2013 Summer Course

on Optical Oceanography, Remote Sensing,
Radiative Transfer Theory, and HydroLight

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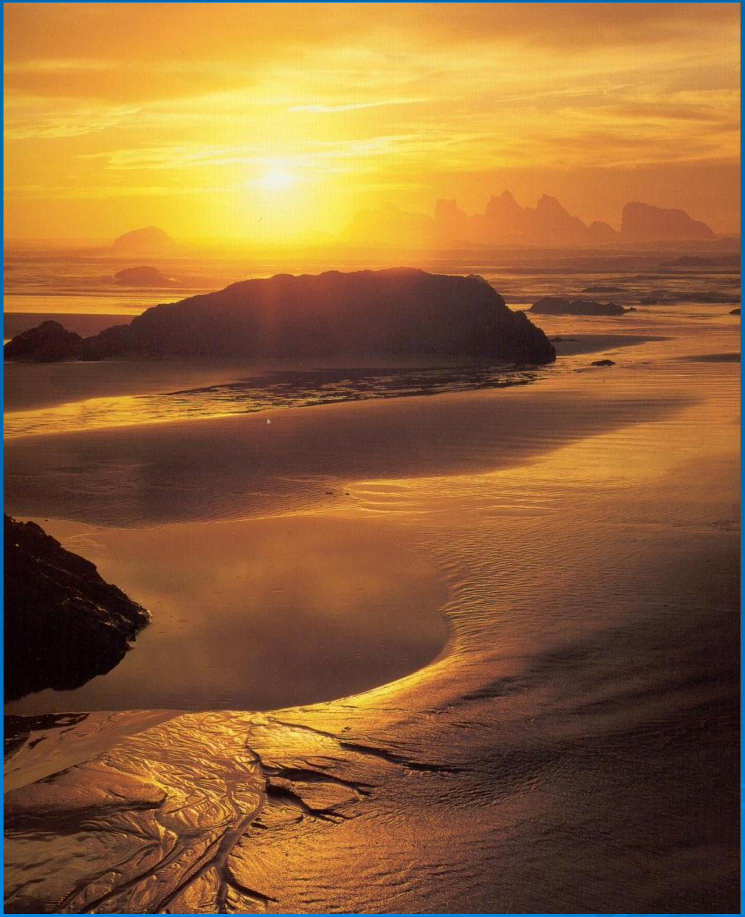
The Volume Scattering Function and Models for Scattering

Delivered at the Darling Marine Center,
University of Maine
July 2013

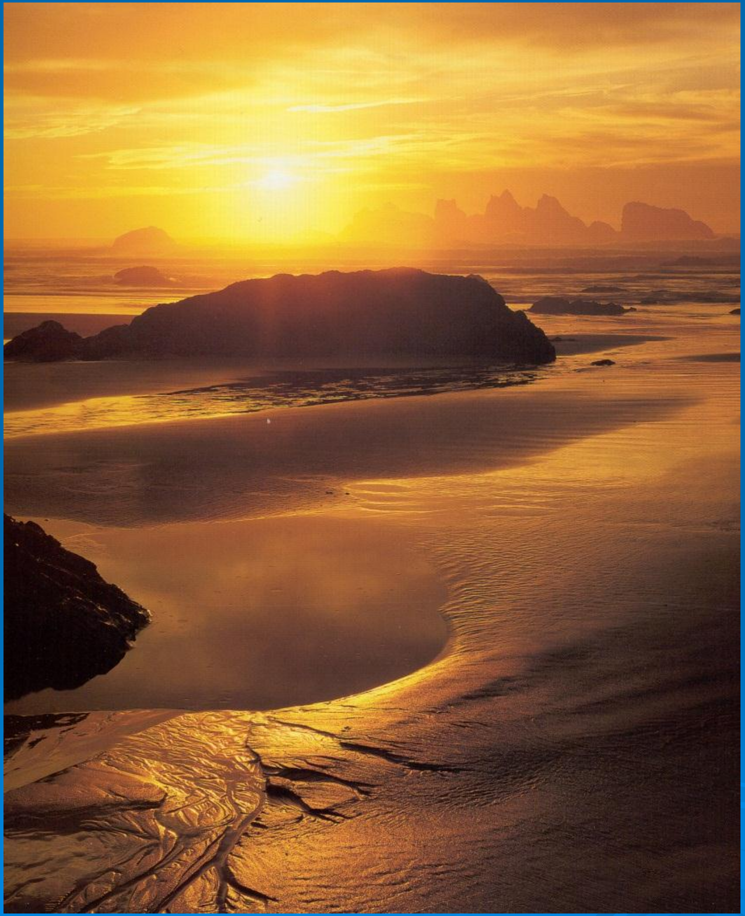
Why is Scattering Important?

The world with scattering

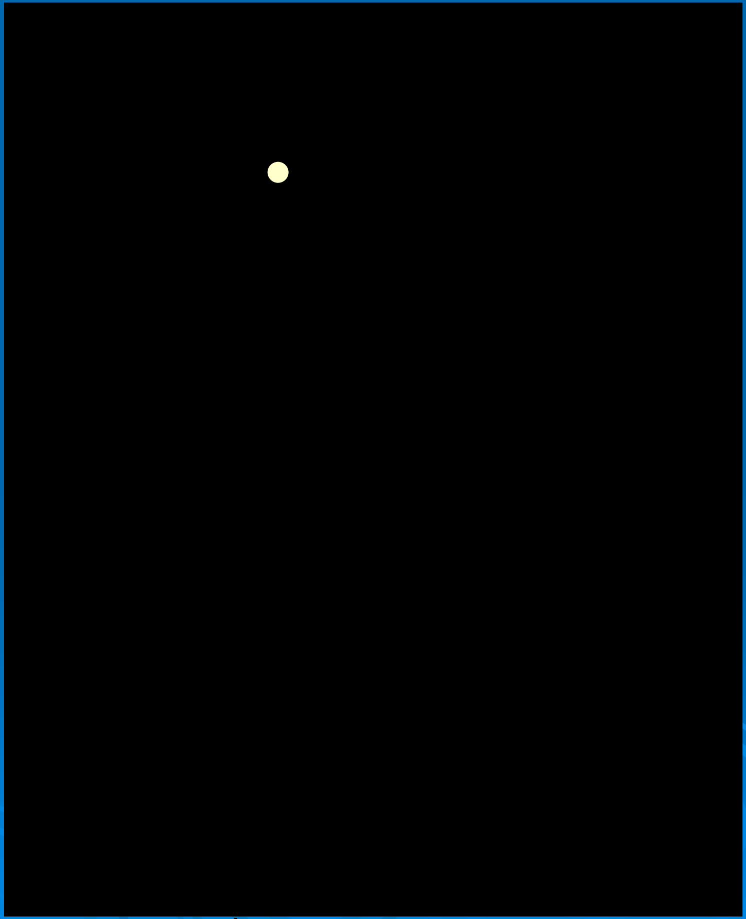


Why is Scattering Important?

The world with scattering



The world without scattering

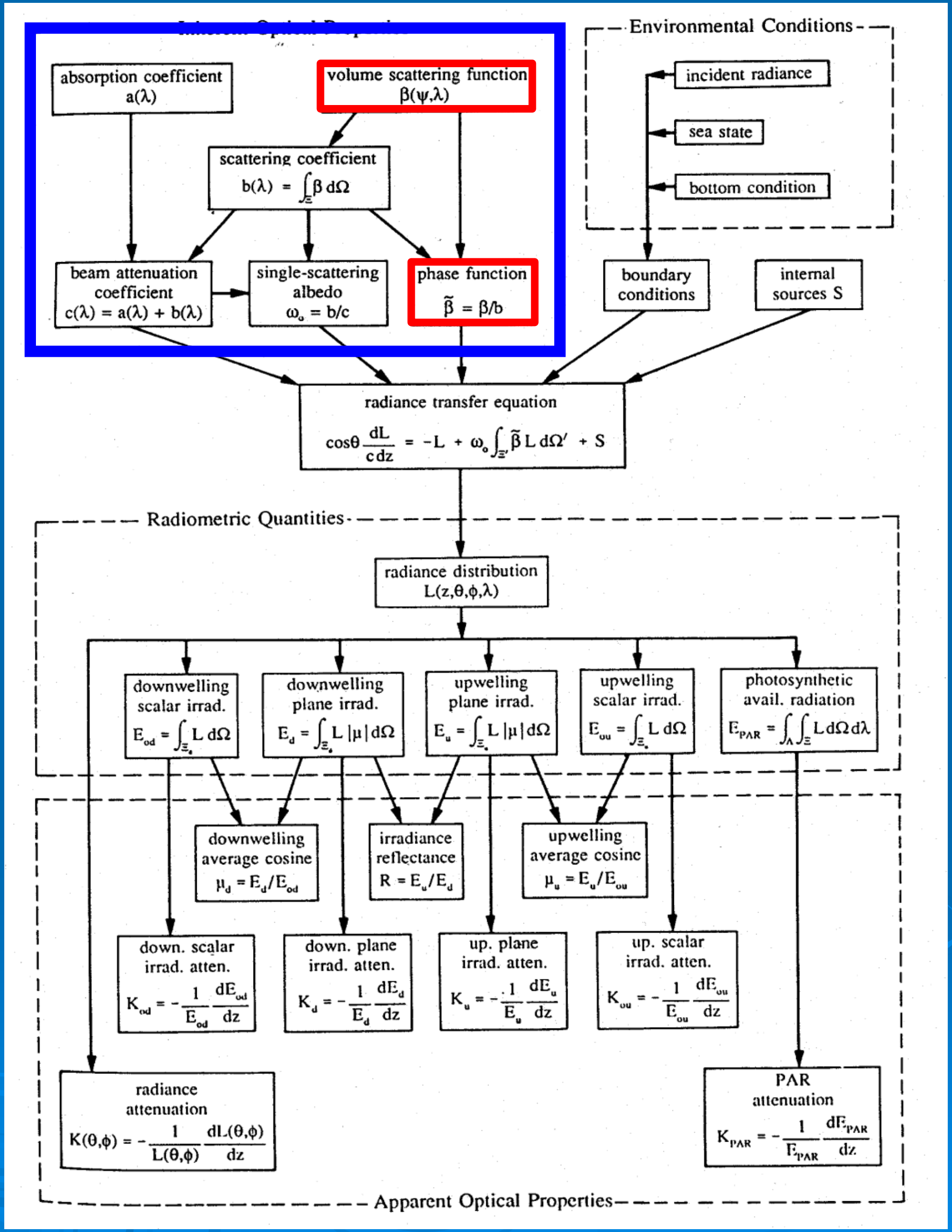


Inherent Optical Properties (IOPs)

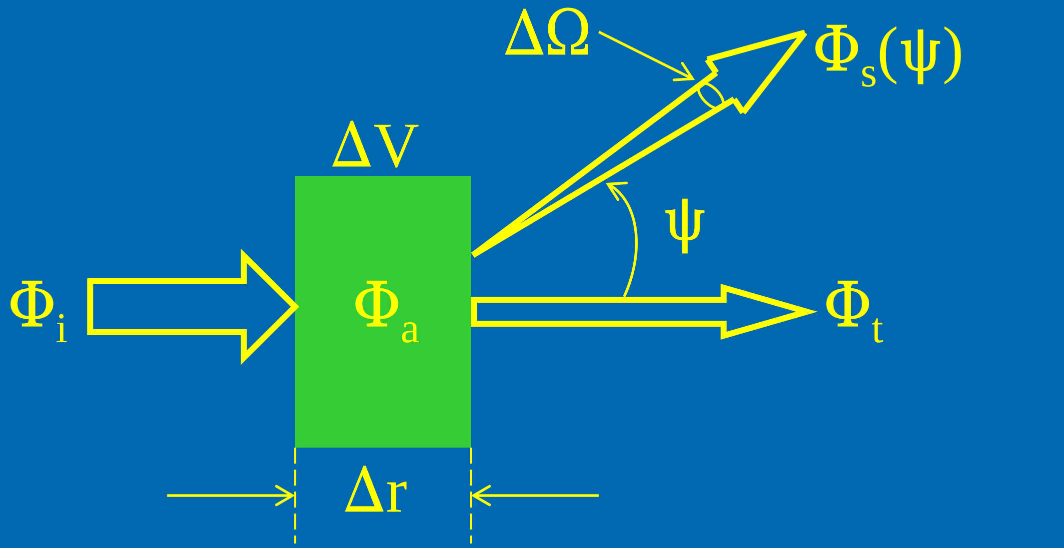
- depend on the concentration, size distribution, and compositions of the particulate and dissolved material in the water (and on the water itself)

- do NOT depend on the light field in the water (therefore, can measure in situ or on a water sample)

The two fundamental IOPs are the absorption coefficient and the volume scattering function—all others can be derived from these two



The Volume Scattering Function (VSF)



It is assumed that Δr is small enough that only single scattering occurs in ΔV (mfp = 1/*c*)

$$\beta(\psi,\lambda) \equiv \mathop{\text{Limit}}_{\Delta r\to 0}\;\mathop{\text{Limit}}_{\Delta\Omega\to 0}\; \frac{\Phi_s(\psi,\lambda)}{\Phi_i(\lambda)\,\Delta r\,\Delta\Omega}$$

$$= \mathop{\text{Limit}}_{\Delta V\to 0}\; \frac{I_s(\psi,\lambda)}{E_i(\lambda)\,\Delta V} \qquad [\mathrm{m}^{-1}\,\mathrm{sr}^{-1}]$$

The VSF tells you everything you need to know about how a volume of matter scatters light (ignoring polarization)

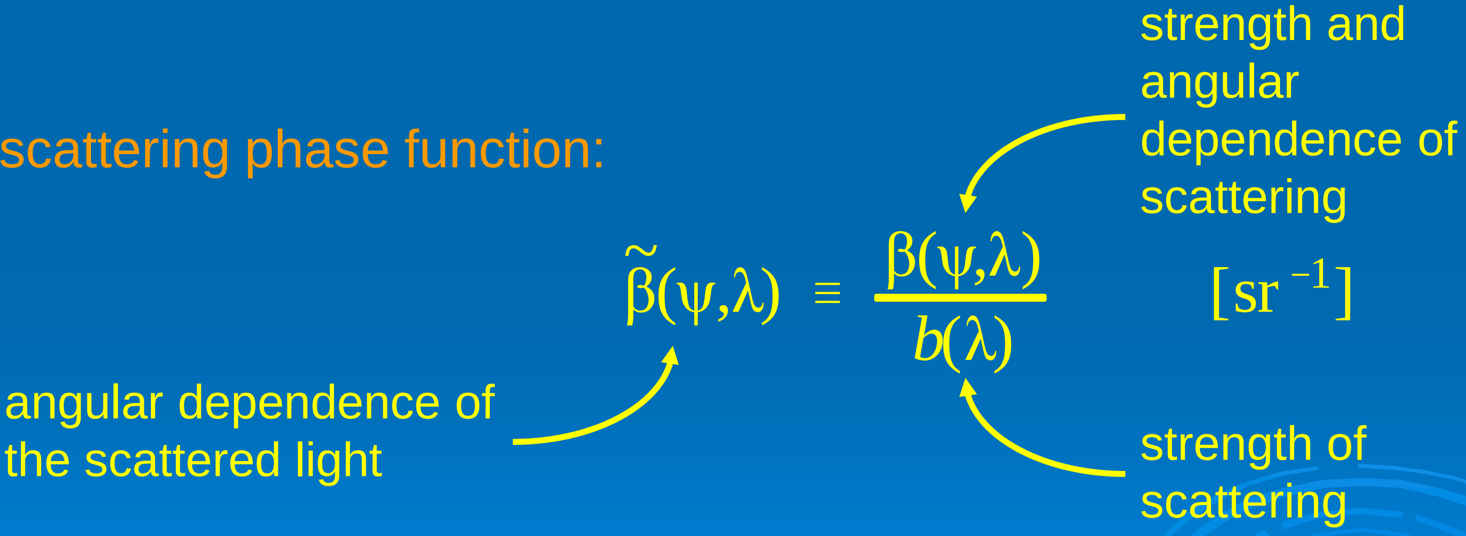
Other Measures of Scattering (1)

The total scattering coefficient:

$$b(\lambda) = \int_{4\pi} \beta(\psi,\lambda)\, d\Omega = 2\pi \int_{0}^{\pi} \beta(\psi,\lambda) \sin\psi \, d\psi = c - a \qquad [\mathrm{m}^{-1}]$$

b tells how much light is scattered, without regard for the direction of the scattering

The scattering phase function:



$$\tilde{\beta}(\psi,\lambda) \equiv \frac{\beta(\psi,\lambda)}{b(\lambda)} \qquad [\mathrm{sr}^{-1}]$$

strength and angular dependence of scattering

angular dependence of the scattered light

strength of scattering

The phase function gives the angular pattern of the scattered light, without regard for the magnitude of the scattering

Other Measures of Scattering (2)

The backscatter coefficient:

$$b_b(\lambda) \equiv 2\pi \int_{\pi/2}^{\pi} \beta(\psi,\lambda) \sin\psi \, d\psi \qquad [\mathrm{m}^{-1}]$$

b_b tells how much light is scattered through ψ = 90 to 180 deg

The backscatter fraction: $B = b_b / b$

B gives the fraction of the total light scattered that is scattered through 90 to 180 deg

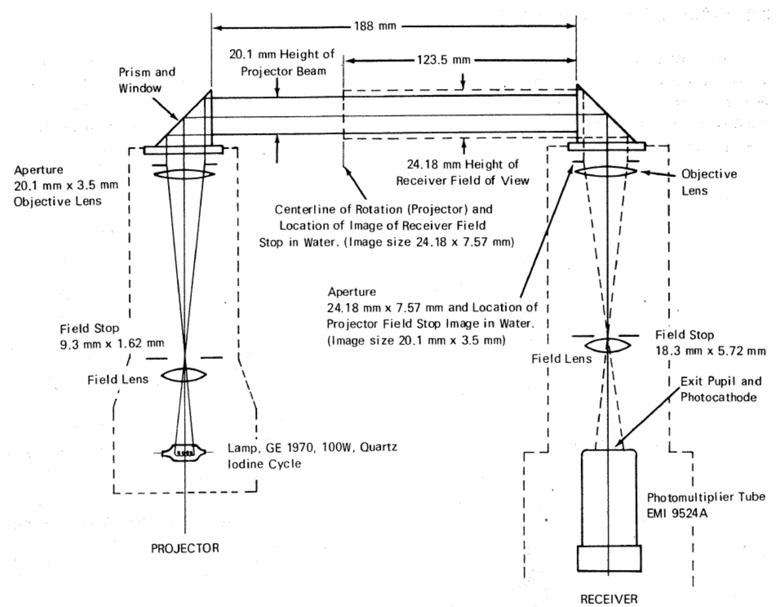
The albedo of single scattering: $\omega_o = b / (a + b)$

ω_o gives the fraction of the light scattered (vs. absorbed) in any interaction with matter; also called *the probability of photon survival*

Measurement of the VSF

A number of instruments have been developed over the years to measure the VSF *in situ*. However, each is unique in design and none is commercially available. Therefore, the VSF is seldom measured. This may change within the next decade, as new commercial instruments are developed.

We do not have time to discuss particular VSF instruments, so I'll only show some example data.



Petzold, SIO Rept 72-78, 1972

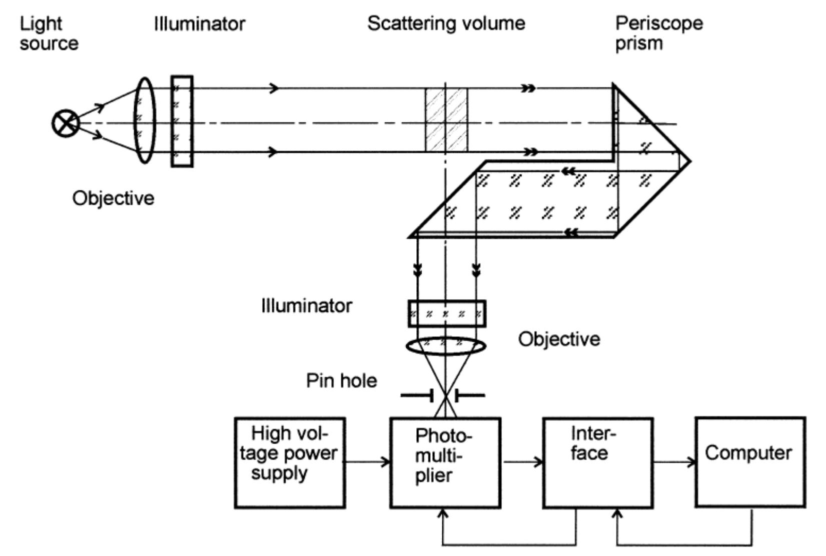
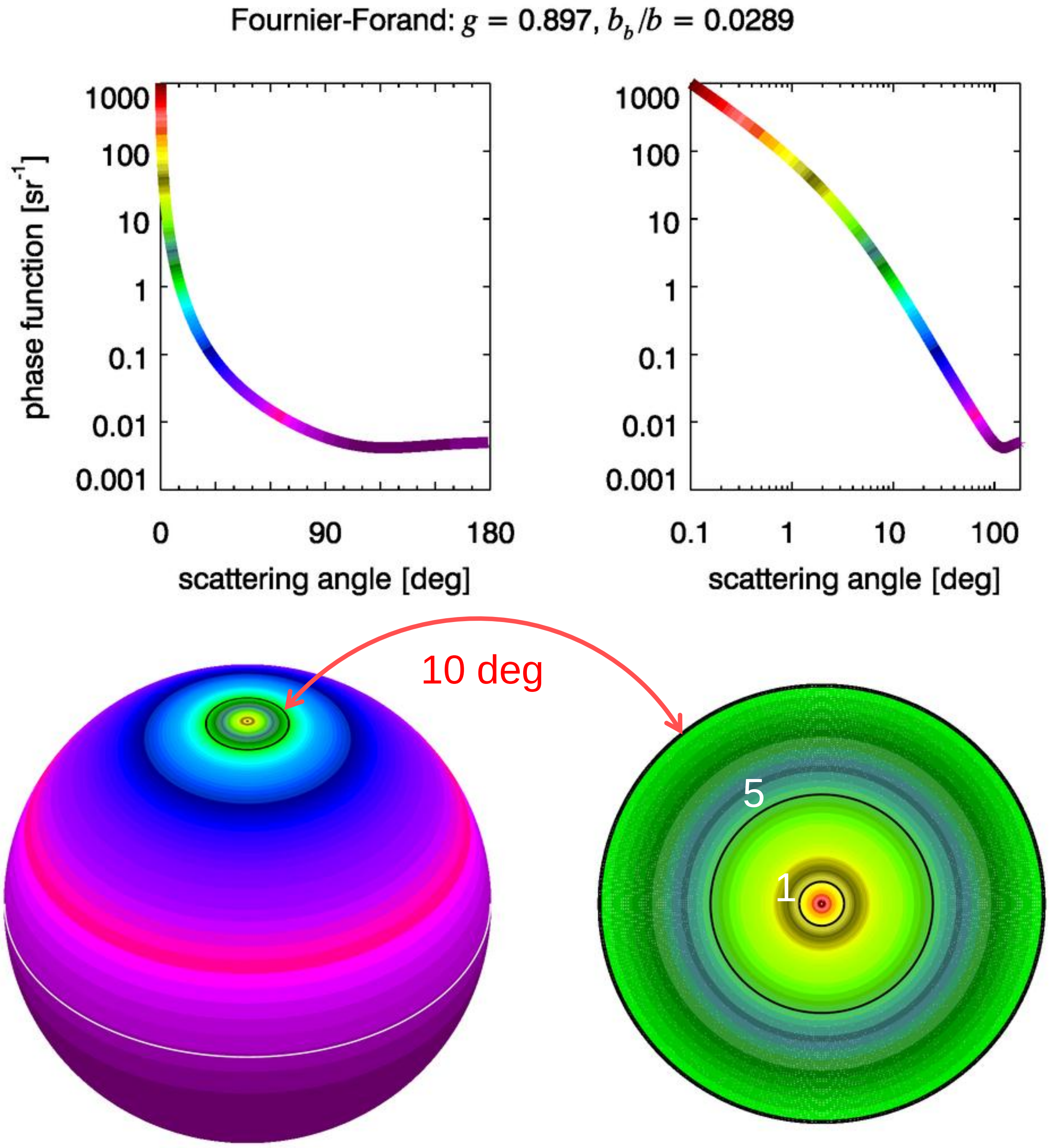


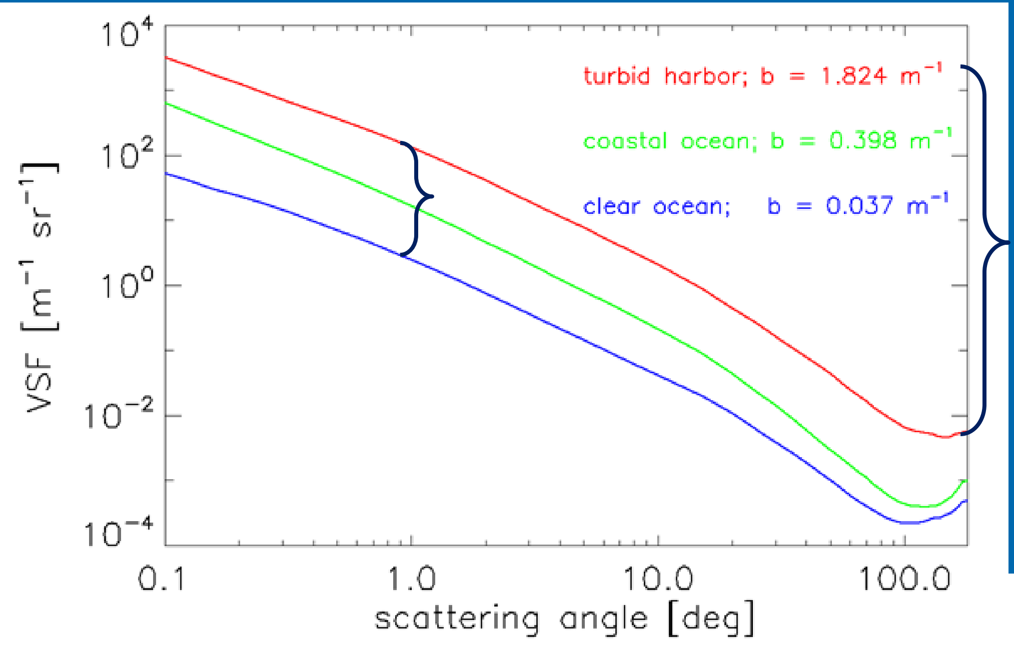
FIG. 1. General schematic diagram of the principle of measurement of the volume scattering function in the ocean. The optical filter (532 nm) is placed before the photomultiplier.

Lee & Lewis, J Atm Ocean Tech, 2003

Visualizing the VSF



Variability in the VSF

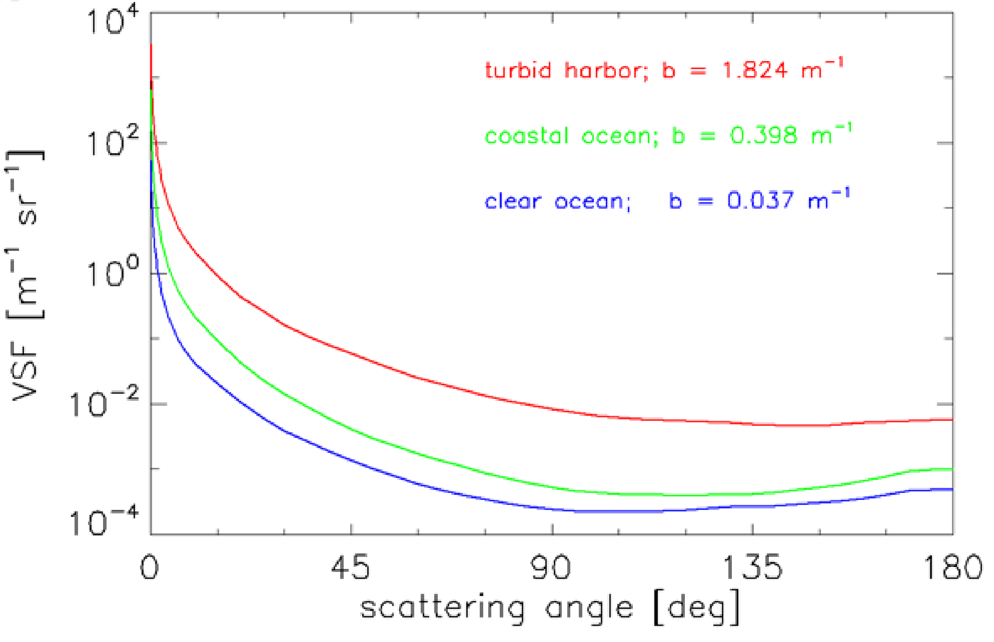


6 orders of magnitude variation between small and large scattering angles, for a given VSF

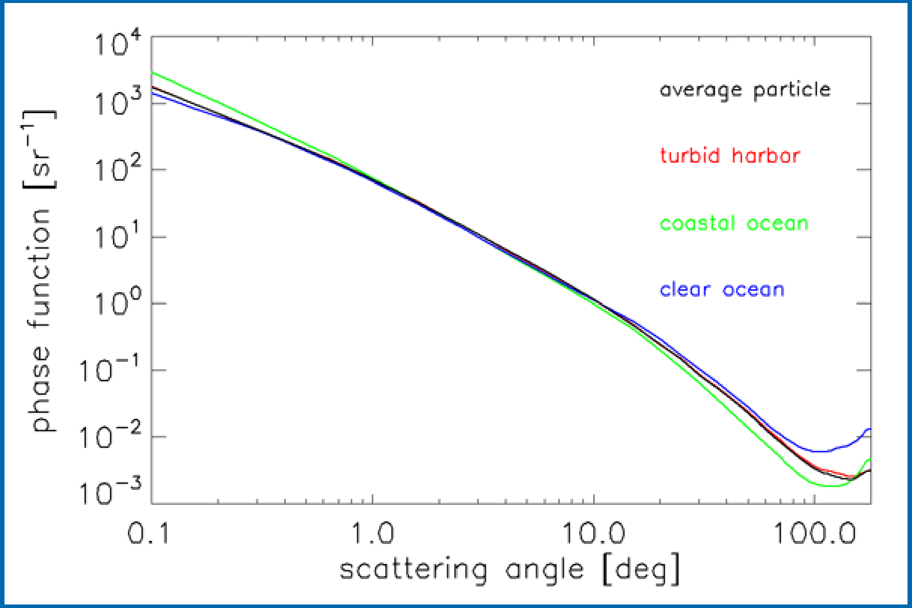
Petzold's data, see www.oceanopticsbook.info/view/scattering/petzolds_measurements

2 orders of magnitude variation between different water types

www.oceanopticsbook.info/view/overview_of_optical_oceanography/visualizing_vsfs



An Idea



It would simplify optical oceanography if most of the variability in the VSF were contained in the scattering coefficient *b*, so that a common phase function could be used for all “particles.”

highly variable

$$\tilde{\beta}(\psi,\lambda) \equiv \frac{\beta(\psi,\lambda)}{b(\lambda)}$$

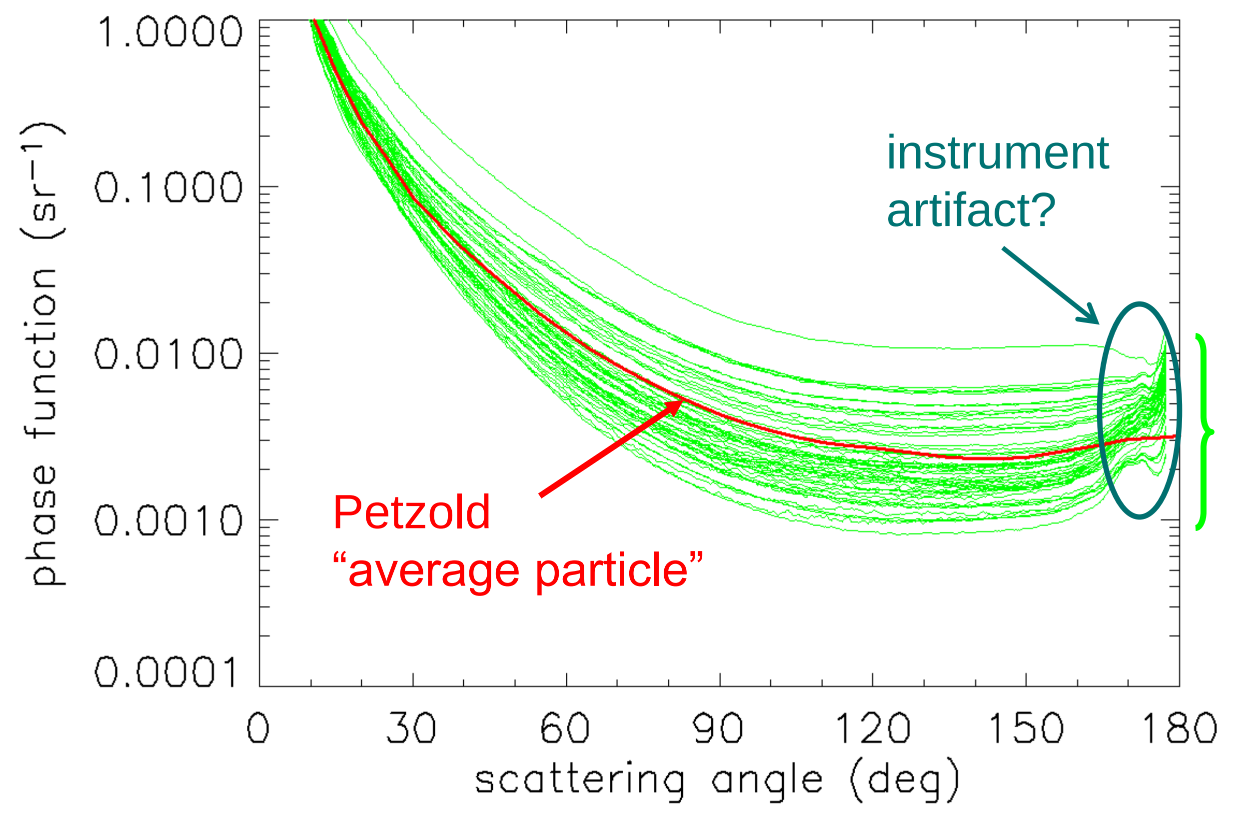
maybe not
too variable

highly variable

Is this a good assumption?

Variability in Phase Functions

62 phase functions measured in coastal New Jersey waters,
λ = 530 nm (VSM data courtesy of E. Boss, M. Lewis, et al.)



instrument artifact?

Petzold
“average particle”

Order-of-magnitude variability

Note: Petzold may be fine on average, but way off in any particular instance.

Variability in Phase Functions

When running HydroLight, you may have measurements of absorption and scattering from an ac-S, for example, but you rarely have measurements of the VSF or the scattering phase function. Therefore, you must guess what phase function to use.

Using the wrong phase function can mean that the HydroLight predictions are much different (factor of 2 to 10) than measured light variables. This is a very common problem in comparing HydroLight predictions with measurements (e.g., of remote-sensing reflectance).

Warning

In the ocean, you can almost never neglect absorption compared to scattering, or scattering compared to absorption.

When measuring absorption, you always have to correct for scattering (e.g., the "scattering correction" for raw ac-9 or ac-S measurements).

When measuring scattering, you always have to correct for absorption.

This means that you need to measure both absorption and scattering simultaneously, and then (sometimes iteratively, sometimes with best guesses, e.g. about the shape of the phase function) correct one against the other. It's not a simple process.

Scattering Depends Strongly on the Particle Size Distribution

Photo by Ensign John Gay, US Navy. The plane was traveling at 1,200 km/hr just 25 m above the sea surface. This photo won first prize in the science and technology division in the World Press Photo 2000 contest, which drew more than 42,000 entries worldwide.

Models for Scattering

First look at data and models for individual components

- water
- phytoplankton (algae)
- CDOM (negligible scattering)
- NAP
    - CPOM (detritus)
    - CPIM (minerals)

Then put the pieces together to get an IOP model for use in HydroLight

The VSF and the Scattering Phase Function

$$\beta = \sum_{i=1}^{N} \beta_i$$

VSFs are additive

$$\tilde{\beta} = \sum (b_i/b)\tilde{\beta}_i$$

phase functions must be weighted by the fraction of component scattering

What components make sense for β?

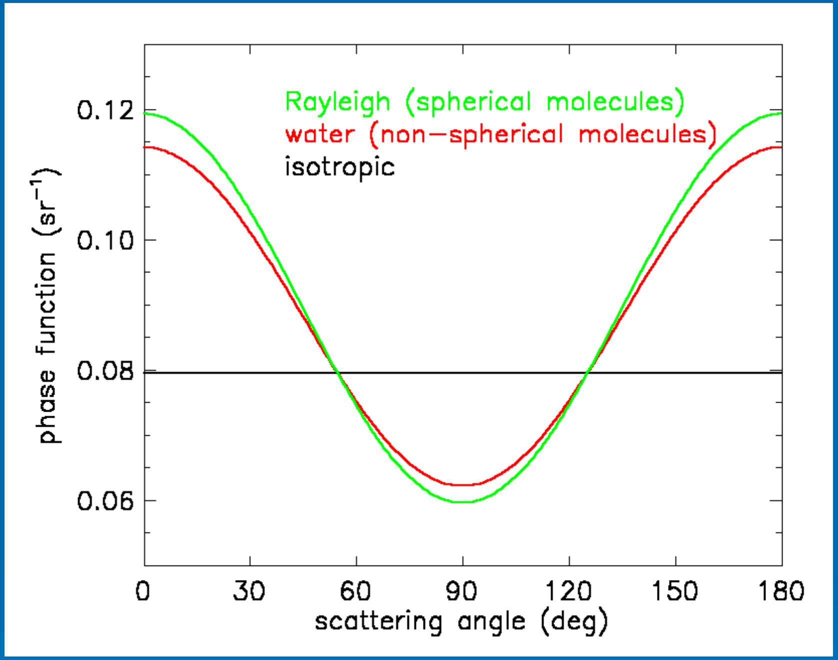
$$= b_w/b\ \tilde{\beta}_w + b_\phi/b\ \tilde{\beta}_\phi + b_{CPOM}/b\ \tilde{\beta}_{CPOM} + b_{CPIM}/b\ \tilde{\beta}_{CPIM}$$

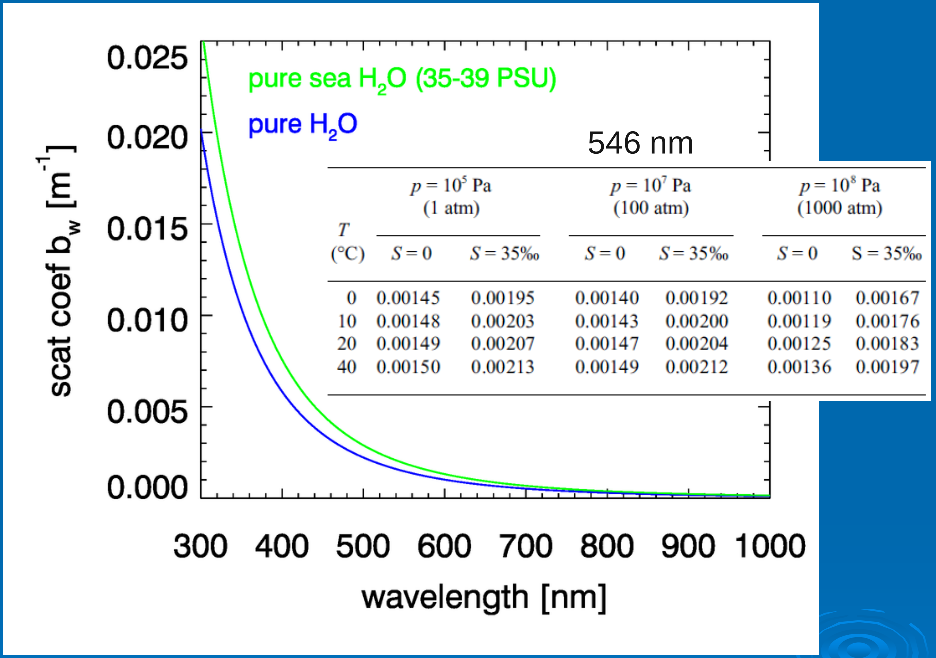
$\tilde{\beta}_i$ is a phase function representative of the *i*th component

b_i/b = fraction of total scattering by particle type *i*

Scattering by Pure Sea Water

scattering by pure water is the only IOP that can be computed from fundamental physics; all others come from measurement





T (°C)	$p = 10^5$ Pa (1 atm)		$p = 10^7$ Pa (100 atm)		$p = 10^8$ Pa (1000 atm)	
	$S = 0$	$S = 35‰$	$S = 0$	$S = 35‰$	$S = 0$	$S = 35‰$
0	0.00145	0.00195	0.00140	0.00192	0.00110	0.00167
10	0.00148	0.00203	0.00143	0.00200	0.00119	0.00176
20	0.00149	0.00207	0.00147	0.00204	0.00125	0.00183
40	0.00150	0.00213	0.00149	0.00212	0.00136	0.00197

phase function

$\tilde{\beta}_w(\lambda,\psi) = 0.06225(1+0.835\cos^2\psi)$

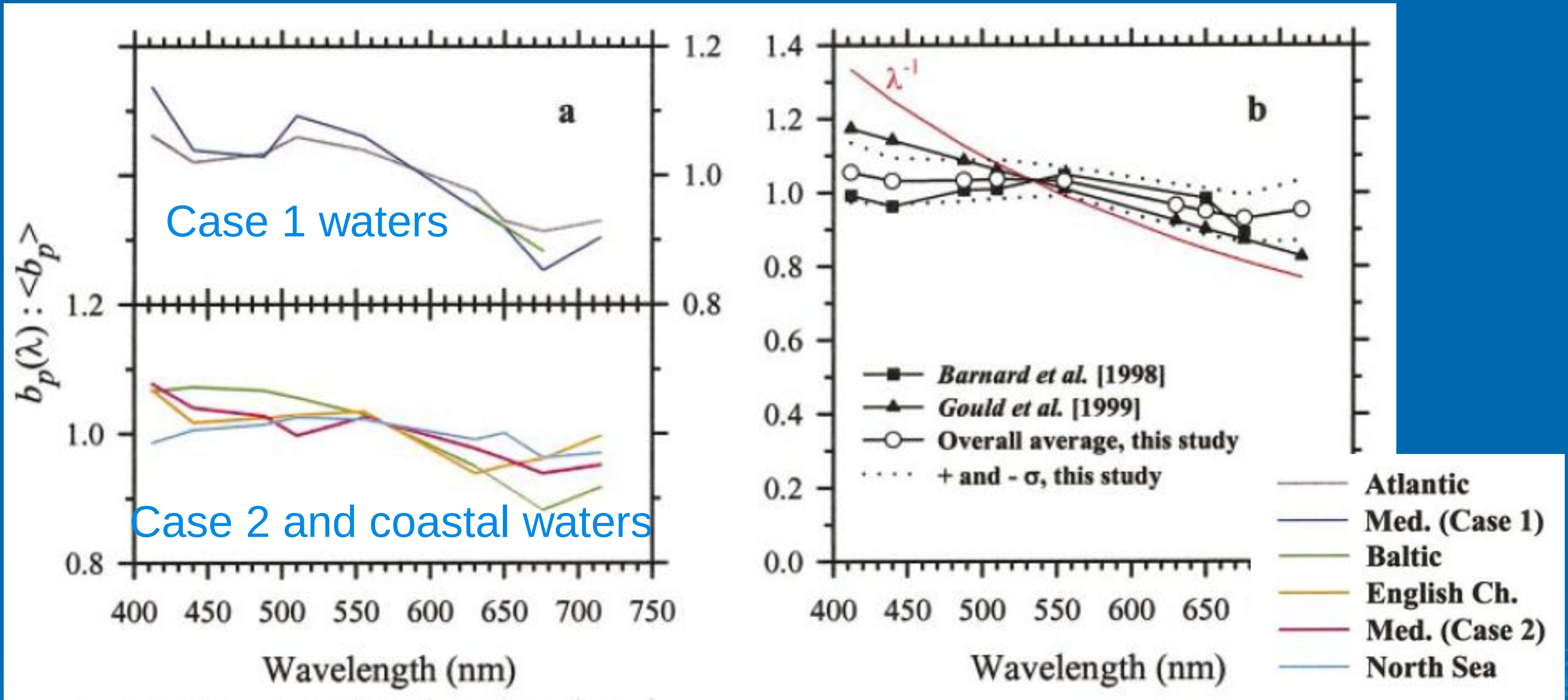
scattering coef spectrum

$b_w(\lambda) = 16.06\ \beta_w(\lambda_o,90^o)\ (\lambda/\lambda_o)^{-4.32}$

water volume scattering function

$\beta_w(\lambda,\psi) = b_w(\lambda_o,90^o)\ (\lambda/\lambda_o)^{-4.32} * (1+0.835\cos^2\psi)$

Wavelength Dependence of Scattering by Particles (Phytoplankton and NAP)

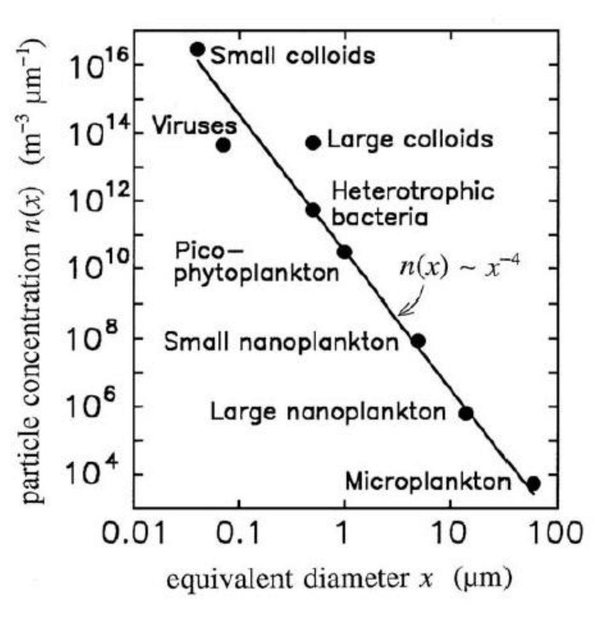


Babin et al. 2003, *Limnol. Oceanogr*. 48(2), 843-859

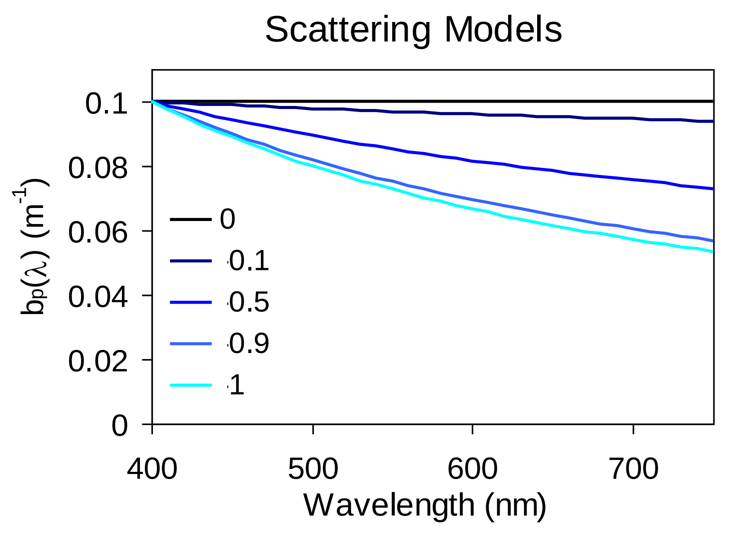
Models for Scattering by Particles

Historically, scattering was hard to measure, so scattering often was modeled using Mie theory (which is exact only for homogeneous spheres) and a Junge size distribution, which gives a power law:

$$b_p(\lambda) = b_p(\lambda_o)\left(\frac{\lambda}{\lambda_o}\right)^{-n}$$



n = 0 to 1, depending on the size distribution (large particles have a small n, small particles have a large n)



$b = c - a$

What do we know about c and a?

Models for Scattering by Phytoplankton and NAP

A power law gives a better fit to beam attenuation than to scattering

$$c_p(\lambda) = c_p(\lambda_o)\left(\frac{\lambda}{\lambda_o}\right)^{-n}, \quad \textit{Boss et al.}\ 2001$$

so get *b* from *c* - *a*

$$b_{NAP}(\lambda) = c_{NAP}(\lambda_o)\left(\frac{\lambda}{\lambda_o}\right)^{-n} - a_{NAP}(\lambda)$$

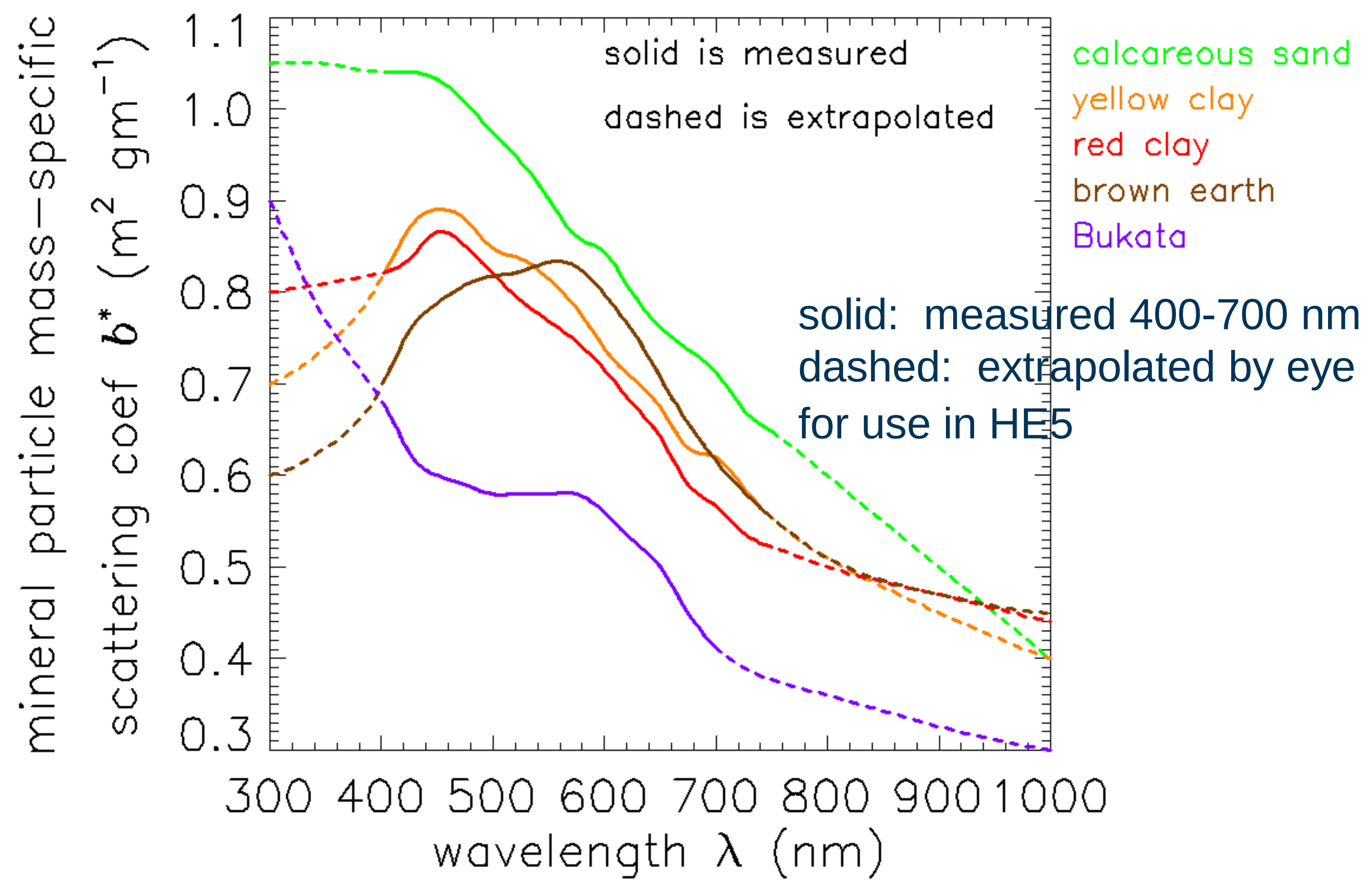
a smoothly varying function of wavelength

$$b_\phi(\lambda) = c_\phi(\lambda_o)\left(\frac{\lambda}{\lambda_o}\right)^{-n} - a_\phi(\lambda)$$

not so smoothly varying function

Scattering by Minerals

(measured and extrapolated spectra used in HE5)



solid: measured 400-700 nm
dashed: extrapolated by eye
for use in HE5

Ahn, PhD dissertation, 1999; Bukata, 1995

Models for Backscattering by Phytoplankton and NAP

Mie theory shows that particle backscattering has same spectral shape as scattering (approximately true for nonspherical, inhomogeneous particles). Therefore the backscatter fraction $B_p = b_{bp}/b_p$ is often assumed to be independent of wavelength.

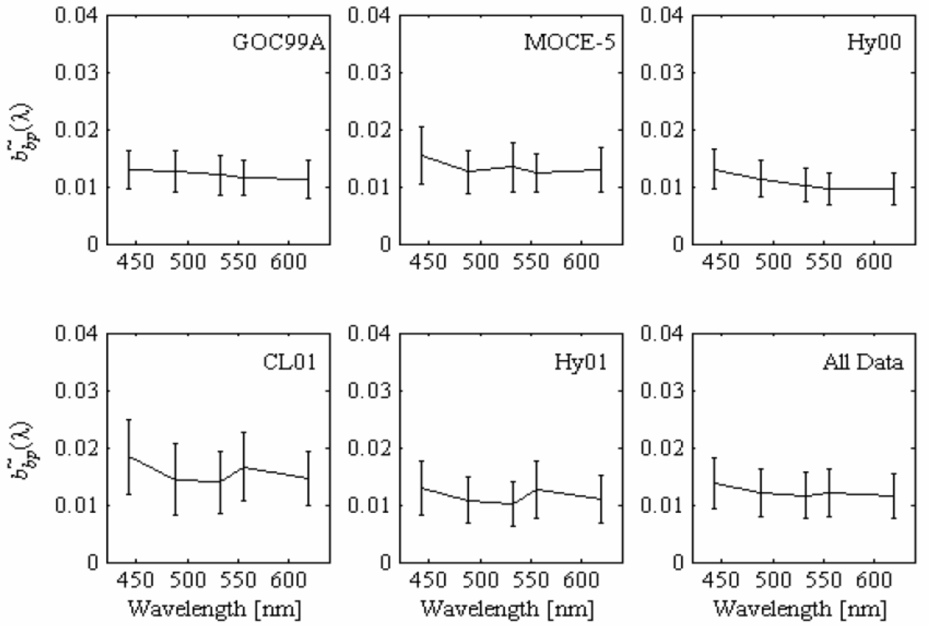


Fig. 3. Mean particulate backscattering ratio spectra for individual data sets, with one standard deviation shown for each data point. The spectra are as follows: GOC99A (top, left), MOCE-5 (top, middle), HyCODE 2000 (top, right), Crater Lake 2001 (bottom, left), HyCODE 2001 (bottom, middle), and the mean of all of the data sets (bottom, right).

Whitmire et al, Optics Express, 2007

Models for Backscattering by Phytoplankton and NAP

So we end up with…

$$b_{bp}(\lambda) = \boxed{\frac{b_{bp}}{b_p}} \underbrace{\left(c_p(\lambda_o) \left(\frac{\lambda}{\lambda_o} \right)^{-n} - a_p(\lambda) \right)}_{\text{model or data for } b_p(\lambda)}, \quad \mathit{Roesler\ and\ Boss}\ 2003$$

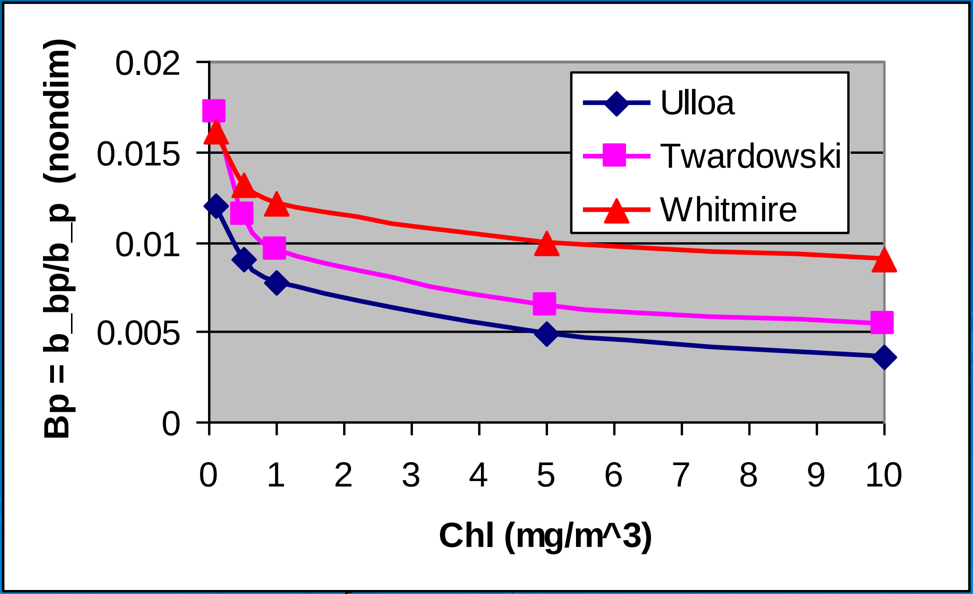
model $B_p = b_{bp}/b_p$ (often assume independent of λ).

Models for Backscattering by Phytoplankton and NAP

Various people have published simple models for B_p as a function of Chl, e.g.

$B_p = 0.01[0.78 - 0.42 \log_{10} Chl]$ (Ulloa, et al, 1994)
$B_p = 0.0096\, Chl^{-0.253}$ (Twardowski et al., JGR, 2001, Case 1 water)
$B_p(555\text{ nm}) = 0.0121 Chl^{-0.125}$ (Whitmire et al., Opt. Exp, 2007)

The predictions vary widely because
* the models are fits to different data sets
* scattering does not correlate well with *Chl*



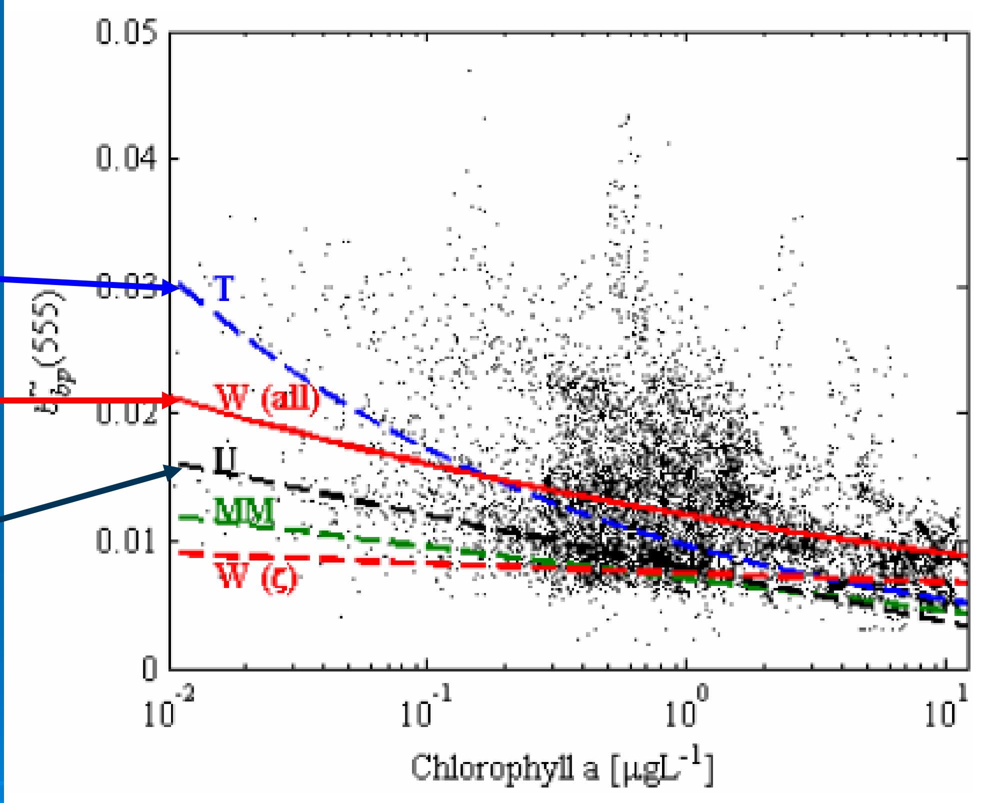
Models for Backscattering by Phytoplankton and NAP

Although there are several “best fit” models for B_p, the variability in B_p vs *Chl* makes them almost useless, even in Case 1 waters.

$B_p = 0.0096\ Chl^{-0.253}$

$B_p(555\text{ nm}) = 0.0121 Chl^{-0.125}$

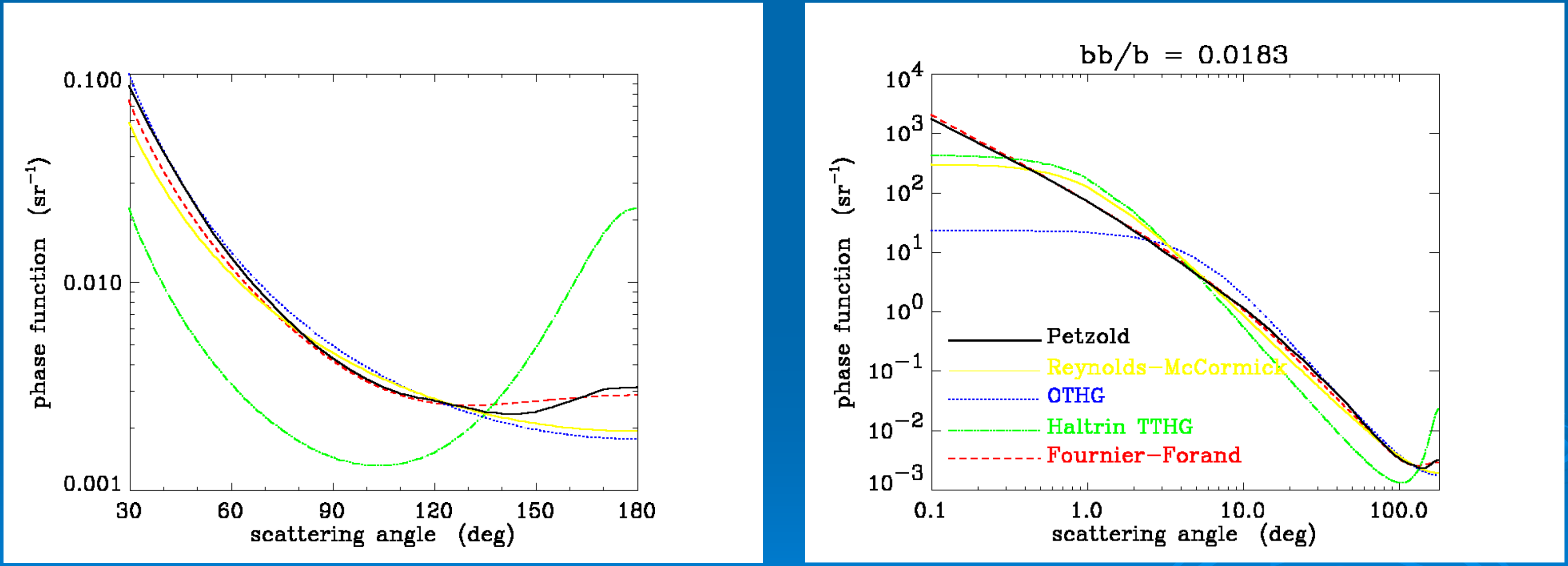
$B_p = 0.01[0.78 - 0.42 \log10 Chl]$



Whitmire et al., Opt. Exp, 2007

Analytic Models for Phase Functions

There are many analytic phase function models. Most of these were developed for non-oceanographic studies (atmospheric optics, astronomy, etc.). Although the shapes are roughly like ocean phase functions, there are usually large differences at very small and/or large scattering angles.

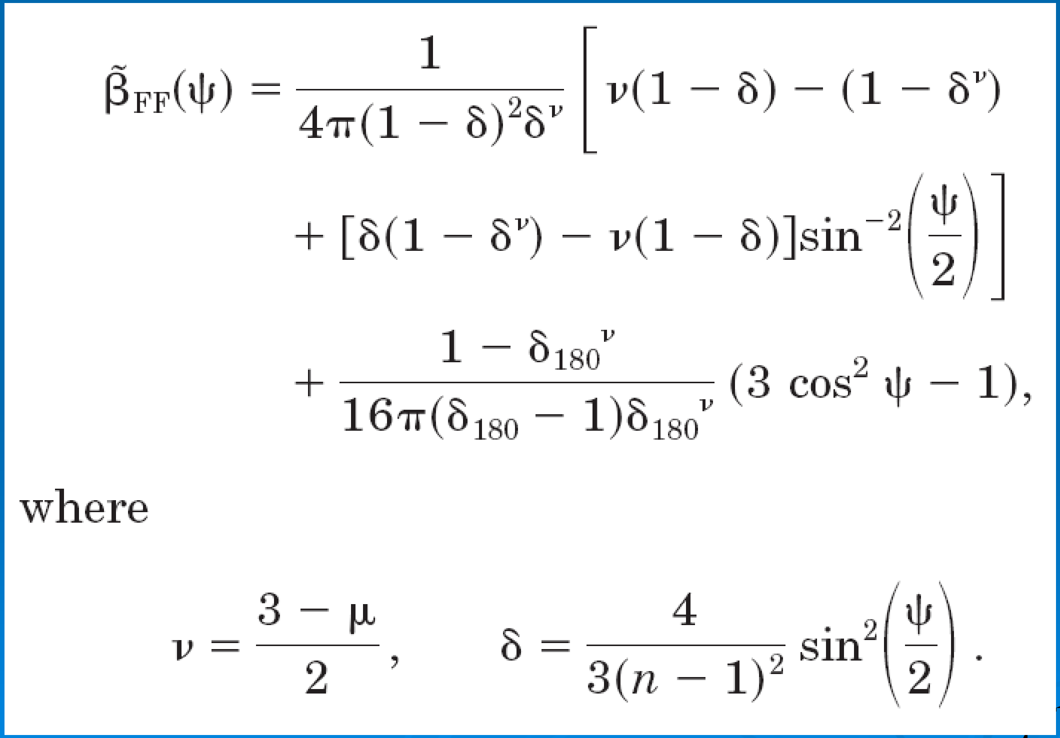


Petzold is measured. The others are analytic models. Only the Fournier-Forand phase function does a good job of matching Petzold over all scattering angles.

The Fournier-Forand Phase Function

Derived from Mie theory

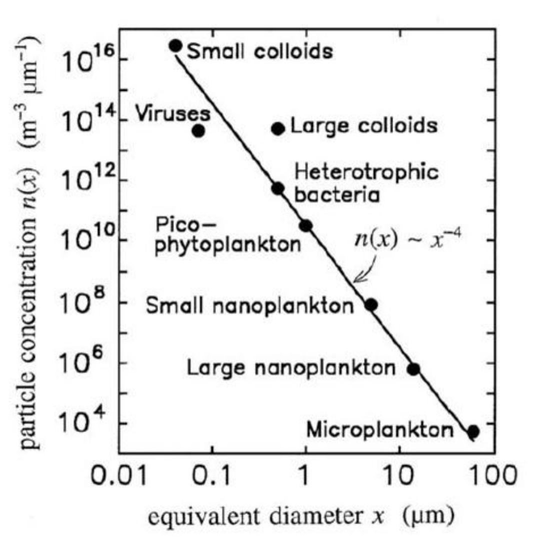
- homogeneous spheres with real refractive index, *n*
- hyperbolic (Junge) size distribution with slope, -μ
- integrate over particles sizes from 0 to infinity



$$\tilde{\beta}_{\mathrm{FF}}(\psi) = \frac{1}{4\pi(1-\delta)^2\delta^\nu}\left[\nu(1-\delta)-(1-\delta^\nu) + [\delta(1-\delta^\nu)-\nu(1-\delta)]\sin^{-2}\left(\frac{\psi}{2}\right)\right] + \frac{1-\delta_{180}{}^\nu}{16\pi(\delta_{180}-1)\delta_{180}{}^\nu}(3\cos^2\psi-1),$$

where

$$\nu = \frac{3-\mu}{2}, \qquad \delta = \frac{4}{3(n-1)^2}\sin^2\left(\frac{\psi}{2}\right).$$



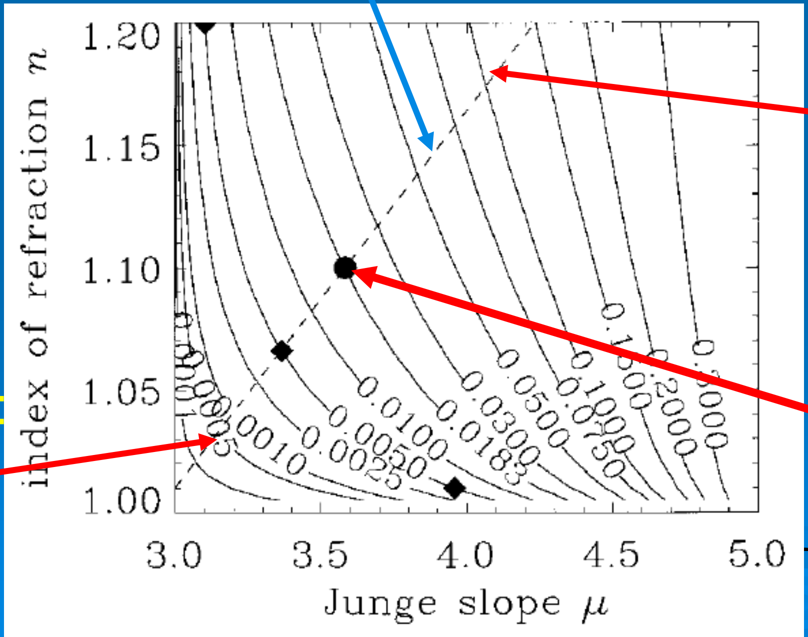
from Mobley et al., 2002

The Fournier-Forand Phase Function

n and μ can be related to the backscatter fraction B_b

$$B_p = 1 - \frac{1 - \delta_{90}{}^{\nu+1} - 0.5(1 - \delta_{90}{}^{\nu})}{(1 - \delta_{90})\delta_{90}{}^{\nu}}$$

When selecting a F-F pf by the backscatter fraction, H uses values along the dotted line



Mobley et al., 2002

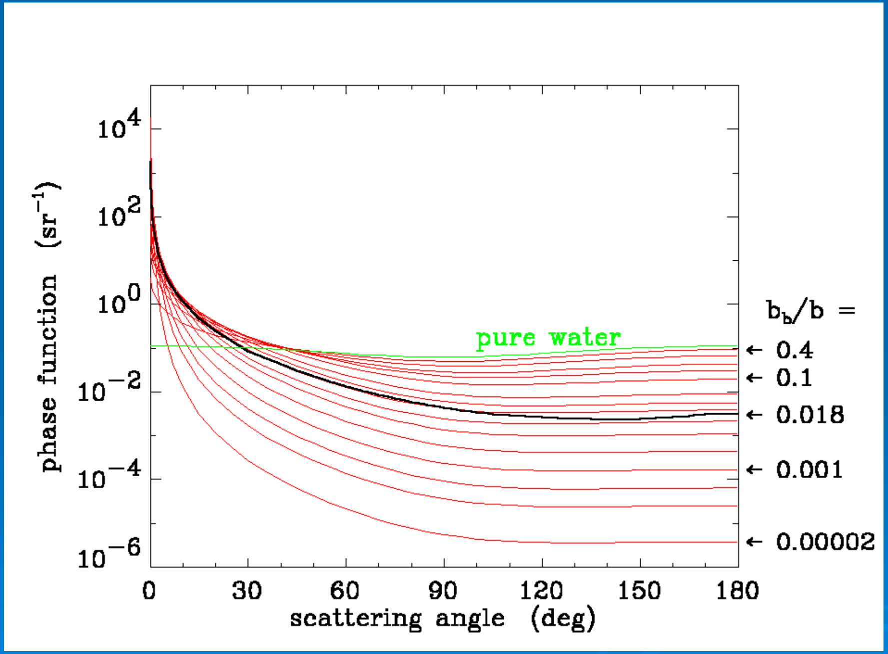
minerals:
$n > 1.15$
large B_p

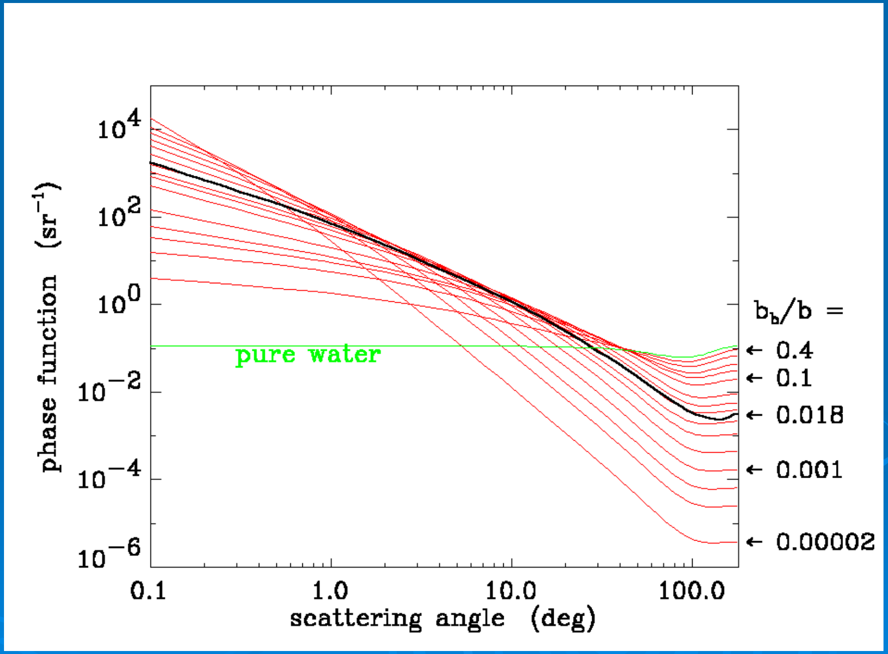
Petzold "turbid harbor", probably a mixture of phytoplankton and minerals

phytoplankton:
$n < 1.05$
small B_p

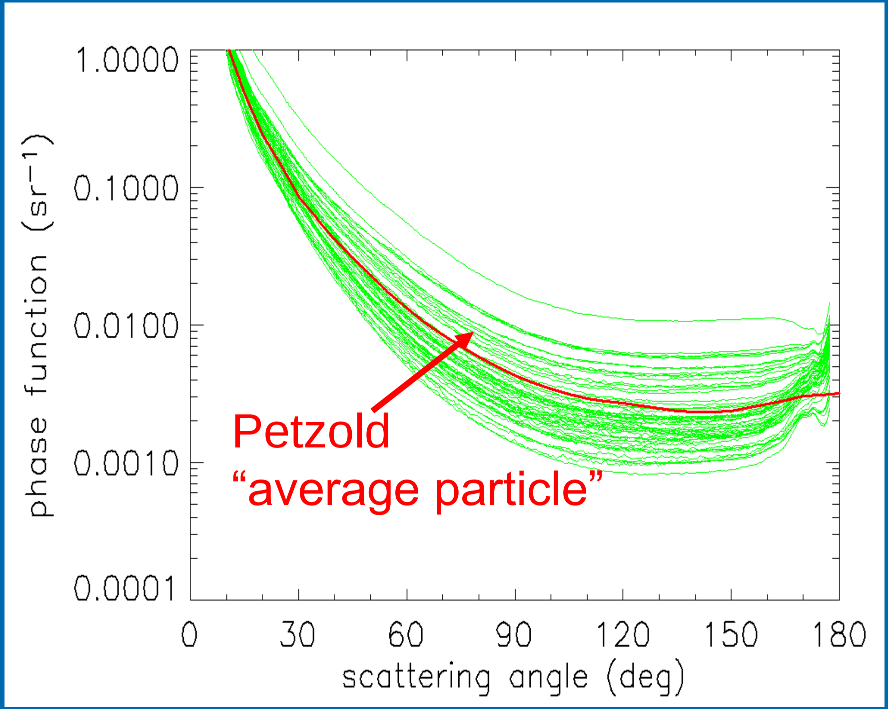
The Fournier-Forand Phase Function

The HydroLight database has a large number of Fournier-Forand phase functions for various backscatter fractions b_b/b. These are interpolated to get the F-F pf for any value of b_b/b, to model any particular component.

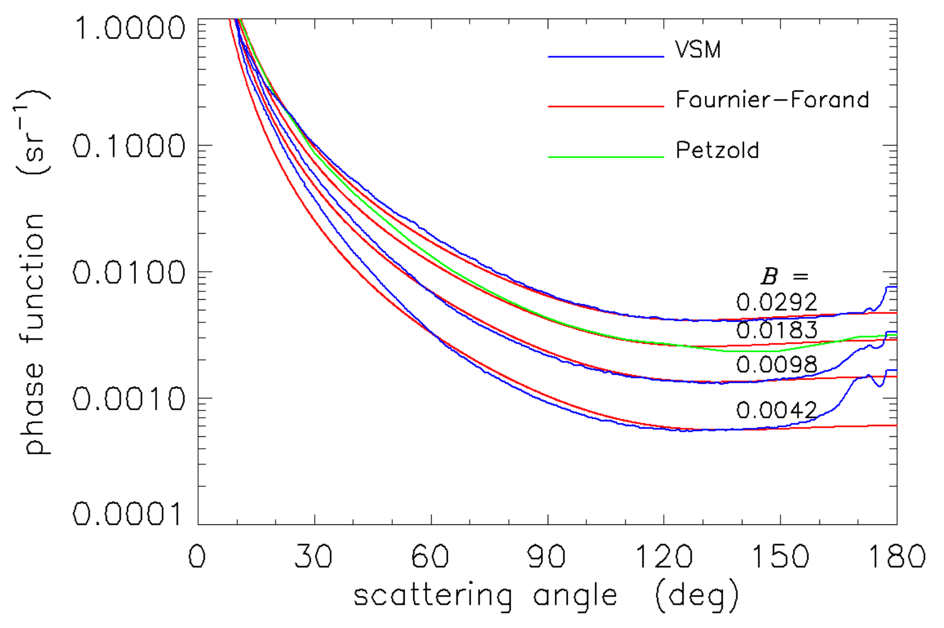




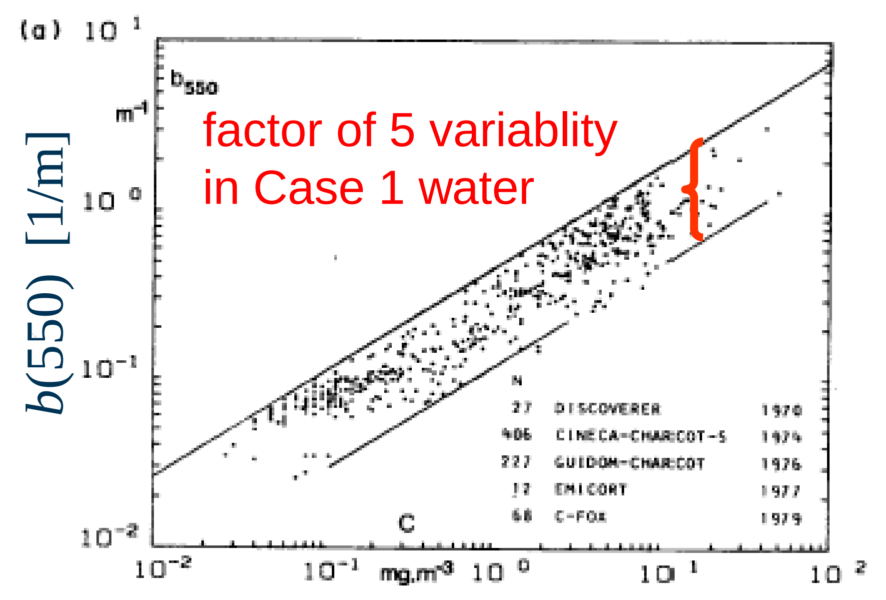
The Fournier-Forand Phase Function



The FF phase function does an adquate job (at least for my purposes) of fitting most measured phase functions



Example: Scattering as a Function of *Chl*

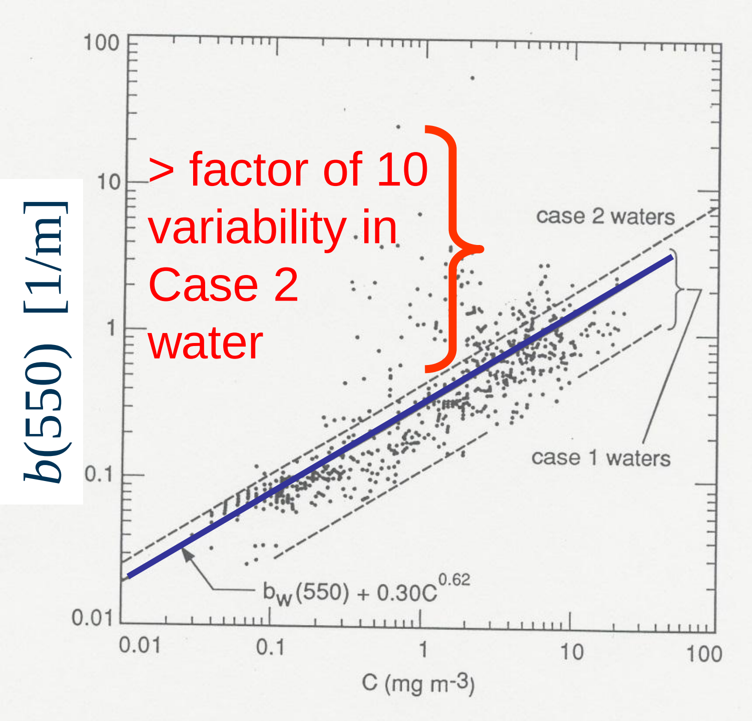


factor of 5 variablity in Case 1 water

$b(550)$ [1/m]

Chl [mg m^{-3}]

Morel 1987, DSR



> factor of 10 variability in Case 2 water

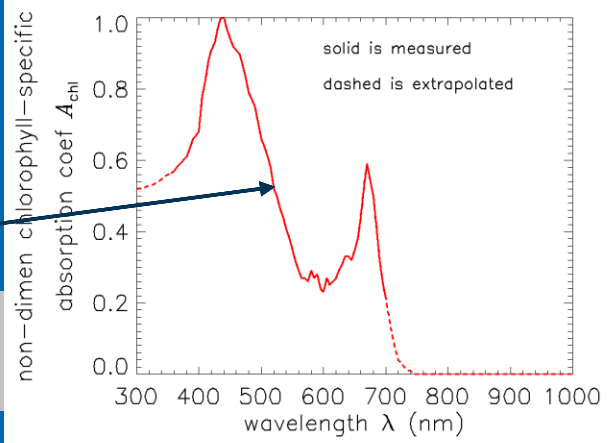
$b(550)$ [1/m]

Chl [mg m^{-3}]

The “classic” Case 1 model for scattering (Gordon and Morel, 1983) just fits a straight line through these data: $b(550) = 0.30 Chl^{0.62}$. This may be good *on average*, but can be very inaccurate for a particular water body! Scattering does not correlate well with Chl, even in Case 1 water. Why?

The “Classic” Case 1 IOP Model in HE5

Pick the pure water spectrum (usually Pope and Fry data). Then particle absorption and scattering are given by



$$a_p(z,\lambda) = 0.06\, \overbrace{a_c^{*\prime}(\lambda)}\, Chl(z)^{0.65}$$

$$a_{CDOM}(z,\lambda) = 0.2\,[a_w(440) + a_p(z,440)]\exp[-0.014\,(\lambda - 440)]$$

$$b_p(z,\lambda) = 0.30\, Chl(z)^{0.62}\left(\frac{550}{\lambda}\right)$$

The user then picks a particle phase function, e.g. a Fournier-Forand pf with a given backscatter fraction B_p. For guidance, can use one of the simple B_p models, but may be very inaccurate.

The “New” Case 1 IOP Model in HE5

Based on papers by Bricaud et al. (1998) for absorption and Morel et al. (2002) for scattering. For details, see www.oceanopticsbook.info/view/optical_constituents_of_the_ocean/__level_2/a_new_iop_model_for_case_1_water

All IOPs are determined by the *Chl* value.

$$a_p(z,\lambda) = A_p(\lambda)\, Chl(z)^{E_p(\lambda)}$$

a_p^* shape helps describe pigment packaging. $A_p(\lambda)$ and $E_p(\lambda)$ are tabulated.

$$b_p(z,\lambda) = 0.416\, Chl(z)^{0.766}\left(\frac{\lambda}{550}\right)^{v}$$

λ dependence of b_p now depends on *Chl*

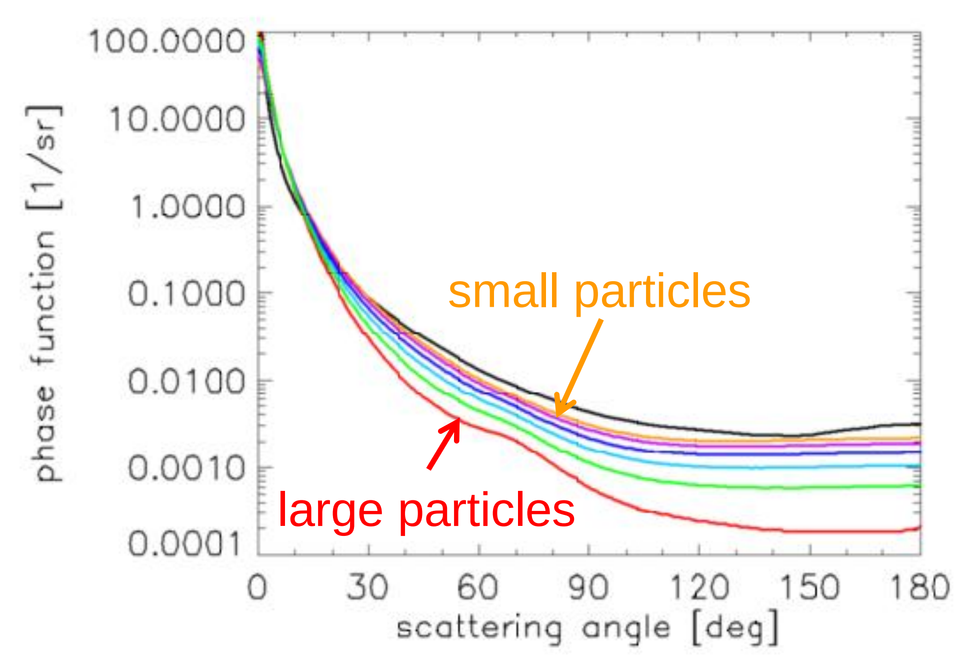
$$\begin{aligned} v &= 0.5\left[\log_{10} Chl(z) - 0.3\right] \quad \text{for } 0.02 \le \text{Chl} < 2 \\ &= 0 \quad \text{for Chl} \ge 2 \end{aligned}$$

The “New” Case 1 IOP Model in HE5

The particle phase function is now determined by the *Chl* value:

$$\tilde{\beta}_p(\psi, Chl) = \alpha_s(Chl)\,\tilde{\beta}_{small}(\psi) + (1 - \alpha_s)\,\tilde{\beta}_{large}(\psi)$$

$$\alpha_s = 0.855\left[0.5 - 0.25\log_{10} Chl\right]$$



Phase functions for small (orange) and large (red) particles as given by Morel et al. (2002). Phase functions for *Chl* = 0.01 (purple), 0.1 (blue), 1.0 (teal), and 10.0 (green), and the Petzold average particle phase function (black) are shown.

Never Forget ...

All IOPs are extremely variable, even for a particular component like phytoplankton or mineral particles. There is no “phytoplankton absorption spectrum,” and it’s even worse for scattering.

Every phytoplankton species, and every nutrient condition and light adaptation condition for a given species, has different absorption and scattering spectra. The same is true for minerals, CDOM, etc.

This variability makes it extremely hard to model IOPs, and extremely hard to know what IOPs to use as input to HydroLight, unless you measured them (which is impossible to do for every situation). Models are always approximate. They can be good on average, but terrible in any specific case.

When HydroLight gives the “wrong answer,” it is almost always because the input IOPs do not correspond to the IOPs of the water body being simulated. Garbage in, garbage out.

Never Forget ...

When using any model for IOPs, think about:

- What data were used to develop the model?
- Global relationships are not appropriate regionally
- Regional models are not valid elsewhere (e.g., a model based on North Atlantic data can't be applied to the south Pacific)
- Models based on near-surface data cannot be applied at depth
- Models based on open-ocean data cannot be applied to coastal waters
- Was the model developed to use satellite-retrieved Chl to recover IOPs?
- Where was the division between Case I and II in the underlying data?

When using any model, always think "maybe good for average or typical values, but maybe terrible for my water body."

There are No Perfect IOP Models, but There is a Perfect Building

The Hall of Prayer for Good Harvests at the Temple of Heaven, Beijing. Photo by Curt Mobley.