Analytical solutions for light interaction with a sphere

Definitions: $x \equiv \pi D/\lambda$-size parameter
$m=n+in'$-index of refraction relative to medium
$\rho \equiv 2x(n-1)$-phase lag suffered by ray crossing the sphere along its diameter
$\rho' \equiv 4xn'$-optical thickness corresponding to absorption along the diameter
$\beta \equiv \tan^{-1}(n'/(n-1))$
D- Diameter
λ-wavelength in medium (=wavelength in vacuum/index of refraction of medium relative to vacuum)

Rayleigh regime: $x<<1$ and $|m|x<<1$
$Q_a=4x\ \mathrm{Im}\{(m^2-1)/(m^2+2)\}$ note: proportional to λ^{-1}
$Q_b=8/3\ x^4\ |(m^2-1)/(m^2+2)|^2$ note: proportional to λ^{-4}
$Q_c= Q_a+ Q_b$
$Q_{bb}= Q_b/2$
Phase function: $<\beta>=0.75(1+\cos^2\theta)$

Rayleigh-Gans regime: $|m-1|<<1$ and $\rho<<1$
$Q_a=8/3\ x\ \mathrm{Im}\{(m-1)\}$ note: proportional to λ^{-1}
$Q_b=|m^2-1|\ [2.5+2*x^2-\sin(4x)/4x-7/16(1-\cos(4x))/x^2+(1/(2x^2)-2)\{\gamma+\log(4x)-Ci(4x)\}]$,
where $\gamma=0.577$ and $C_i(x)=-\int_x^\infty \frac{\cos(u)}{u}du$
$Q_c= Q_a+ Q_b$
For $x<<1$: $Q_b=32/27\ x^4\ |m-1|^2$, $Q_{bb}= Q_b/2$
For $x>>1$: $Q_b=2\ x^2\ |m-1|^2$, $Q_{bb}= 0.31|m-1|^2$

Anomalous diffraction (VDH): $x>>1$, $|m-1|<<1$ (ρ can be $>>1$)
$Q_c=2-4\exp(-\rho\tan\beta)[\cos(\beta)\sin(\rho-\beta)/\rho+(\cos\beta/\rho)^2\cos(\rho-2\beta)]+4(\cos\beta/\rho)^2\cos2\beta$
$Q_a=1+2\exp(-\rho')/\rho'+2(\exp(-\rho')-1)/\rho'^2$
$Q_b= Q_c-Q_a$

Geometric optic: $x>>>1$
$Q_c=2$
Absorbing particle: $Q_b=1$, $Q_a=1$
Exactly Non-absorbing particle: $Q_b=2$, $Q_a=0$
Angular scattering cross section: $\widehat{\beta}_{diff}(\theta)=\frac{Gx^2}{16\pi}\left[\frac{2J_1(x\sin\theta)}{x\sin\theta}\right]^2(1+\cos\theta)^2$, where G is the cross sectional area ($\pi D^2/4$).