Backscatter corrected Fournier-Forand phase function for remote sensing and underwater imaging performance evaluation

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ABSTRACT

A new more general phase function is derived which models more accurately the refracted and scattered components of the phase function. Modeling accurately the backscatter contribution is absolutely required when evaluating and predicting the performance of oceanographic remote sensing systems and underwater imaging systems[1], as the veiling glare from the in water scattering particles is the dominant first order source of image degradation. The function is further generalized to account for the effect of shelled organisms with two different indices of refraction for the bulk material and for the outer shell. It explains some of the puzzling discrepancies noticed by many workers who tried to model the oceanic phase functions using a single overall index of refraction. It shows why the index fitting parameter generally used in the phase function for ocean waters is always higher than the bulk index of refraction of the particles measured in the laboratory.

Keywords: Scattering, phase function, imaging

1. INTRODUCTION

A previously developed physics based oceanic phase function model[2] used only an approximation to the diffraction term to obtain an analytic expression to the phase function for Junge distributions of particles with small relative index of refraction. In the current work, a way was found to simply separate the diffraction, refraction and reflection term and combine them in a new analytic expression for the phase function. This new approach allows one to simply evaluate the contribution of the shell to the light scattered in the back hemisphere. The results clearly show that the shell reflection controls completely the scattering at large angles, while the index of the bulk material controls the medium to small forward angle scattering. The separation of the shell and bulk contribution allows model fits to oceanic phase functions that match experimental results with indices of refraction for the shell and bulk material that are in the range of measured properties of the individual organisms. Using these new results a formula is derived for a single approximate effective index of refraction that can be used in a standard Fournier-Forand phase function.

2. PARTITIONING THE PHASE FUNCTION

When analyzing the various contributions to scattering, it is particularly instructive to consider the large particle limit. In that limit the various contributions to the total scattering function can be simply separated out. The various mutual interference terms become negligible. This separation leads to relatively simple formulations for the separate terms of the scattering function and the ways of combining them become obvious. This approach is particularly suitable when the purpose is to produce formulae that are applicable to large ensembles of particles since the mutual interference terms will almost always be very nearly cancelled out due to the required integration over the size distributions.

If we follow this approach we must consider three terms. The first term is the scattering due to the diffraction of light around the particle. In the large particle limit , according to the Babinet theorem, this term has a total scattering cross-section value equal to the geometric scattering cross-section. The second term is the scattering due to the light refracted by its passage through the particle. This term would also be equal to the geometric cross-section if one did not account for the amount of light reflected by the particle at its various boundaries. In fact it is equal to the geometric cross-section

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minus the amount of reflected light. The final term is the reflected light. For the oceanic environment, the evaluation of these various terms is considerably simplified by the fact that the relative index of refraction is very near one. One can then use the anomalous diffraction approximation for the diffraction term, the simple analytic approximation for the refraction term and the small index asymptotic limit for the various refracted terms.

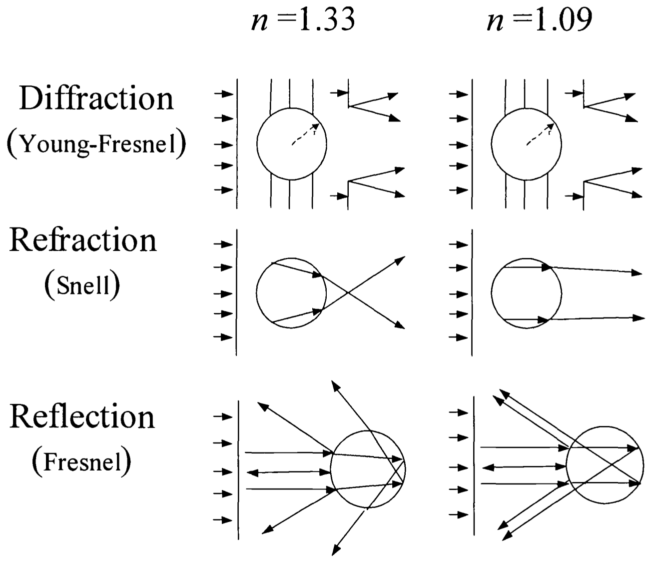


Fig 1. The phase function averaged over an ensemble of particles can be separated into its component parts, diffraction, refraction, reflection, and the interference terms can be neglected to first order. When the relative index of refraction is near unity (n=1.09), as is the case for water borne particulates, the physics further simplifies and simple approximate expressions can be obtained for all three terms.

We thus have for an individual particle:

$$\sigma_{scat}(\theta) = \sigma_{scat}^{diff}(\theta) + \sigma_{scat}^{refr}(\theta) + \sigma_{scat}^{refl}(\theta)$$

ω is the total amount of light reflected by the particle's front and back surfaces. For a spherical particle one obtains the following results.

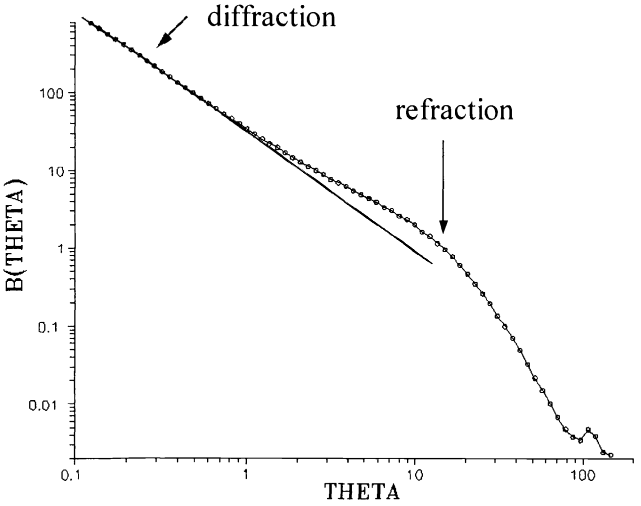
$$\sigma_{scat}^{diff}(\theta) = \left(\frac{\hat{s}}{4}\right)\left[2\frac{\mathrm{J}_1^{\,2}(\mathrm{x}\,2\sin\theta/2)}{(2\sin\theta\;2)^2}\right]$$

$$\sigma_{scat}^{refr}(\theta) = \left(\frac{\hat{s}}{4}\right)(1-\omega\;)\mathrm{F}(\mu,\theta)$$

$$F(\mu,\theta) = \left[2\frac{4\mu^2}{\left[(2\sin\theta/2)^2 + 4\mu^2\right]^2}\right], \quad \mu = n-1$$

$$\sigma_{scat}^{refl}(\theta) = \sigma_{scat}^{refl,front}(\theta) + \sigma_{scat}^{refl,back}(\theta)$$

The term $\hat{s}/4$ is one quarter of the surface area of the particle. This represents the value of the mean projected area for randomly oriented convex objects. The refracted term is the standard geometric optics approximation for small index[3]. The reflected term is the sum of the reflection from the front and back surfaces of the particles. As noted before, all interference terms and resulting cross terms are neglected in this approximation.



Mie code results for spheres.
n=1.1
m=3.5

Fig 2. The separation between the effect of diffraction and refraction can clearly be seen in the near forward angle on this graph of the phase function of a Junge distribution of spheres with an inverse power exponent of 3.5 and a relative index of refraction of 1.1. The straight line is the pure diffractive part. The points are the result of an extensive exact Mie code calculation. The refractive term shows up in the formation of a characteristic knee near 10°. The addition of the refractive term from this study would give an excellent fit. It should be noted that the refractive term is strongly shape dependant.

3. EVALUATION OF THE REFLECTED TERM

$|r_n|^2$ is the reflection coefficient as a function of angle for parallel and perpendicular polarization. The reflection from the first surface is straightforward to compute.

$$\sigma_{scat}^{refl,front}(\theta) = \left(\frac{\hat{s}}{4}\right)\left[\frac{1}{4}\left(|r_1|^2 + |r_2|^2\right)\right]$$

The reflection from the second or back surface of the particle is computed by assuming that the rays going through the particle have not been deflected to first order. This assumption is consistent with the small departure from one of the index of refraction and with the basic assumptions of the anomalous diffraction approximation.

$$\sigma_{scat}^{refl,back}(\theta) = \frac{\hat{s}}{4}\left\{\frac{1}{4}\left[|r_1|^2\left(1-|r_1|^2\right)^2 + |r_2|^2\left(1-|r_2|^2\right)^2\right]\right\}$$

For the sake of simplicity we will assume that the index of refraction is real and that the particles do not absorb. Generalizing to an absorbing particle is straightforward as follows:

$$r_1 = \frac{\cos\theta_i - n\cos\theta_t}{\cos\theta_i + n\cos\theta_t}$$

$$r_2 = \frac{n\cos\theta_i - \cos\theta_t}{n\cos\theta_i + \cos\theta_t}$$

$$\sin\theta_i = n\sin\theta_t \quad , \quad \theta = \pi - 2\theta_i .$$

Additional factors required to compute the scattering due to reflection are $C_{scat}^{refl,front}$ and $C_{scat}^{refl,back}$, the total angle integrated reflective cross-sections, which are in turn required to obtain expressions for the different $\omega_{i,j}$ (total mean reflectivity) factors for the various surfaces involved in the reflective term. These factors are as follows:

$$C_{scst}^{refl,front} = \left(\frac{\hat{s}}{4}\right)\int_0^{\pi}\left[\frac{1}{4}\left(|\mathrm{r}_1|^2+|\mathrm{r}_2|^2\right)\right]\sin(\theta)\,d\theta$$

$$= \left(\frac{\hat{s}}{4}\right)\frac{\omega_{1,0}+\omega_{2,0}}{2}$$

$$C_{scst}^{refl,back} = \frac{\hat{s}}{4}\int_0^{\pi}\left\{\frac{1}{4}\left[|\mathrm{r}_1|^2\left(1-|\mathrm{r}_1|^2\right)^2+|\mathrm{r}_2|^2\left(1-|\mathrm{r}_2|^2\right)^2\right]\right\}\sin(\theta)\,d\theta$$

$$= \frac{\hat{s}}{4}\frac{\omega_{1,2}+\omega_{2,2}}{2}$$

$$\omega_{1,0} = \frac{1}{3}\frac{(3\mathrm{n}-1)(\mathrm{n}-1)}{(\mathrm{n}+1)^2}$$

$$\omega_{2,0} = \frac{1}{(\mathrm{n}^2+1)^3(\mathrm{n}^2-1)^2}\times$$

$$\left\{(\mathrm{n}^4-1)-(\mathrm{n}^6-4\mathrm{n}^5-7\mathrm{n}^4+4\mathrm{n}^3-\mathrm{n}^2-1)\right.$$

$$\left.+2\mathrm{n}^2\left[(\mathrm{n}^2-1)^4\ln\left(\frac{\mathrm{n}-1}{\mathrm{n}(\mathrm{n}+1)}\right)+(\mathrm{n}^2+1)^4\ln(\mathrm{n})\right]\right\}$$

$$\omega_{1,2} = \frac{8}{35}\frac{\left[35\mathrm{n}^3+21\mathrm{n}^2+7\mathrm{n}+1\right](\mathrm{n}-1)}{(\mathrm{n}+1)^6}$$

$$\omega_{2,2} \approx \frac{\omega_{1,2}}{\omega_{1,0}}\omega_{2,0} = \frac{24}{15}\frac{\left[35\mathrm{n}^3+21\mathrm{n}^2+7\mathrm{n}+1\right]}{(3\mathrm{n}+1)(\mathrm{n}+1)^4}\omega_{2,0}$$

$$\omega = \frac{\omega_{1,0}+\omega_{2,0}}{2}+\frac{\omega_{1,2}+\omega_{2,2}}{2}$$

4. EFFECT OF SHAPE IN REFLECTION

We now have in hand all the terms required to evaluate the various contributions to the phase function. We should first note that both the refractive and reflective terms are completely size independent. Both depend primarily on the value of the index of refraction. Furthermore, to first order it can be shown that the refractive term is also shape independent. This remarkable conclusion comes about from a direct application of the following theorem of Van de Hulst[3]:

"The scattering pattern caused by reflection on large convex particles with random orientation is identical with the scattering pattern on a large sphere made of the same material and with the same surface condition. "

If the assumption is made that the rays passing through the particle will not be significantly deflected, then the following corollary immediately applies:

"In the case where the index of refraction is near enough unity that the deflection of the light inside the particle is negligible, the same scattering pattern will be made by the back surface of the particle as the front surface. "

5. MODELING THE EFFECT OF SHELLED PARTICLES

The results to date apply to a particle with a single index of refraction. For many years some puzzling results have been obtained when attempts were made to model the oceanic phase function by scattering codes that used a single averaged index of refraction to describe the scattering particles. The derived index of refraction was found to be much larger than the actual index measured from samples of individual scattering organisms. Many of the organisms present in the water column are in fact thin shells filled with liquid. The index of refraction of the shell material can be much larger than the index of the bulk material. For large particles, the contribution to scattering in the back hemisphere is in general dominated by the light reflected from the front and back surfaces of the particle. The amount of reflection and therefore the backscatter is dominated by the Fresnel reflections and this is controlled by the values of the difference between the indices of refraction of the different surfaces. The bulk or mean volume averaged index of the particles can be much smaller than the real index of the thin shell. This effect explains most of the noted discrepancies.

To handle the effect of a thin-shelled particle with different indices of refraction for the shell and the bulk of the particle, the simplest approach consists of replacing the index by a set of effective indices. One of the indices is designed to handle properly the refractive and diffractive terms and the other is designed to handle the reflective term.

The appropriate effective index of refraction for the reflective terms of a thin-shelled particle is one that gives the same total reflectivity as the shell thickness goes to zero. n_s is the index of refraction of the shell material and n_c is the index of refraction of the core material.

$$\frac{(n-1)^2}{(n+1)^2} = \frac{(n_s-1)^2}{(n_s+1)^2} + \frac{\left(\dfrac{n_s}{n_c}-1\right)^2}{\left(\dfrac{n_s}{n_c}+1\right)^2}$$

$$n = \frac{1+\Delta}{1-\Delta} \quad , \quad \Delta = \sqrt{\frac{(n_s-1)^2}{(n_s+1)^2} + \frac{\left(\dfrac{n_s}{n_c}-1\right)^2}{\left(\dfrac{n_s}{n_c}+1\right)^2}}$$

This effective index therefore applies to all the reflective terms in the phase function. For the refractive terms the effective index must give the appropriate phase difference rather than match the reflectivity. In the refractive and diffractive terms, the appropriate value of the effective index is therefore:

$$n = n_s f_s + n_c(1-f_s)$$

where f_s is the fraction of the particle thickness or radius occupied by the shell.

6. PHASE FUNCTION FORM FOR SHELLED PARTICLES

Following our previous arguments we therefore obtain the following terms for a normalized phase function. Consider first the refractive phase function term for a sphere.

$$p_{refr}(\theta,\lambda) = \frac{1}{2}(1-\omega_t)2\frac{4(n-1)^2}{\left[(2\sin\theta/2)^2+4(n-1)^2\right]^2}$$

With the index having the value appropriate for the refractive term. Note that the value of the index to use in evaluating ω_t is however the reflective effective index. The reflective term is obtained as follows:

$$p_{refl}(\theta,\lambda)=\frac{1}{2}\omega_t \mathrm{F}_{refl}(\mathrm{n},\theta)$$

$$\omega_t=\frac{\omega_{1,0}+\omega_{2,0}}{2}+\frac{\omega_{1,2}+\omega_{2,2}}{2}$$

$$\begin{aligned}\mathrm{F}_{refl}(n,\theta)=&\frac{\omega_{1,0}\left|\mathrm{r}_1(\theta)\right|^2}{4\omega_t}+\frac{\omega_{2,0}\left|\mathrm{r}_2(\theta)\right|^2}{4\omega_t}\\&+\frac{\omega_{1,2}\left|\mathrm{r}_1(\theta)\right|^2\left(1-\left|\mathrm{r}_1(\theta)\right|^2\right)^2}{4\omega_t}+\frac{\omega_{2,2}\left|\mathrm{r}_2(\theta)\right|^2\left(1-\left|\mathrm{r}_2(\theta)\right|^2\right)^2}{4\omega_t}\end{aligned}$$

with the effective index obviously having the value appropriate for the reflective term.

The diffractive term is the only term that is explicitly dependent on the particle size distribution. We will use for it the result that we previously obtained for an inverse power law distribution of particles[2]. This type of distribution is characteristic and representative of oceanic particle size distributions. In this case the appropriate index is the refractive term index.

$$p_{diff}(n,\theta,\lambda)=\frac{1}{2}\frac{(\lambda/2\pi)^{3-m}\int_0^\infty \pi\,\mathrm{x}^2 Q_{scat}(n,\mathrm{x})f(\theta,\mathrm{x})\left(\frac{k}{\mathrm{x}^m}\right)d\mathrm{x}}{(\lambda/2\pi)^{3-m}\int_0^\infty \pi\,\mathrm{x}^2 Q_{scat}(n,x)\left(\frac{k}{\mathrm{x}^m}\right)d\mathrm{x}}$$

$$\mathrm{x}=\frac{2\pi\,\mathrm{r}}{\lambda}\quad,\quad \rho=2(\mathrm{n}-1)\mathrm{x}$$

$$Q_{scat}(\mathrm{x})\approx\frac{\rho^2/2}{1+\rho^2/4}$$

$$\frac{f(\theta,\mathrm{x})}{2\pi}\approx\frac{1}{4\pi}\frac{1+4\mathrm{x}^2/3}{(1+u^2\mathrm{x}^2/3)^2}$$

$$\begin{aligned}p_{diff}(n,\theta,\lambda)=&\frac{1}{2}\frac{1}{4\pi}\frac{1}{(1-\delta)^2\delta^v}\\&\left(\left[v(1-\delta)-(1-\delta^v)\right]+\frac{4}{u^2}\left[\delta(1-\delta^v)-v(1-\delta)\right]\right)\end{aligned}$$

$$v=\frac{3-m}{2}\quad,\quad \delta=\frac{u^2}{3(n-1)^2}\quad,\quad u=2\sin(\theta/2)$$

The complete normalized phase function is now obtained by simply summing all three terms.

$$p(n,\theta,\lambda) = p_{diff}(n,\theta,\lambda) + p_{refr}(n,\theta) + p_{refl}(n,\theta)$$

In this equation for shelled particles, where the index of refraction of the shell is much higher than the index of the core material, the appropriate effective indices must be used. If the shell is thin, the index for the diffractive and refractive terms approaches the core index of refraction while the index for the reflective term approaches the higher valued shell index of refraction.

7. CONCLUSIONS

The natural separation into a reflected term involving mostly the shell index of refraction and the diffracted and refracted terms involving mostly the core index of refraction, allows one enough freedom to model oceanic particles with indices of refraction values that now approach those expected from the basic models of the indices of refraction of phyto-plankton. The backscatter portion of the phase function is controlled by the reflective term while the forward scatter portion is dominated by the diffracted and refracted terms. Both these terms can be modeled independently by using different values for the shell and core indices of refraction.

We have gained more insight and precision for the evaluation of oceanic phase functions but have unfortunately lost in the process the relative simplicity of the standard Fournier-Forand phase function. We are currently working on an approximation that would allow one to compute a single effective index of refraction that could be used in the standard Fournier-Forand form to approximate faithfully the results of the present analysis.

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