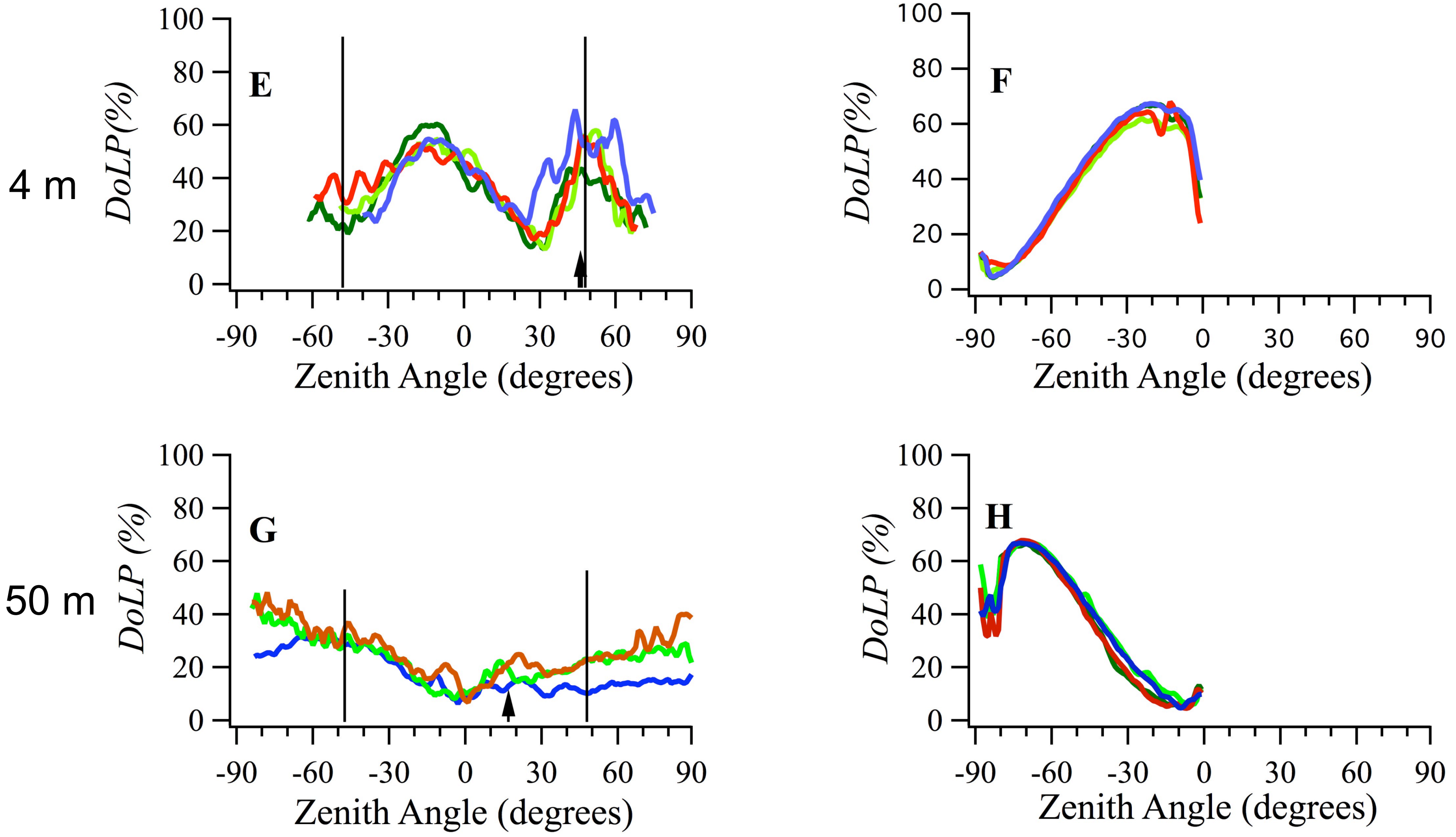
Downwelling spectral variation, coastal

1 m

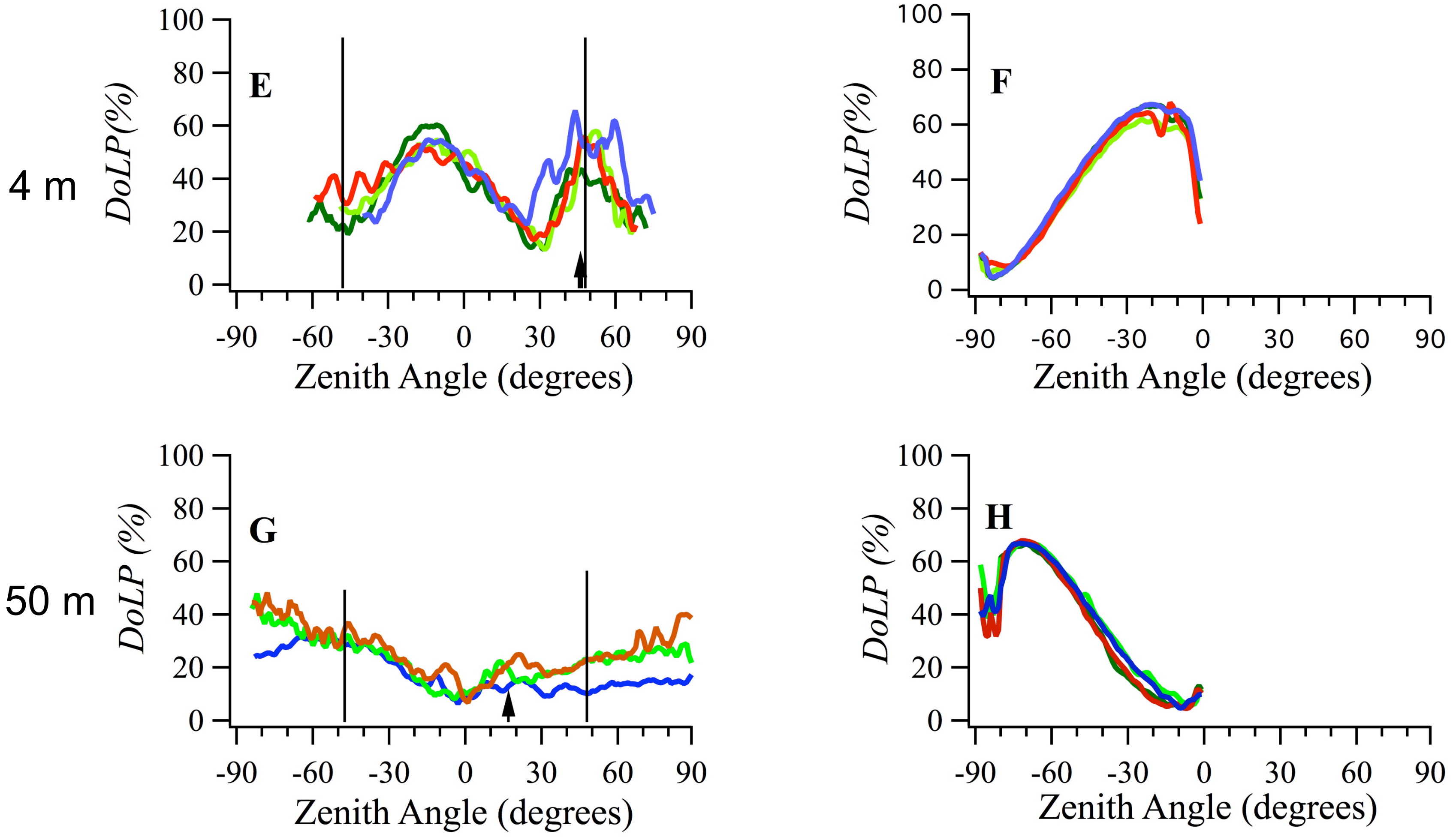
30 m



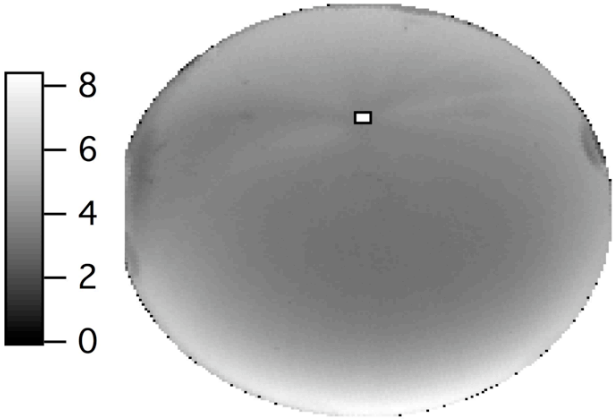
Downwelling spectral variation, clear water



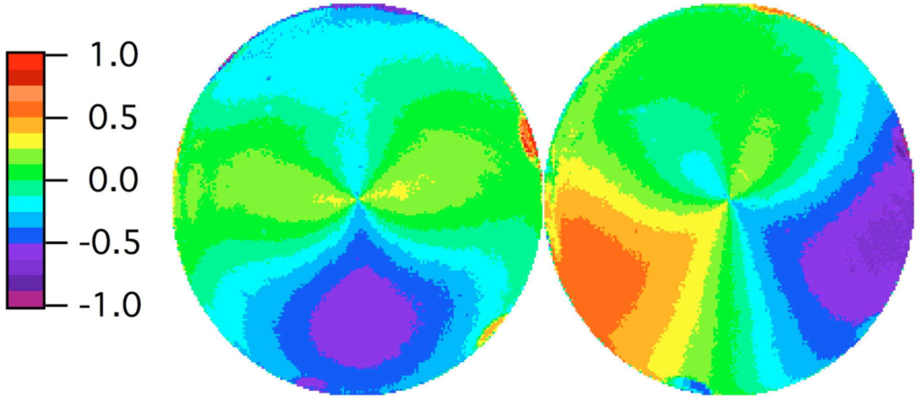
Downwelling spectral variation, clear water



Example upwelling images



Radiance



Q/I U/I

The radiance units are mW cm^{-2} nm^{-1} sr^{-1}, Q/I and U/I are dimensionless

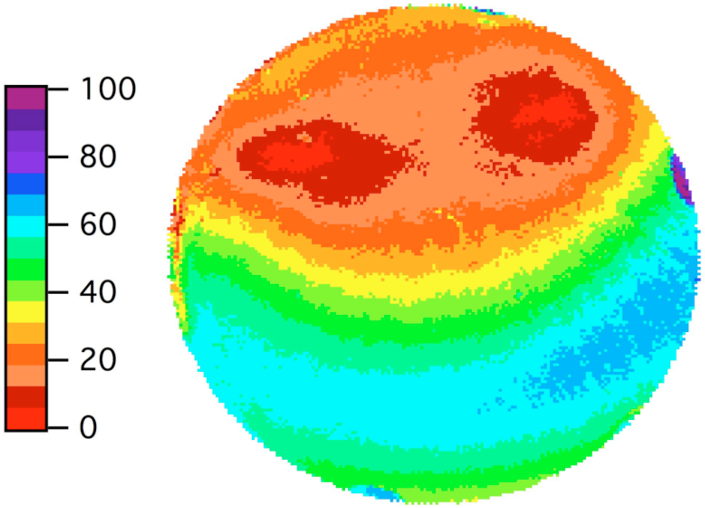
Hawaii (December 2, 2005, 20.83^{0} N, 157.18^{0} W, 10:25 local time).

Wavelength is 436 nm

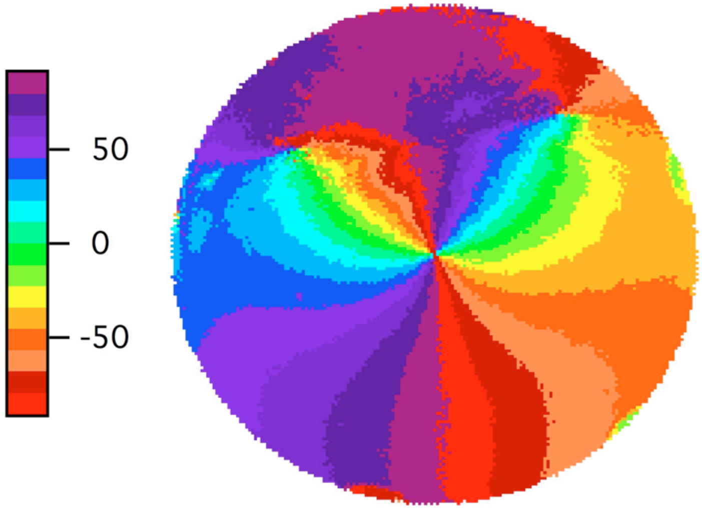
solar zenith angle is 51^{0}.

Measured with POLRads instrument (recent article in OE)

Example continued

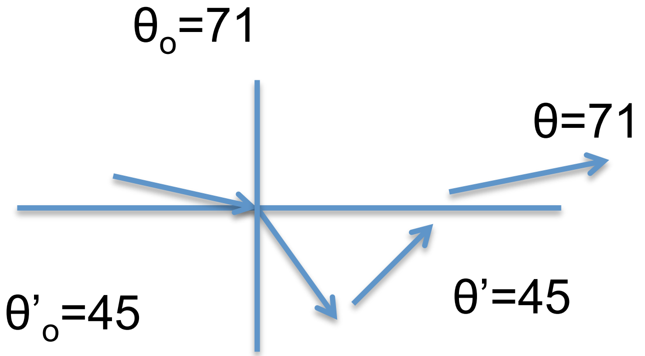


Degree of Polarization



Plane of Polarization

Note high degree of polarization, 90 degrees to refracted solar position, probably lost in glitter pattern, however still significant polarization in other areas.



Other recent applications of polarization:

Use of polarized light for enhanced imaging: scattered light is more polarized than light from target (many people, early work by Gilbert, AO, 1967)..keeps getting rediscovered.

Polarized detection by animals..and polarized light camouflage (Molly Cummings, UT, Roger Hanlon, MBL).

Depolarization due to minerals?

Your instrument may have a polarization sensitivity (old RSR instruments, spectrometers without scrambling)

The satellite instrument is polarized, to greater or lesser extent.

Might be more information in the polarization signal, new sensors will be developed (one just sank…APS on Glory) especially important for atmospheric aerosols.