

UNIVERSITY
OF MIAMI



Polarization in Sky and water

At least an hours worth of it

Ken Voss, Ocean Optics Summer class, 2015

Polarization important as another parameter of the light field you can measure for additional information....or it may be a problem in your measurement. Simplest light field....plane wave

Can describe plane wave propagating in z direction as:

$$E_l(z,t) = a_l \cos(\omega t - kz + \delta_l)$$

$$E_r(z,t) = a_r \cos(\omega t - kz + \delta_r)$$

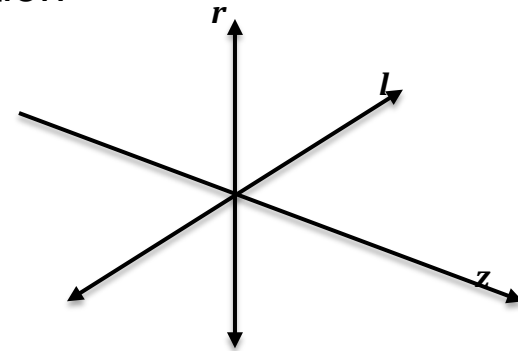
Let: $\delta = \delta_r - \delta_l$

Some simple cases can be seen:

$\delta=0$, (or a_l or $a_r=0$) light is linearly polarized

$\delta=\pi/2$, $a_l = a_r$ light is circularly polarized

Everything else is called elliptically polarized



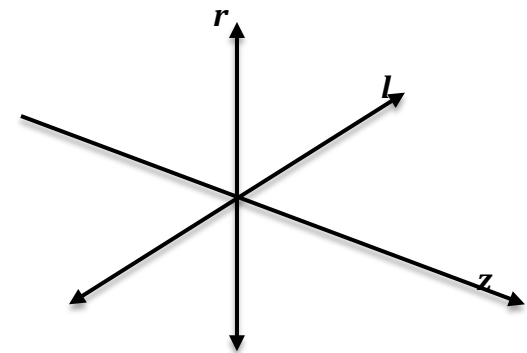
Now in general, ω , for visible light is on the order of 6×10^{14} Hz, we can't really measure these oscillations directly, in general. Also the natural light field is not a single plane wave, but a superposition of many waves. So we need a more conv. description....the Stokes vector.

$$I = \langle a_l^2 \rangle + \langle a_r^2 \rangle = \langle E_l E_l^* \rangle + \langle E_r E_r^* \rangle = I_l + I_r$$

$$Q = \langle a_l^2 \rangle - \langle a_r^2 \rangle = \langle E_l E_l^* \rangle - \langle E_r E_r^* \rangle = I_l - I_r$$

$$U = \langle 2a_l a_r \cos \delta \rangle = \langle E_l E_r^* \rangle + \langle E_r E_l^* \rangle = I_{45} - I_{-45}$$

$$V = \langle 2a_l a_r \sin \delta \rangle = i(\langle E_l E_r^* \rangle - \langle E_r E_l^* \rangle) = I_{RCP} - I_{LCP}$$



Some Simple Stokes Vectors:

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Unpolarized light

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ I \\ 0 \\ 0 \end{bmatrix}$$

Polarized along l

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ -I \\ 0 \\ 0 \end{bmatrix}$$

Polarized along r

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ 0 \\ I \end{bmatrix}$$

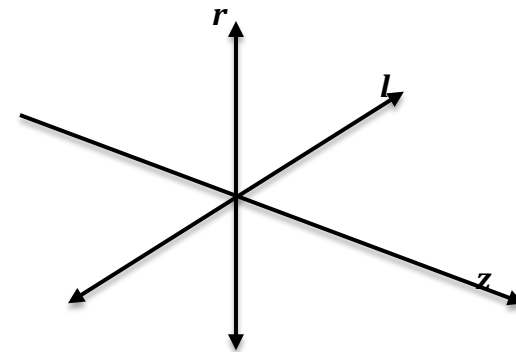
Right Circularly polarized light

$$I = I_l + I_r$$

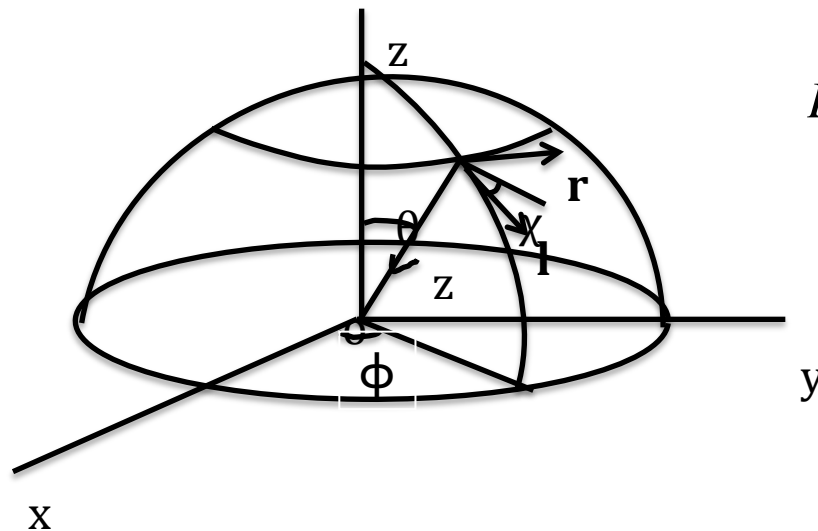
$$Q = I_l - I_r$$

$$U = I_{45} - I_{-45}$$

$$V = I_{RCP} - I_{LCP}$$



Can't figure out where to put this slide...so here it is....



Degree of polarization

$$DoP = \frac{(Q^2 + U^2 + V^2)^{1/2}}{I}, \quad 0 \leq DoP \leq 1$$

Plane of polarization

$$\tan 2\chi = \frac{U}{Q}, \quad -90^0 \leq \chi \leq 90^0$$

For completeness, ellipticity

$$\sin 2\beta = \frac{2a_l a_r \sin \delta}{a_l^2 + a_r^2} = \frac{V}{(Q^2 + U^2 + V^2)^{1/2}}$$

Can transform these Stokes vectors in a linear process, using Mueller Matrices:

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

Modify incoming Stokes Vector to outgoing Stokes Vector.

Examples, linear polarizer

$$\frac{1}{2} \begin{bmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{I+Q}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} I+Q \\ I+Q \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

Two crossed polarizers

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = 1/2 \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} 1/2 \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0 = 1/4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 1/4 \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ I \\ 0 \\ 0 \end{bmatrix}$$

Two crossed polarizers, with
another one inbetween

$$\frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{8} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ I \\ 0 \\ 0 \end{bmatrix}$$

Two crossed polarizers, with
another one in between continued

$$= 1/8 \begin{bmatrix} I \\ -I \\ 0 \\ 0 \end{bmatrix} = 1/8 \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ 0 \\ I \\ 0 \end{bmatrix}$$

How about a circular polarizer?

- Need a quarter wave plate after a polarizer

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = 1/2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} 1/2 \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = 1/4 \begin{bmatrix} I \\ 0 \\ 0 \\ I \end{bmatrix} = 1/4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I \\ 0 \\ I \\ 0 \end{bmatrix}$$

To determine the Stokes vectors, need at least 4 carefully determined measurements. To determine Mueller matrix, need 16 measurements:

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

Typically, normalize by M_{11} (which is normal light scattering volume scattering function). This emphasizes the polarization properties of the matrix.

Instruments to measure the Stokes Vector are variations on radiance instruments.

Two types, either single direction at a time (such as 4 co-bore sighted Gershun tubes or 3, since the circular polarization is very small or zero for the most part) or some sort of imaging device, such as fisheye camera with polarizers.

Example Sky instrument

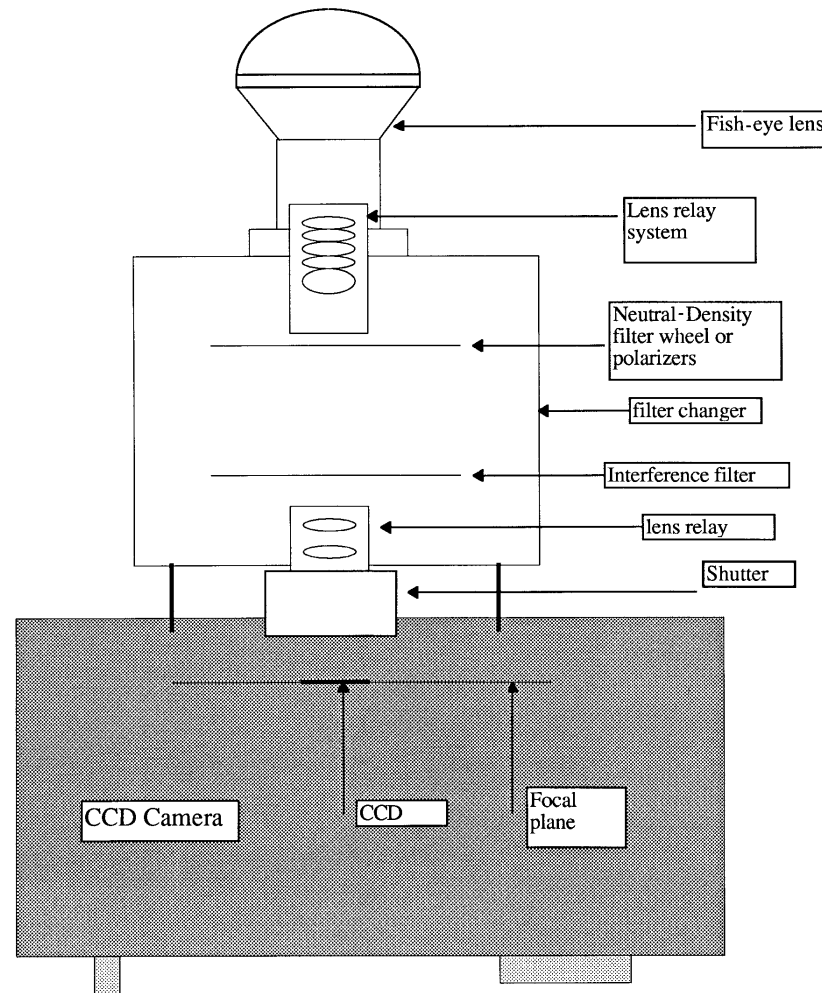
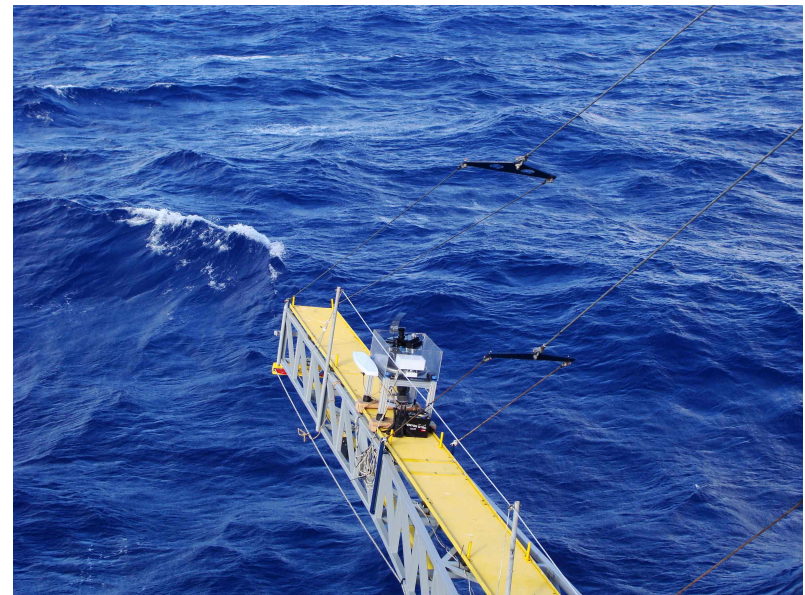
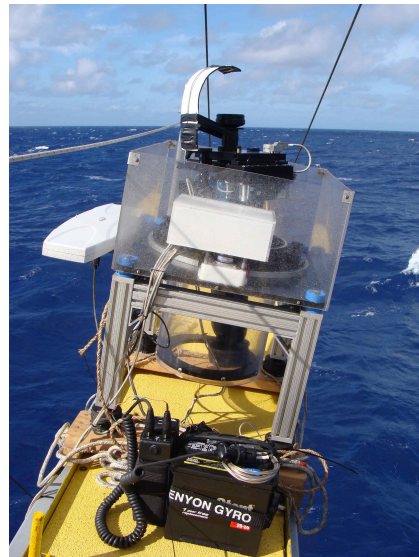


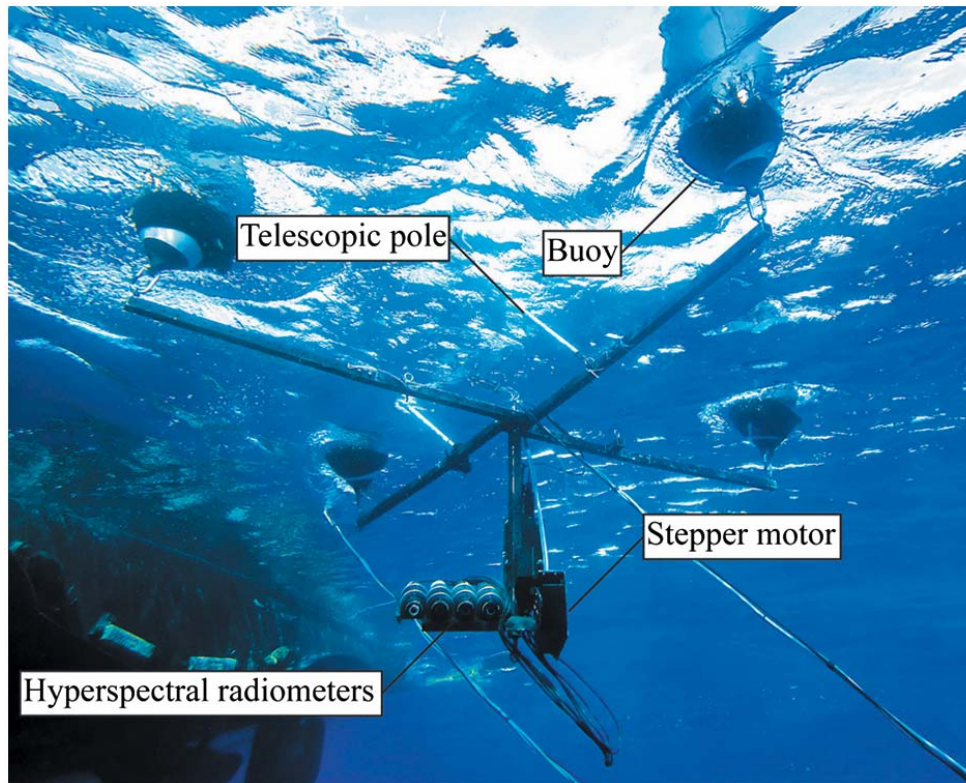
Fig. 1. Block diagram of the RADS-IIP instrument.

Instruments

- Sky scene relatively stable, hence time sequence can be used
 - Example K. J. Voss and Y. Liu, “Polarized radiance distribution measurements of skylight: I. system description and characterization”, 1997, Applied Optics, **36** :6083-6094.
- Must have stabilization to use on ships/moving platforms: Adapt system used by filming crews



In-water...need simultaneous measurements...either with Gershun tube radiometers fitted with polarizers



Estimating particle composition and size distribution from polarized water-leaving radiance, Alberto Tonizzo,1,* Alex Gilerson,1 Tristan Harmel,1 Amir Ibrahim,Jacek Chowdhary, Barry Gross,Fred Moshary, and Sam Ahmed1, Applied Optics, Vol 50, 5047-5058

Or simultaneous fisheye systems, with polarizers:

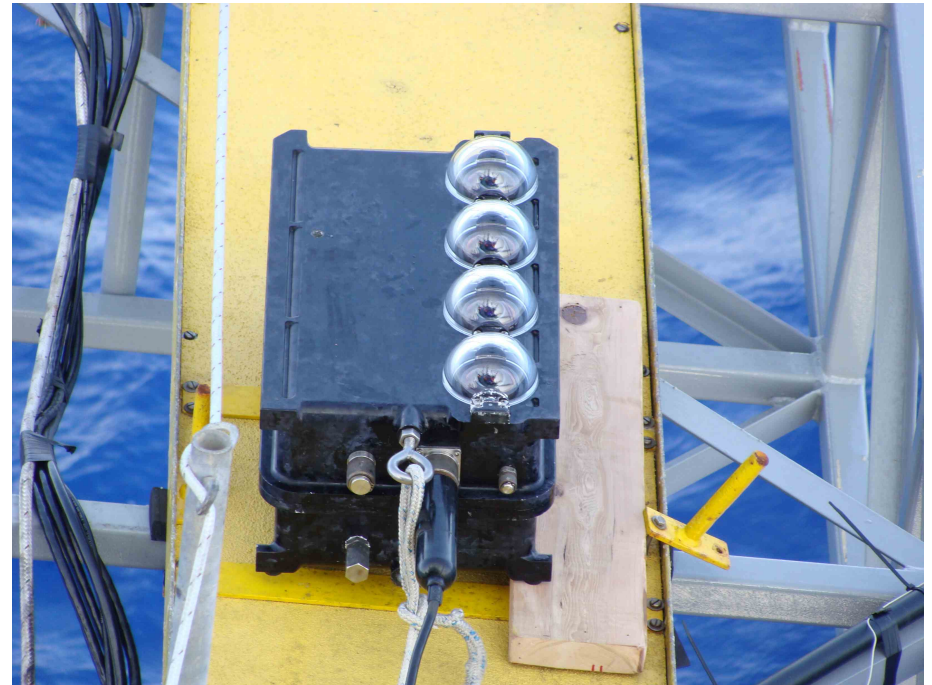
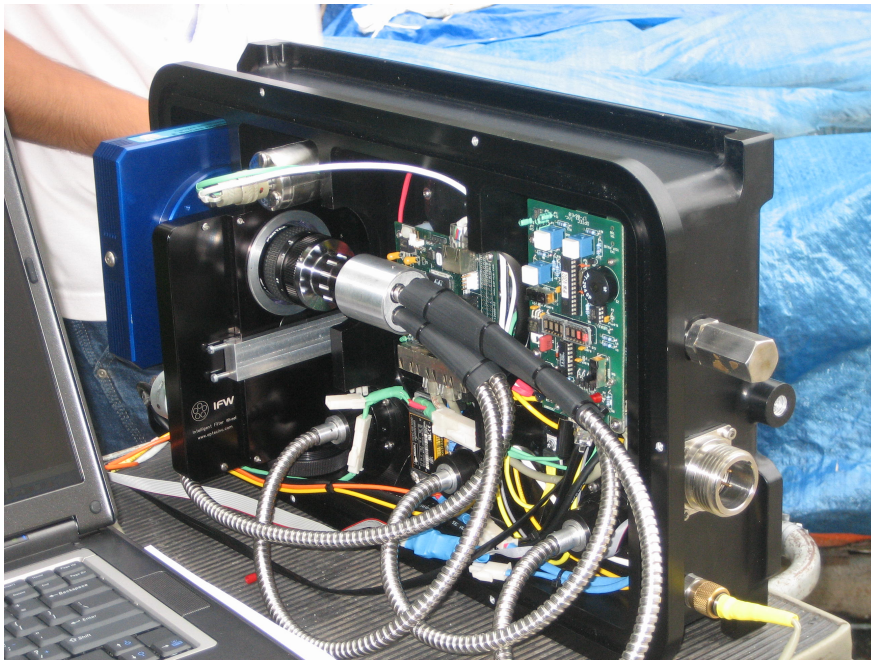


{Polrads

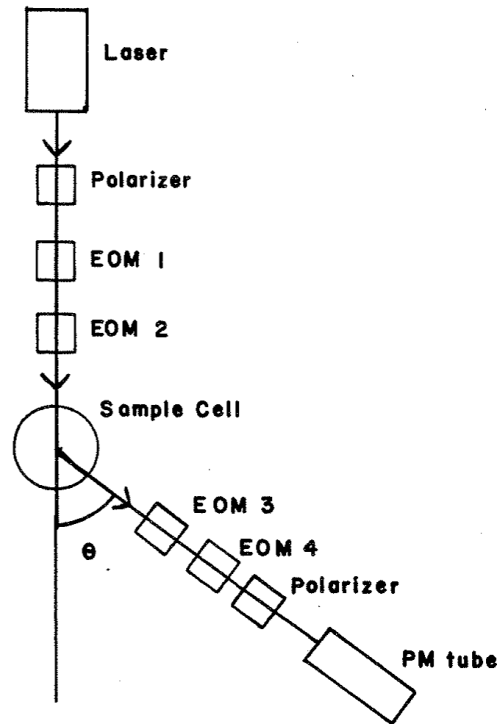
Instruments

DPOL instrument:

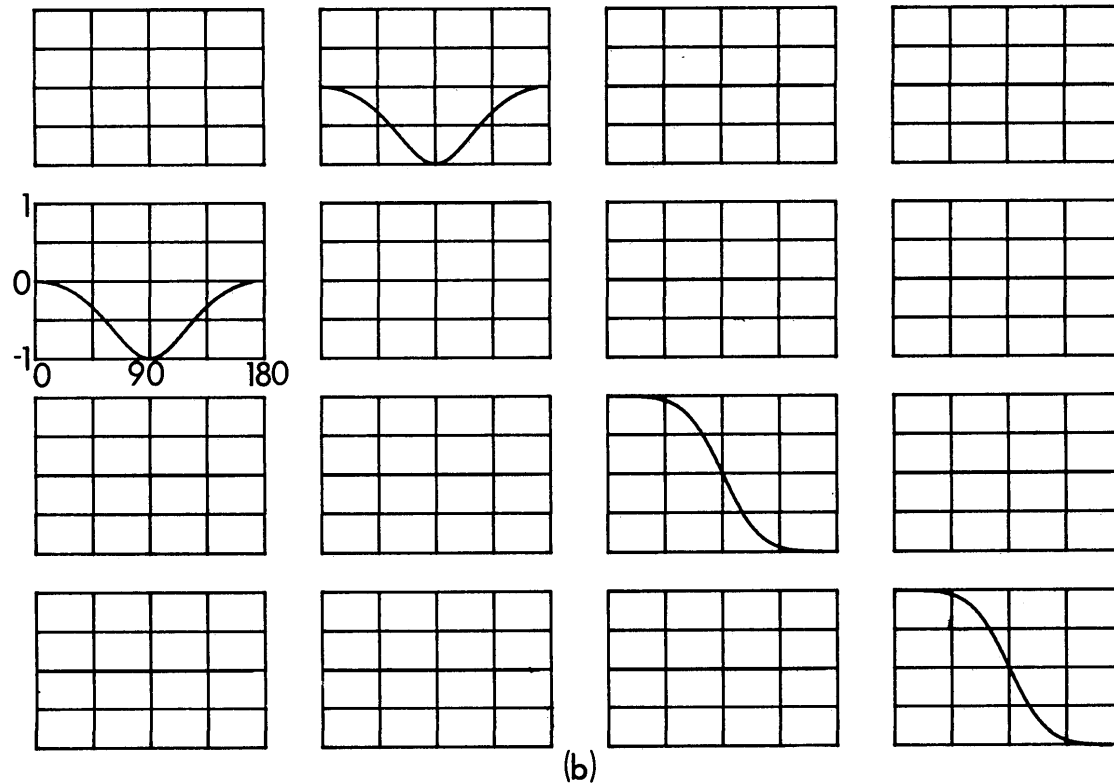
- System has 4 lenses, polarizer's in each
- Fiber Optic bundle collects light from each image into one super image
- Super image focused onto camera through spectral filter changer



To measure the Mueller matrix need 16 measurements (set of 4 different input Stokes vectors permuted with 4 polarization analyzers). Either do this serially or another method was:



Mueller matrix for Rayleigh scattering.



Low index particles also scatter this way (Rayleigh-Gans approx.)

Mueller matrix for Rayleigh scattering. Lets look at some specific angles:

$$M(0^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M(90^\circ) = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M(180^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

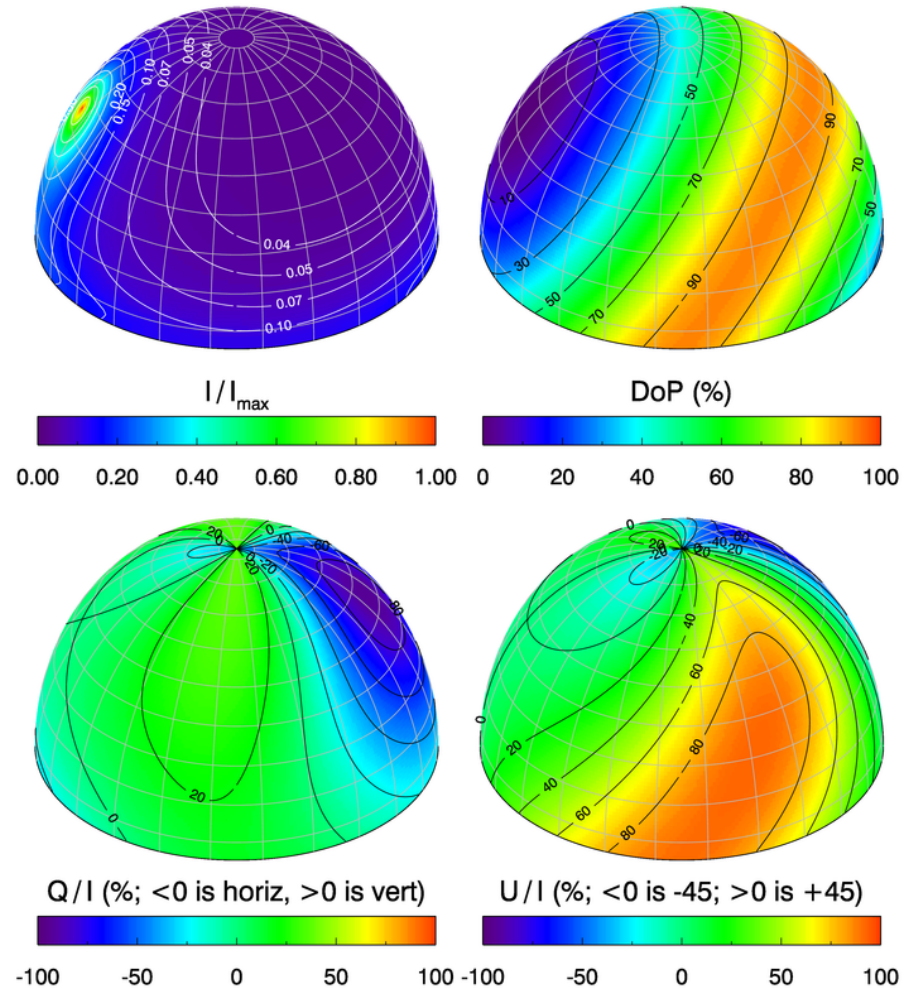
What do these do?

Case for Rayleigh single scattering in Sky,

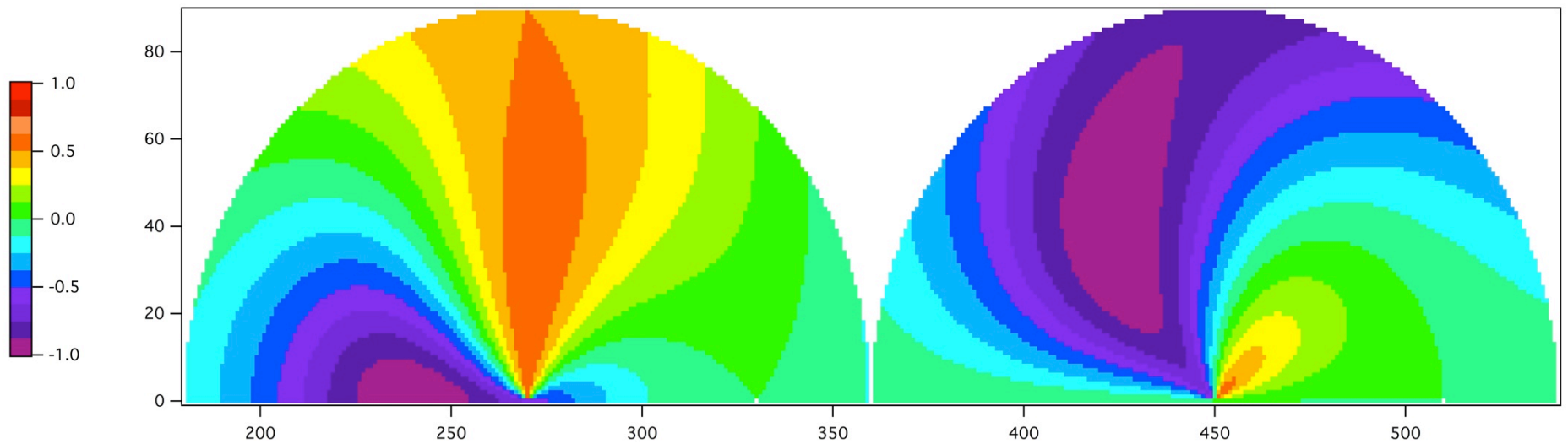
Single-scattering Rayleigh Sky

$$(\theta_{\text{sun}}, \varphi_{\text{sun}}) = (50.0, 0)$$

From Curt



Case for Rayleigh single scattering in Sky.



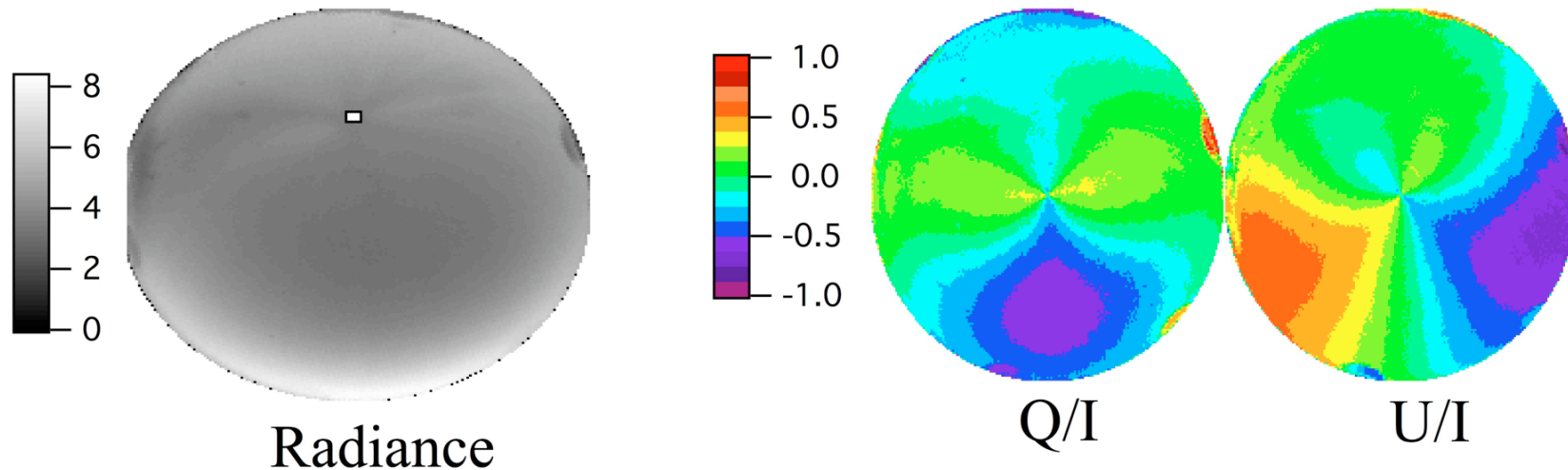
Q/I

U/I

V/I = 0

60 degree zenith angle

Example in-water upwelling images



The radiance units are $mW\ cm^{-2}\ nm^{-1}\ sr^{-1}$, Q/I and U/I are dimensionless

Hawaii (December 2, 2005, 20.83° N, 157.18° W, 10:25 local time).

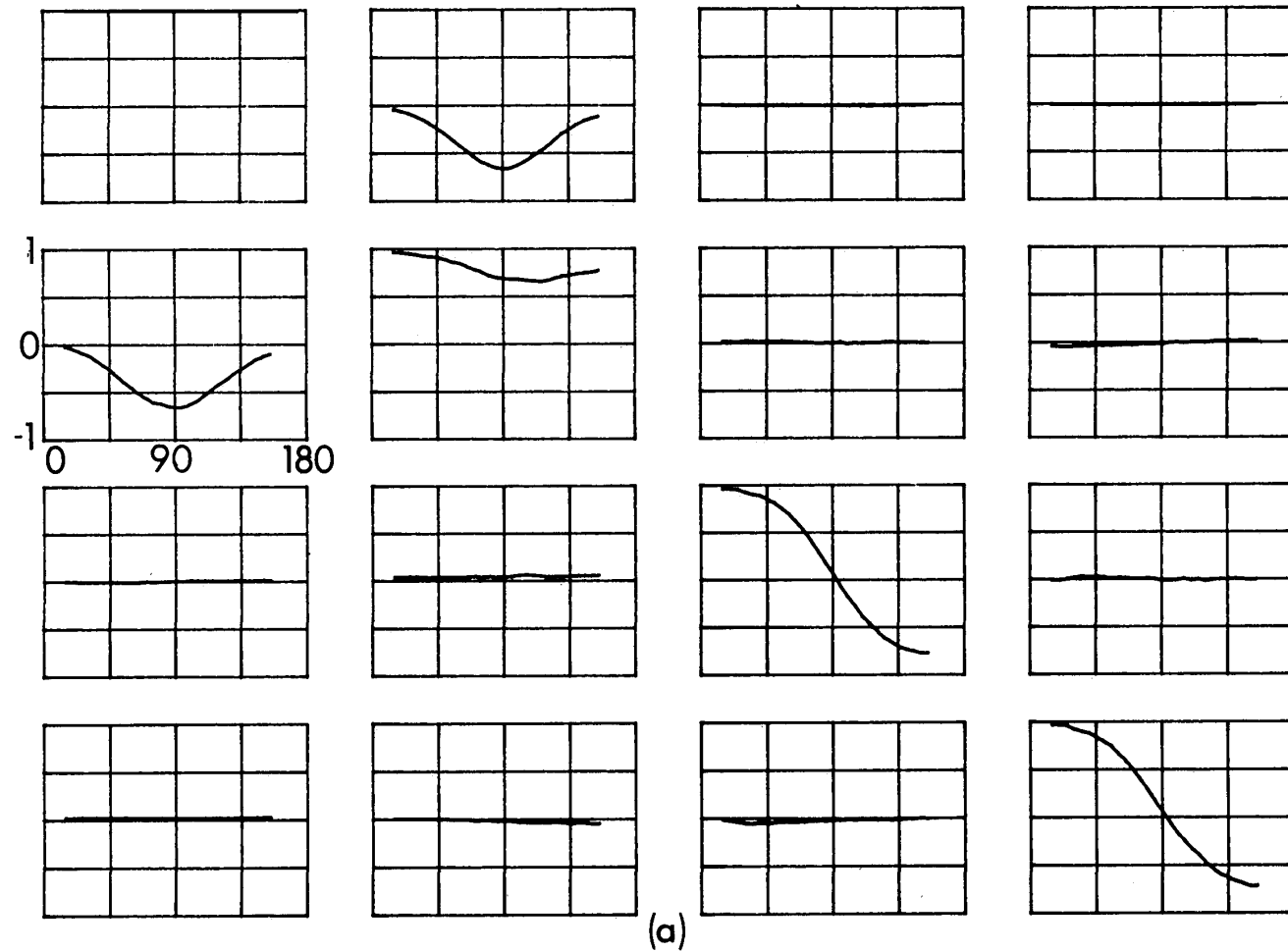
Wavelength is 436 nm

solar zenith angle is 51° .

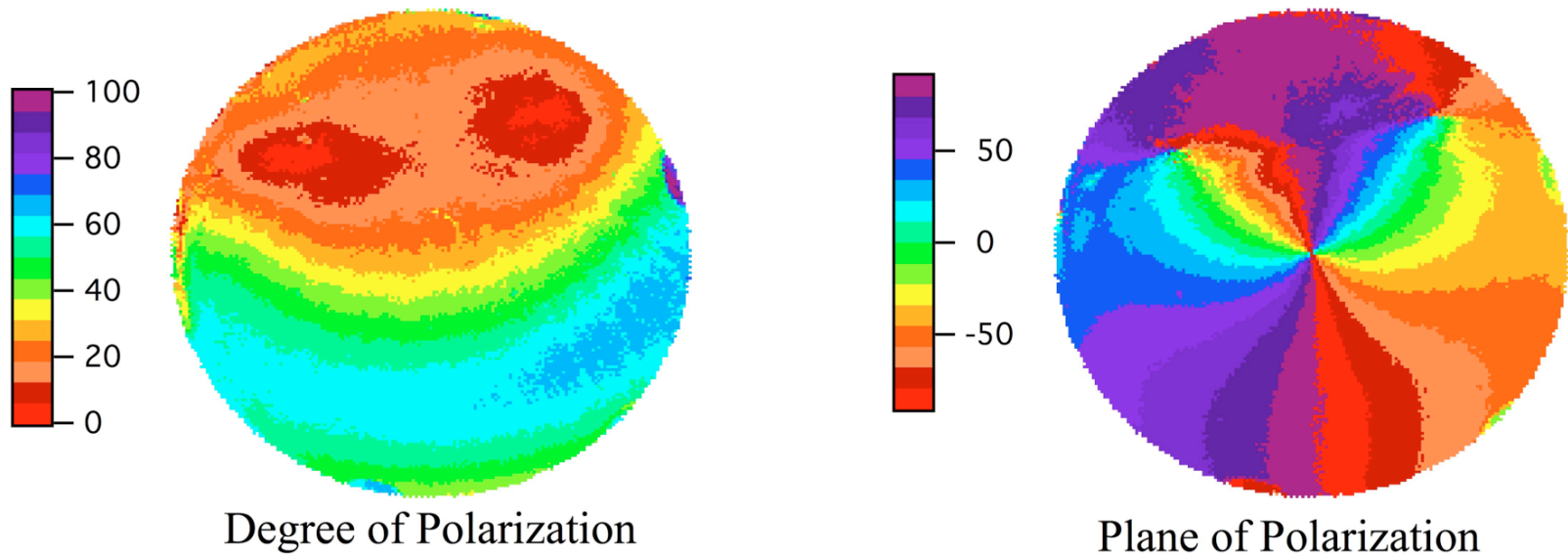
Measured with POLRads instrument (recent article in OE)

Mueller matrix for Ocean water (Voss and Fry, 1984).

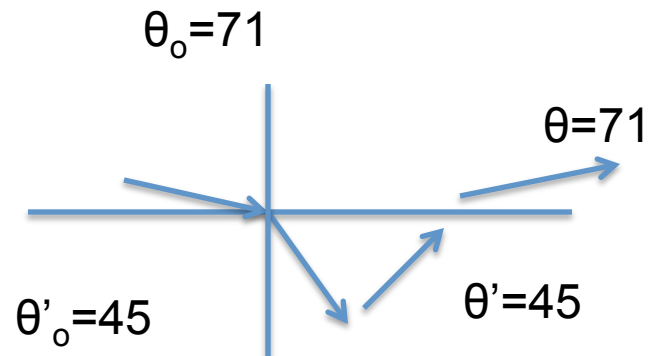
What are the differences?



Example continued



Note high degree of polarization, 90 degrees to refracted solar position, probably lost in glitter pattern, however still significant polarization in other areas.



Why do you care?

Other recent applications of polarization:

Use of polarized light for enhanced imaging: scattered light is more polarized than light from target (many people, early work by Gilbert, AO, 1967)..keeps getting rediscovered.

Polarized detection by animals..and polarized light camouflage (Molly Cummings, UT, Roger Hanlon, MBL).

Depolarization due to minerals?

2 possible remote sensing applications, change in M_{12}/M_{11}

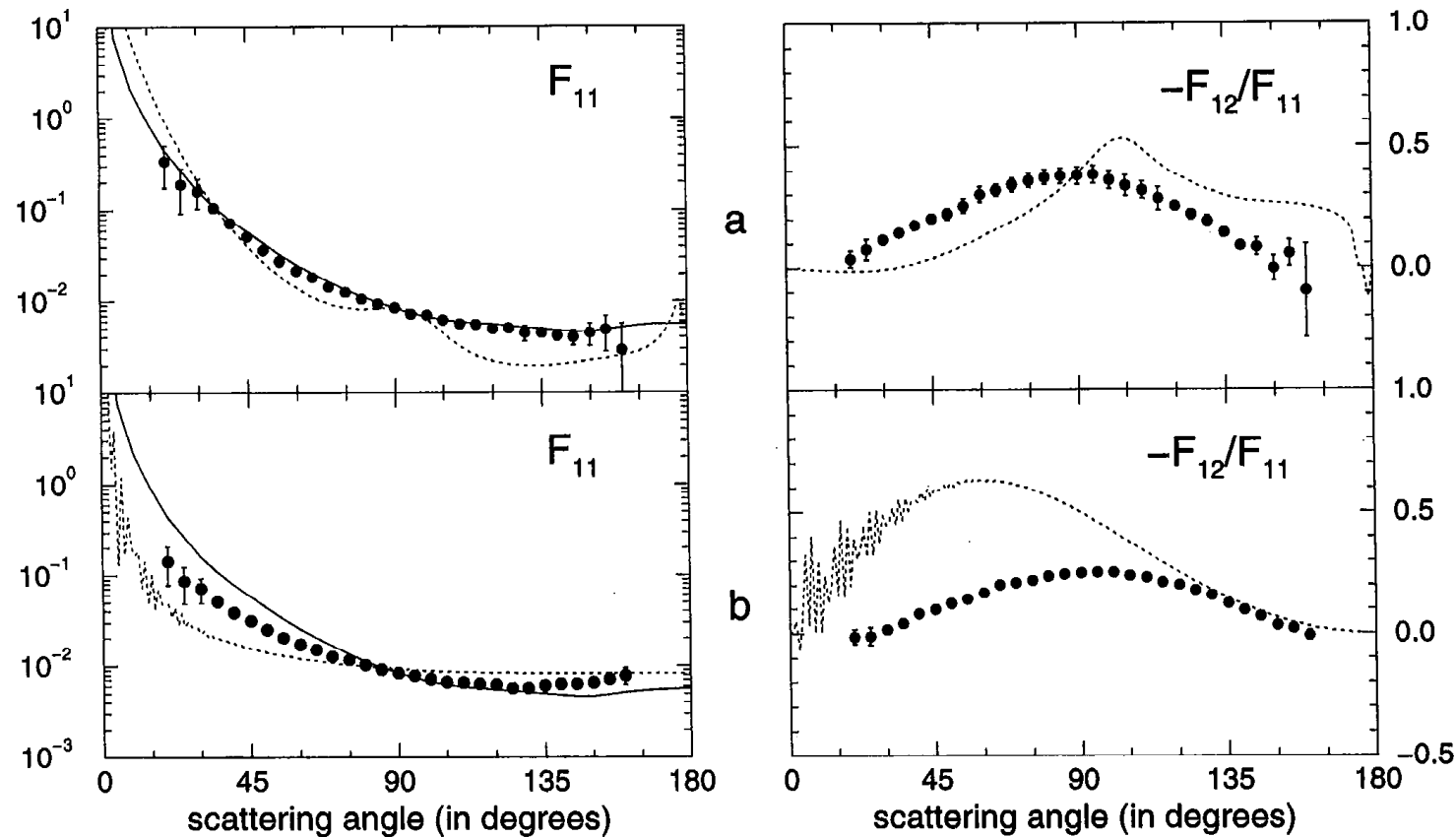


Fig. 9. Same as Fig. 6 for (a) Westerschelde silt with diameters ranging between 3 and 5 μm , and (b) Westerschelde silt with diameters ranging between 5 and 12 μm .

Other ideas about polarization

- Ibrahim et al. 2012, Optics Express

Retrieve c/a

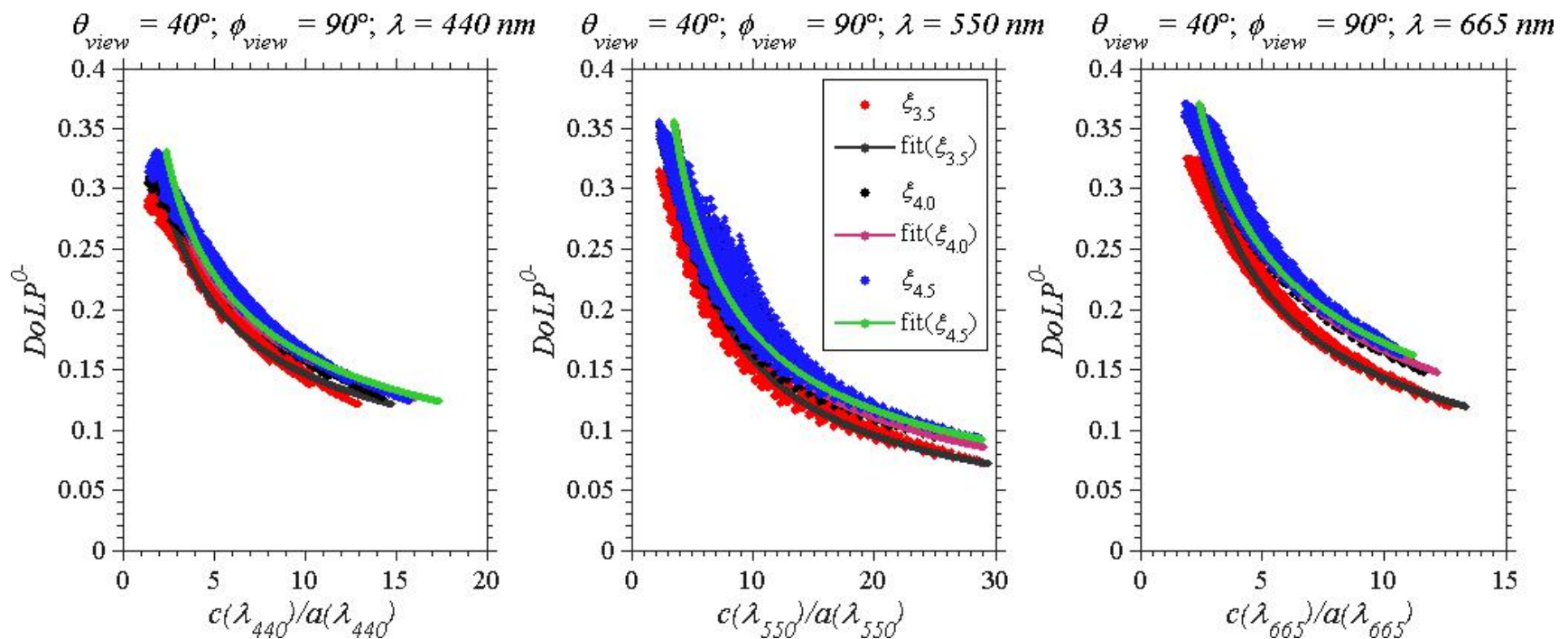


Fig. 8. Fitted relationship between DoLP at $\theta_{view} = 40^\circ$ and $\phi_{view} = 90^\circ$ and c/a ratio at three wavelengths for three different NAP slopes of the particle size distribution (PSD).

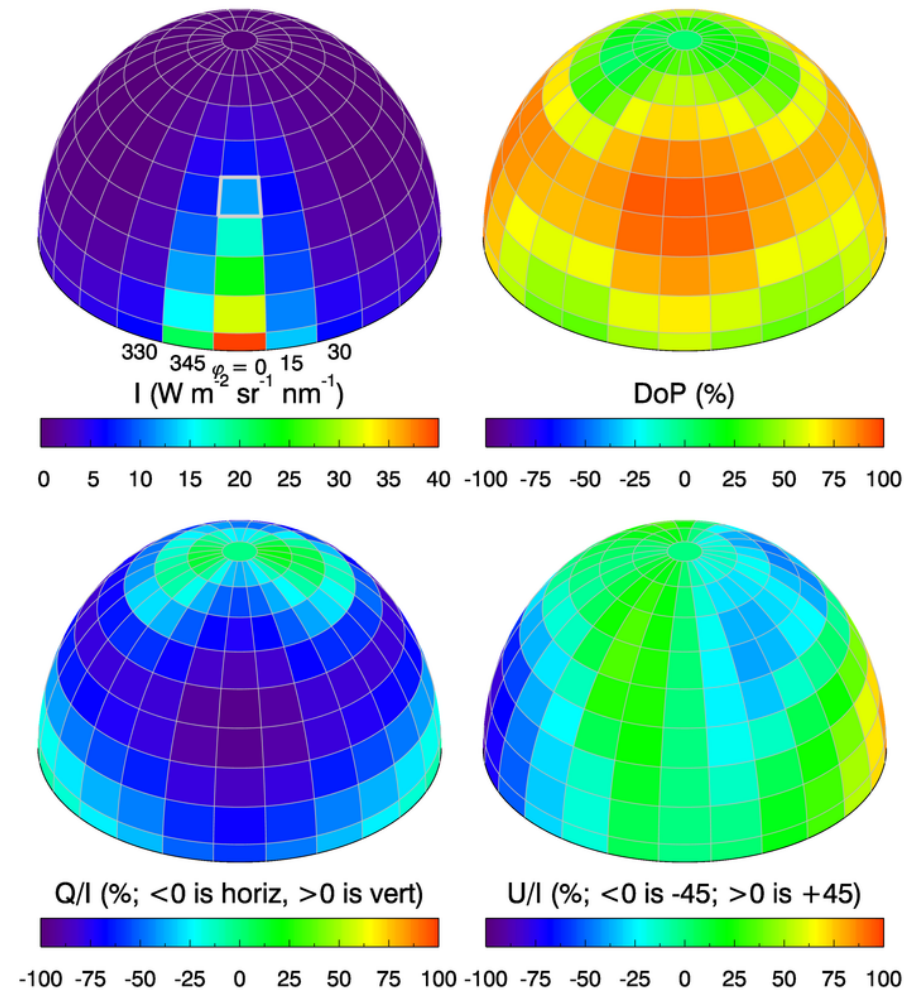
Your instrument may have a polarization sensitivity (old RSR instruments, spectrometers without scrambling)

The satellite instrument is polarized, to greater or lesser extent.

Might be more information in the polarization signal, new sensors will be developed (one just sank...APS on Glory) especially important for atmospheric aerosols.

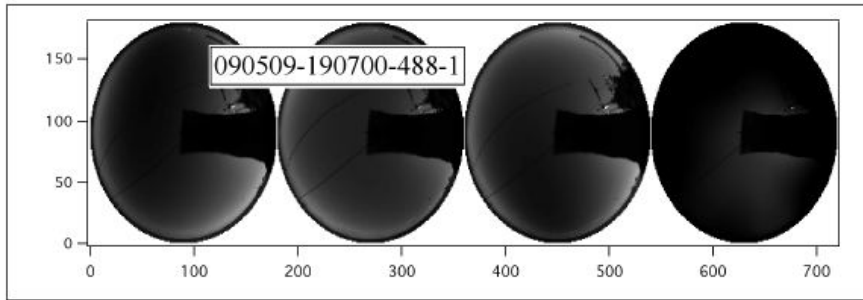
For better or worse, surface reflection highly polarized

Rayleigh Sky; $(\theta_{\text{sun}}, \varphi_{\text{sun}}) = (50.0, 180)$
Reflected Radiance; FFT; $U = 10 \text{ m s}^{-1}$

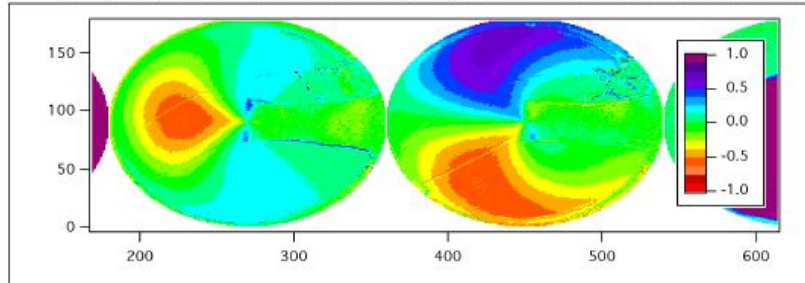


Extra slides

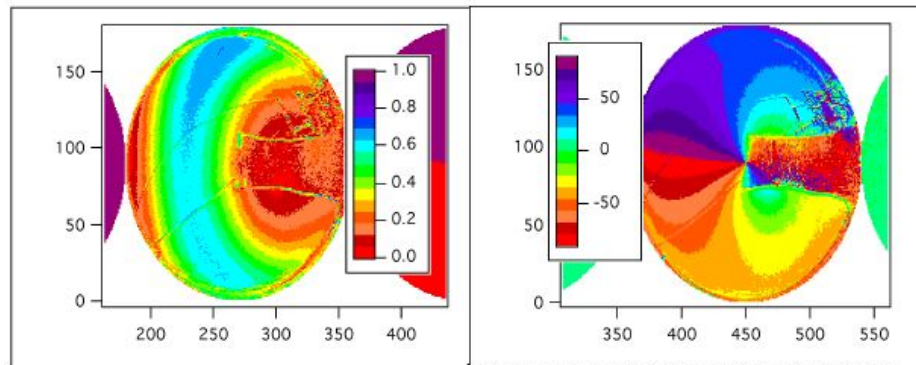
Sky light radiance distribution.



Intensity images, left is intensity from Stokes Calculation, right is intensity from unpolarized image

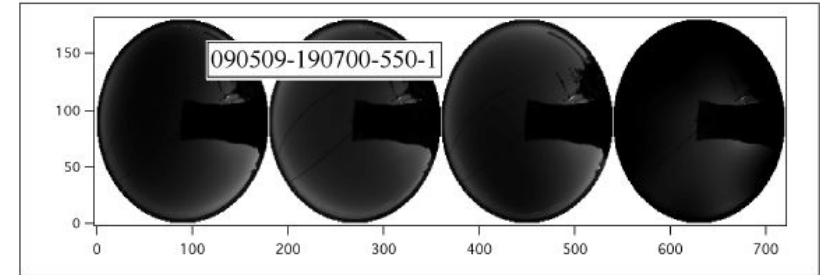


Normalized Stokes vectors Q (on left) and U (on right).

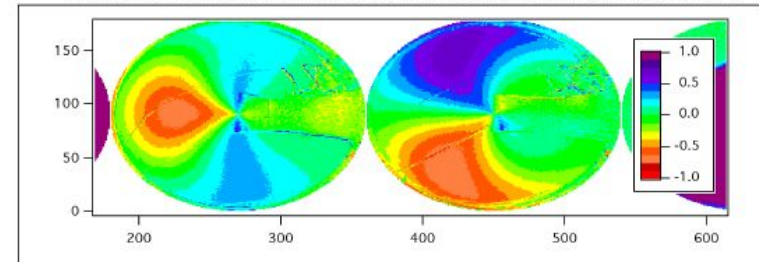


Degree of linear polarization

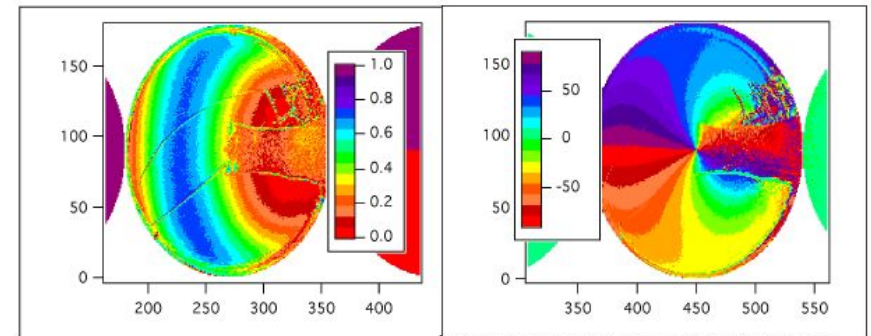
Plane of polarization, in degrees. Reference frame defined relative to plane with zenith and view direction. 0 deg is in the reference plane, positive angles rotate clockwise (looking along direction of propagation) towards the perpendicular to this plane.



Intensity images, left is intensity from Stokes Calculation, right is intensity from unpolarized image



Normalized Stokes vectors Q (on left) and U (on right).

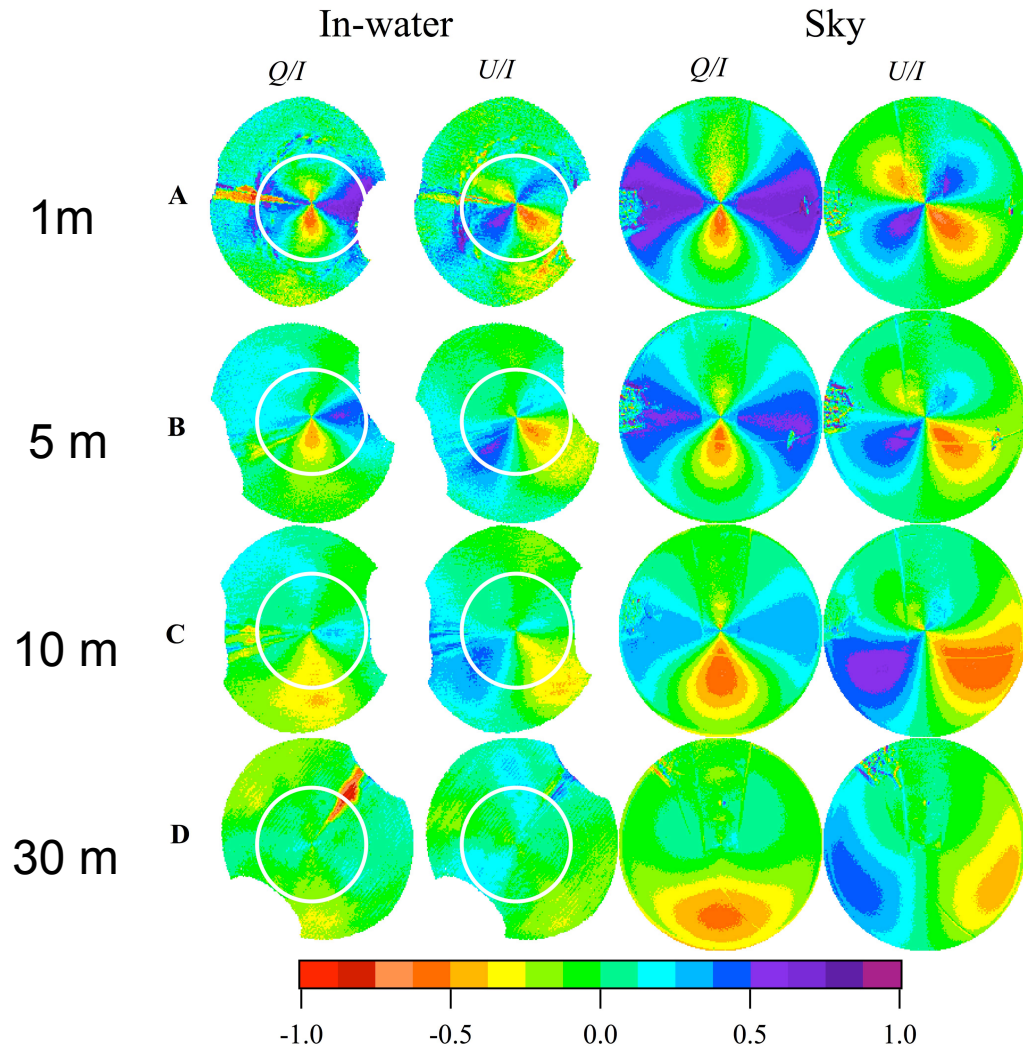


Degree of linear polarization

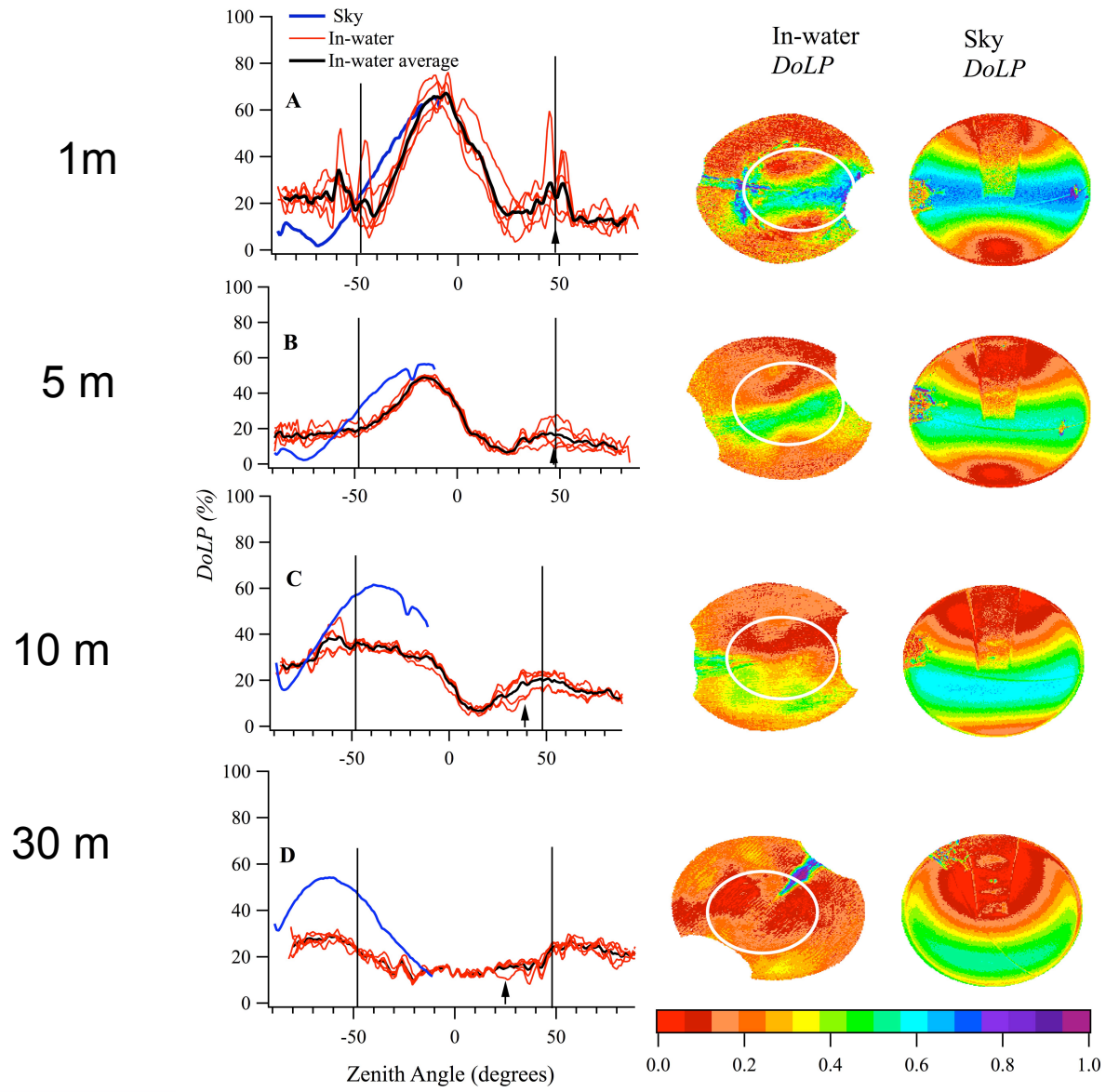
Plane of polarization, in degrees. Reference frame defined relative to plane with zenith and view direction. 0 deg is in the reference plane, positive angles rotate clockwise (looking along direction of propagation) towards the perpendicular to this plane.



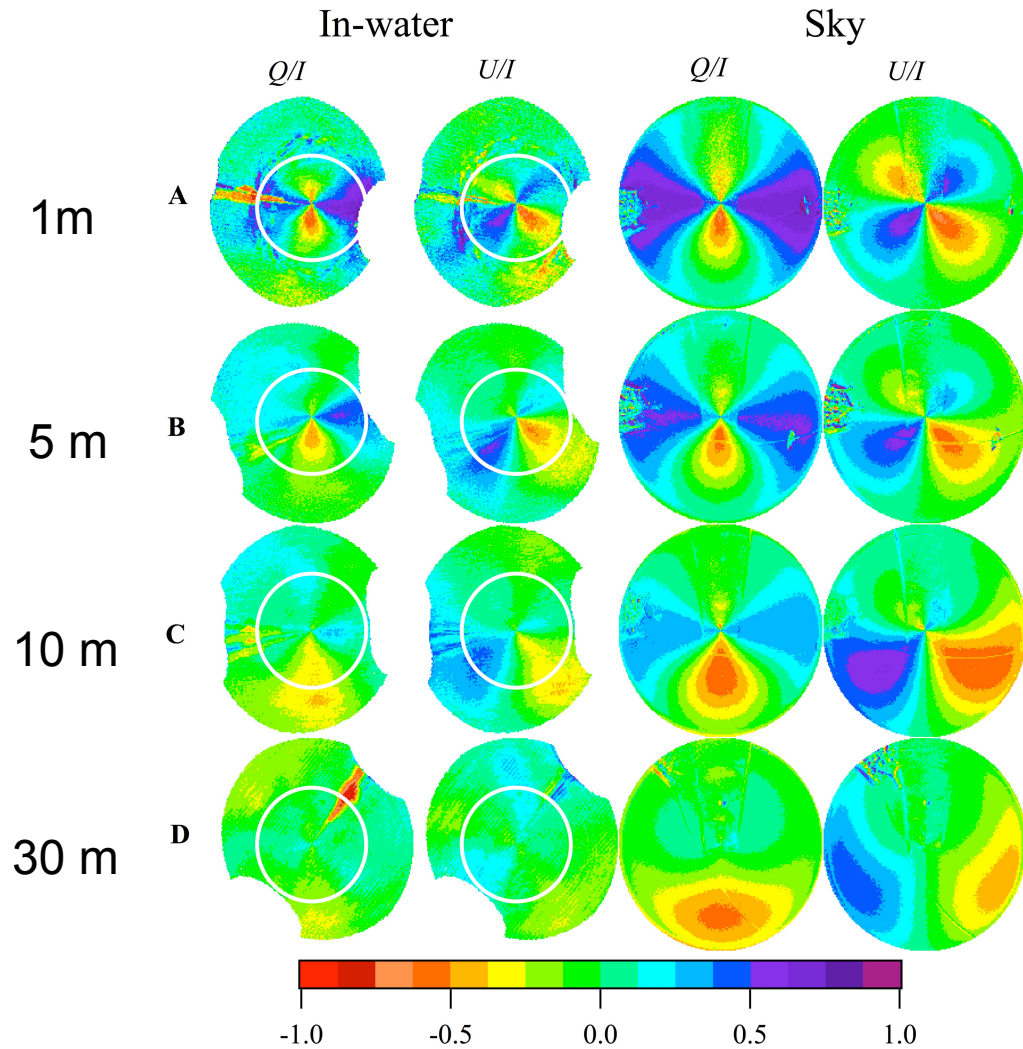
Downwelling radiance distribution.



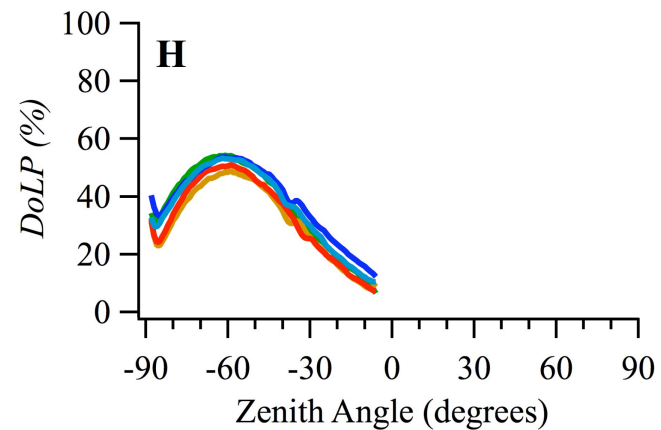
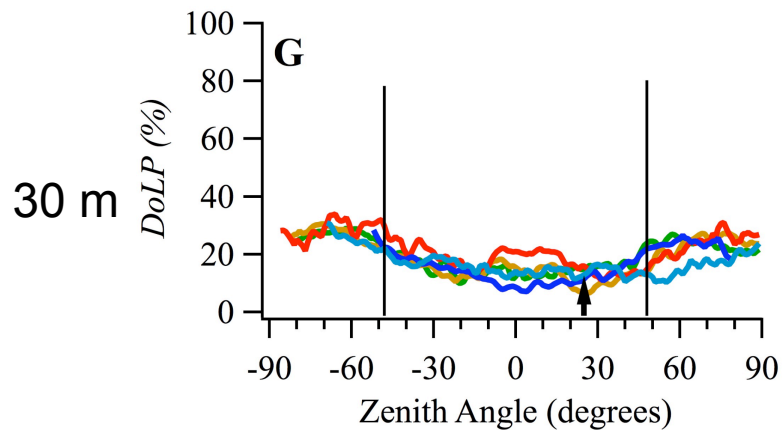
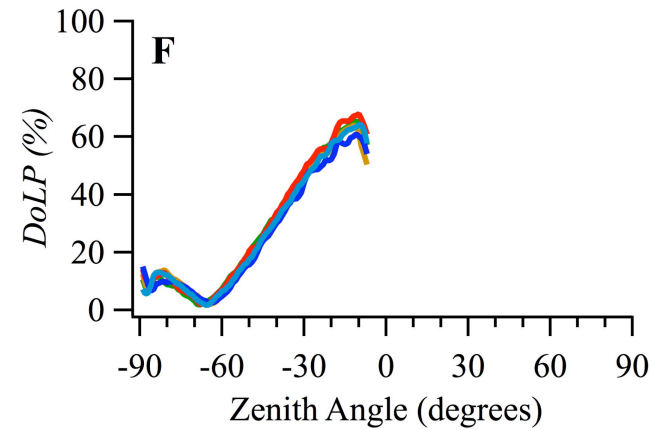
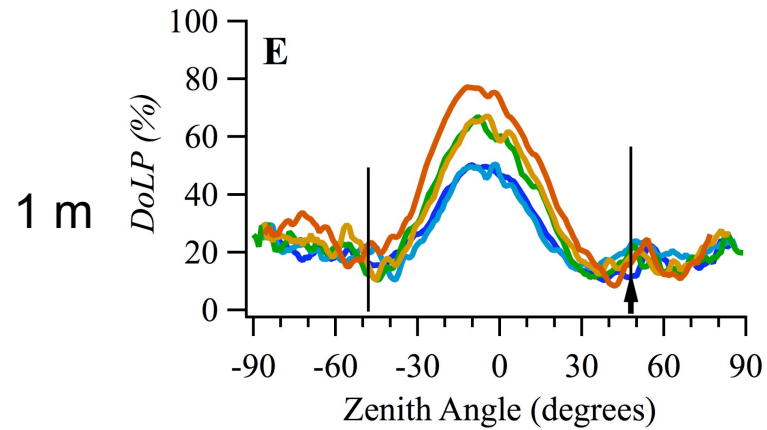
Downwelling radiance distribution.



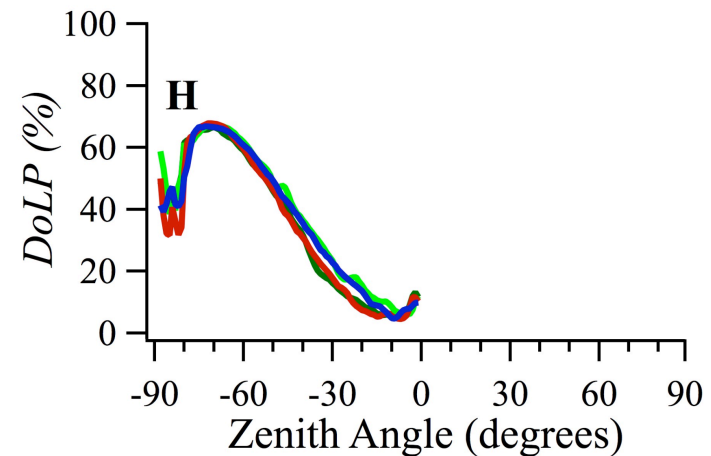
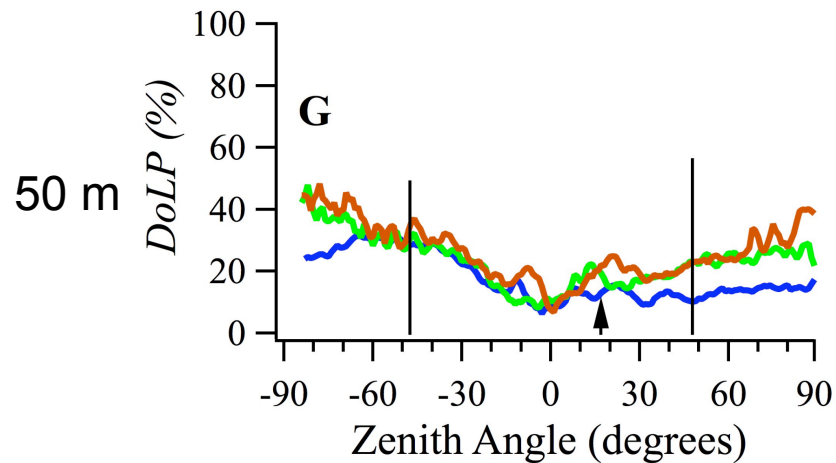
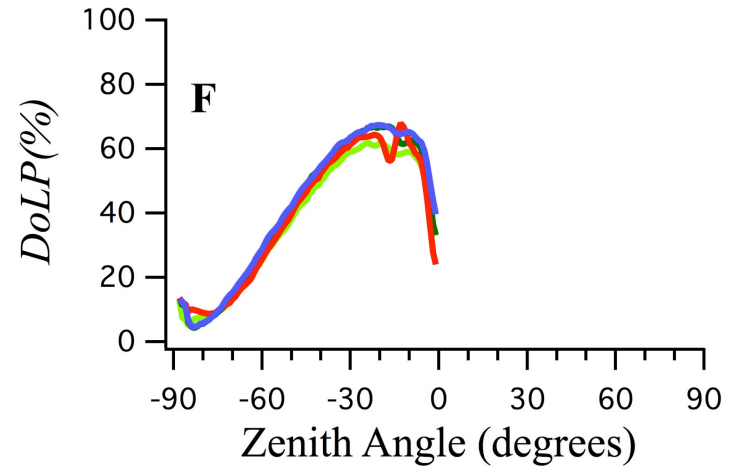
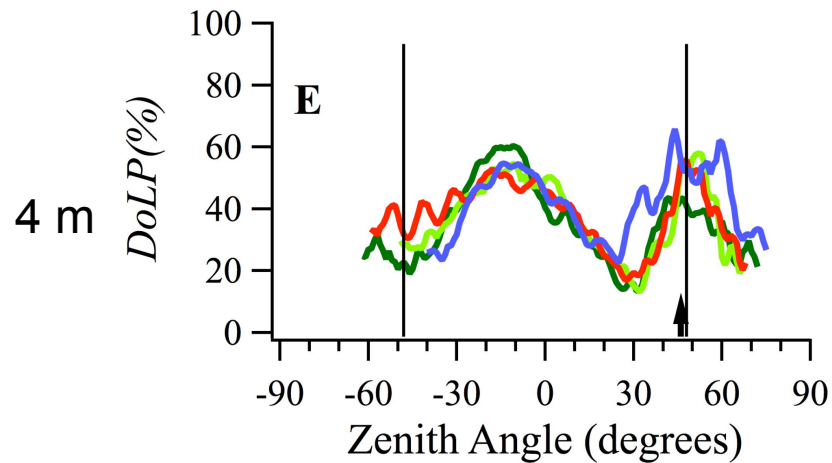
Downwelling radiance distribution.



Downwelling spectral variation, coastal



Downwelling spectral variation, clear water



Downwelling spectral variation, clear water

