Implications of a new phase function for autonomous underwater imaging

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ABSTRACT

Autonomous underwater vehicles do not have sufficient communications bandwidth over long ranges to send back real time images even for monitoring purposes. Autonomous imaging from underwater vehicles will therefore require real-time imaging system performance prediction in order to ensure that the vehicle can position itself at a range that will allow it to take an image of the scene or target of interest at the required resolution and contrast level. Ideally the inherent optical properties of the surrounding waters should be measured on board. This may not be feasible or only a restricted set may be measurable. In order to improve the prediction of the imaging performance, a physics based analytic phase function that could effectively exploit any a-priori or in-situ measured parameters would be extremely helpful. Such a new physics based analytic phase function has been derived and tested against exact scattering codes. Among other features it is sufficiently precise to allow an accurate determination of the backscatter ratio based on an estimate of the mean index of refraction which is by far the dominant parameter. The new formulation shows clearly why the backscatter ratio, which is the dominant factor in determining imaging range, is insensitive to the inverse power of the size distribution and almost entirely controlled by the mean index of refection.

Keywords: Scattering, phase function, imaging, autonomous underwater vehicles

1. INTRODUCTION

There is no current or potential future free space underwater communication link with sufficient bandwidth to transmit images at the rate required to position an autonomous vehicle in real-time. Seriously compounding the problem, almost all the techniques developed to automatically control and optimize camera images taken in air are not applicable in the large majority of underwater environments. The fundamental problem is that in many situations the backgrounds are virtually featureless and there is no way for a camera system to adjust itself by resolving features. For example, there is no moderately reliable a priori way of determining if one is looking at a mud flat or at in water scattering near the vehicle which also generally has the appearance of a featureless fog. The real question to answer in this situation is not “Do I see something?” but rather “If there had been an object or feature of interest to me would I have seen it?”

To answer this last question properly, one must first be able to assess what features an image, with no intervening media to corrupt it, would need to have to allow a human or a computer to recognize the objects or features of interest. Once this is done the optical effect of the water on the image must be evaluated by first measuring the optical properties of the water column and then using an appropriate model to calculate the resultant behavior of the corrupted image as a function of the distance to the target of the vehicle[1]. The ideal tool for determining the complete profile of the inherent optical properties of the water column between the imaging system and the scene of interest is a multiple field of view polarized LIDAR. To accomplish these last two tasks an estimate of the phase function is required. It would obviously be of great use if this estimated phase function was parameterized in terms of the physical properties of the scattering particles such as the mean index of refraction and the inverse power of the size distribution. If the phase function is in this physical form one can bring to bear all the prior oceanographic knowledge one has about the nature of the scattering particles present in the different types of waters to bracket the range of the optical parameters when analyzing the LIDAR returns.

2. NEED FOR AN APPROXIMATE MODEL FOR HYDROSOLS

The very nature of hydrosols prevents the use of the exact scattering models extensively developed for aerosols. A common feature of almost all aerosols is that they are composed of a small solid core that acts a nucleation center and coats itself in water.

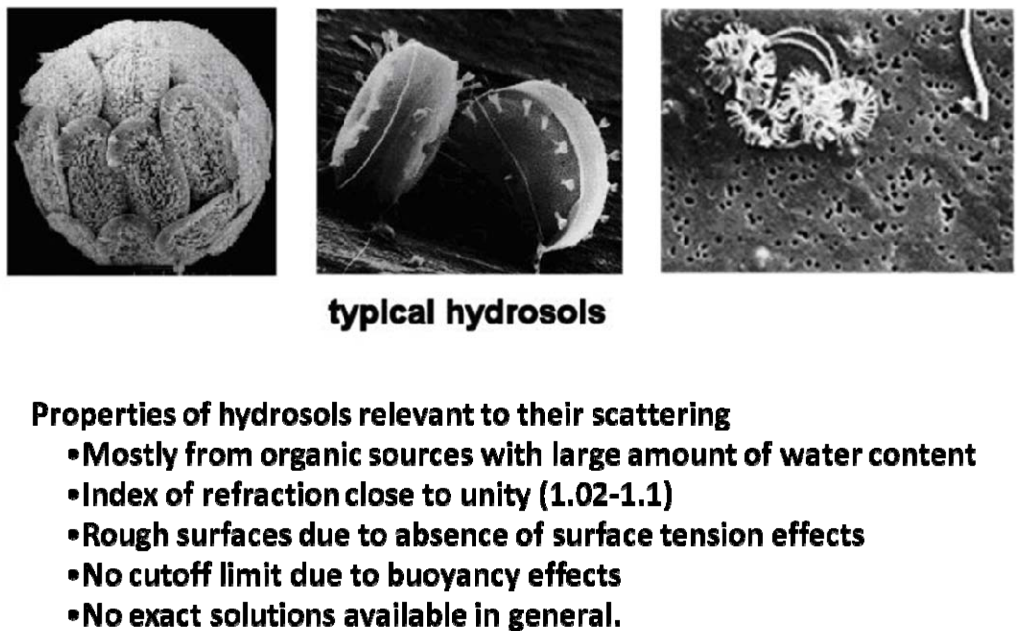

Fig 1. The phase function averaged over an ensemble of particles can be separated into its component parts, diffraction, refraction, reflection, and the interference terms can be neglected to first order. When the relative index of refraction is near unity (n=1.09), as is the case for water borne particulates, the physics further simplifies and simple approximate expressions can be obtained for all three terms.

Surface tension ensures that the surface of all these aerosols even if they are distorted into spheroids is smooth. As shown in figure 1 the absence of surface tension leads to complex structures and surface features for hydrosols. A further complication is that they are generally surrounded by a hard shell of relative index of the order of 1.1 to 1.15 with a soft organic core of relative index of 1.02 to 1.05 The only way to obtain good approximations to the scattering from large ensembles of these structures is to decompose the scattering in its basic components and use the simplest physical formulations possible for these components that account for all the relevant features.

3. PARTIONING THE PHASE FUNCTION

A previously developed physics based oceanic phase function model[2] used only an approximation to the diffraction term to obtain an analytic expression to the phase function for Junge distributions of particles with small relative index of refraction. In further work[3], a way was found to simply separate the diffraction, refraction and reflection term and combine them in a new analytic expression for the phase function. This new approach allows one to simply evaluate the contribution of the shell to the light scattered in the back hemisphere. The results clearly show that the shell reflection controls completely the scattering at large angles, while the index of the bulk material controls the medium to small forward angle scattering. The separation of the shell and bulk contribution allows model fits to oceanic phase functions that match experimental results with indices of refraction for the shell and bulk material that are in the range of measured properties of the individual organisms. Using these new results a formula was derived for a single approximate effective index of refraction that can be used in a standard Fournier-Forand phase function[4]. Two important effects were neglected in this previous work which led to a significant overestimate of the backscattered fraction form both the scattering due to diffraction around the particle and refraction through the particle. The purpose of the present work is to correct these two terms. This is important as the LIDAR signal amplitude depends on the backscattered light.

As stated in the previous analyses[3-4], when evaluating the various contributions to scattering, it is particularly instructive to consider the large particle limit. In that limit the various contributions to the total scattering function can be simply separated out. The various mutual interference terms become negligible. This separation leads to relatively simple formulations for the separate terms of the scattering function and the ways of combining them become obvious. This approach is particularly suitable when the purpose is to produce formulae that are applicable to large ensembles of particles since the mutual interference terms will almost always be very nearly cancelled out due to the required integration over the size distributions.

If we follow this approach we must consider three terms. The first term is the scattering due to the diffraction of light around the particle. In the large particle limit, according to the Babinet theorem, this term has a total scattering cross-section value equal to the geometric scattering cross-section. The second term is the scattering due to the light refracted by its passage through the particle. This term would also be equal to the geometric cross-section if one did not account for the amount of light reflected by the particle at its various boundaries. In fact it is equal to the geometric cross-section minus the amount of reflected light and absorbed light if the particle material absorbs. The final term is the reflected light. At the present time we will only consider the non-absorbing. The extension to absorption is very straightforward but tedious[3]. For the oceanic environment, the evaluation of these various terms is considerably simplified by the fact that the relative index of refraction is very near one. One can then use the anomalous diffraction approximation for the diffraction term, the simple analytic large particle approximation for the refraction term and the small index asymptotic limit for the various refracted terms.

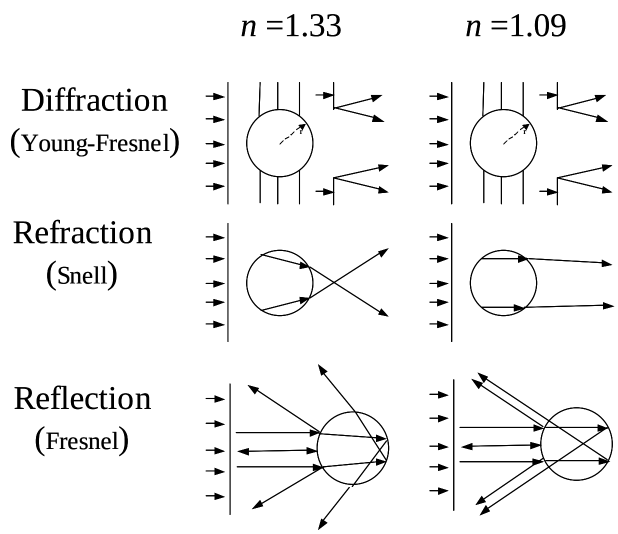

Fig 2. The phase function averaged over an ensemble of particles can be separated into its component parts, diffraction, refraction, reflection, and the interference terms can be neglected to first order. When the relative index of refraction is near unity (n=1.09), as is the case for water borne particulates, the physics further simplifies and simple approximate expressions can be obtained for all three terms.

We thus have for the scattering cross-section an individual particle:

$$\sigma_{scat}(\theta) = \sigma_{scat}^{diff}(\theta) + (1-\omega)\sigma_{scat}^{refr}(\theta) + \sigma_{scat}^{refl}(\theta)$$

ω is the total amount of light reflected by the particle's front and back surfaces. The reflected term needs to be further broken down in terns of reflections from the front and back surfaces.

$$\sigma_{scat}^{refl}(\theta) = \sigma_{scat}^{refl,front}(\theta) + \sigma_{scat}^{refl,back}(\theta)$$

The cross-sections can be expressed in terms of the probabilities of scattering of the various components multiplied by the orientation averaged geometric cross-section of the particle.

$$\sigma_{diff}(\theta) = \left(\frac{\hat{s}}{4}\right) p_{diff}(\mu, n, \theta)$$

$$\sigma_{refr}(\theta) = \left(\frac{\hat{s}}{4}\right)(1-\omega\ )\,p_{refr}(n,\theta)$$

$$\sigma_{scat}^{refl,\,front}(\theta) = \left(\frac{\hat{s}}{4}\right)\left[\frac{1}{8\pi}\left(|\mathrm{r}_1|^2 + |\mathrm{r}_2|^2\right)\right]$$

$$\sigma_{scat}^{refl,back}(\theta) = \frac{\hat{s}}{4}\left\{\frac{1}{8\pi}\left[|\mathrm{r}_1|^2\left(1-|\mathrm{r}_1|^2\right)^2 + |\mathrm{r}_2|^2\left(1-|\mathrm{r}_2|^2\right)^2\right]\right\}$$

$$\omega = \frac{\omega_{1,\,front} + \omega_{2,\,front}}{2} + \frac{\omega_{1,back} + \omega_{2,back}}{2}$$

$$\omega_{i,\,front} = \int_0^{2\pi} d\varphi \int_0^{\pi}\left[\frac{1}{4\pi}\left(|\mathrm{r}_i|^2\right)\right]\sin(\theta)\,d\theta = \int_0^{\pi}\left[\frac{1}{2}\left(|\mathrm{r}_i|^2\right)\right]\sin(\theta)\,d\theta$$

$$\omega_{i,back} = \int_0^{2\pi} d\varphi \int_0^{\pi}\left\{\frac{1}{4\pi}\left[|\mathrm{r}_i|^2\left(1-|\mathrm{r}_i|^2\right)^2\right]\right\}\sin(\theta)\,d\theta = \int_0^{\pi}\left\{\frac{1}{2}\left[|\mathrm{r}_i|^2\left(1-|\mathrm{r}_i|^2\right)^2\right]\right\}\sin(\theta)\,d\theta$$

The term $\hat{s}/4$ is one quarter of the surface area of the particle. This represents the value of the mean projected area for randomly oriented convex objects. $|\mathrm{r}_i|^2$ is the reflection coefficient as a function of angle for parallel and perpendicular polarizations. μ is the inverse power of the Junge particle size distribution and n is the index of refraction of the particle. As noted before, all interference terms and resulting cross terms are neglected in this approximation.

In the case where a surface is rough and acts therefore as a perfect diffuser the angular distribution is completely different and increases in the backscatter direction. The form is well known and is derived in Van de Hulst[5] .

$$\sigma_{rough}(\theta) = \frac{\hat{s}}{4}\omega\ \left(\frac{3}{4\pi}(\sin\theta - \theta\cos\theta)\right)$$

We can characterize to a good approximation the reflection coming from any scattering particle as a linear combination of diffuse and specular components. To do this we will define a roughness factor $1 \le R \le 1$ so that the total contribution to scattering becomes:

$$\sigma_{tot}^{refl}(\theta) = (1 - R)\sigma_{scat}^{refl}(\theta) + R\,\sigma_{rough}(\theta)$$

4. EVALUATION OF THE DIFFRACTED TERM

The term that was missing in the previous evaluations of the diffracted term is the Kirchhoff obliquity factor which ensures that there is no contribution to diffraction in the backward direction. Including this term means that there is no contribution by diffraction in the retro reflection direction and therefore no contribution to the signal received by a LIDAR system. This modified diffractive scattering probability also reduces by an order of magnitude the total contribution of diffraction to the scattering integrated over the entire back hemisphere.

$$obliquity\ = (1+\cos(\theta))^2$$

Note that:

$$u(\theta) = 2\sin\left(\frac{\theta}{2}\right)$$

$$(1+\cos(\theta)) = 2 - \frac{u(\theta)^2}{2}$$

$$\sin(\theta)d\theta = \frac{u(\theta)}{2}d\,u(\theta)$$

With this notation and using the standard Fournier-Forand[3] functional form for the phase function we get the following result:

$$f(\theta)=\frac{N_0}{\left[1+\frac{u^2 x^2}{3}\right]^2}\left(2-\frac{u^2}{2}\right)^2$$

Where N_0 is normalization constant such that:

$$2\pi\int_0^{\pi} f(\theta)\sin(\theta)d\theta=1$$

This implies that:

$$\frac{1}{N_0}=2\pi\int_0^2\frac{1}{\left[1+\frac{u^2 x^2}{3}\right]^2}\left(2-\frac{u^2}{2}\right)^2\frac{u}{2}du$$

$$\frac{1}{N_0}=\frac{6\pi}{x^2}\left(1+\frac{3}{2x^2}+\frac{3}{2x^2}\ln\left[\frac{3}{3+4x^2}\right]+\frac{9}{8x^4}\ln\left[\frac{3}{3+4x^2}\right]\right)$$

This expression has the following limits.

$$N_0\approx\frac{3}{8\pi}\left(1+\frac{32}{27}x^2\right)\quad \lim x\to 0$$
$$N_0\approx\frac{3}{8\pi}\left(\frac{4}{9}x^2\right)\quad \lim x\to\infty$$

We can approximate the normalization factor to an error of 14% by the following expression:

$$N_0\approx\frac{3}{8\pi}\left(1+\frac{4}{9}x^2\right)$$

This finally gives:

$$f(\theta)\approx\frac{3}{8\pi}\left(1+\frac{4}{9}x^2\right)\frac{1}{\left(1+\frac{u^2 x^2}{3}\right)^2}\left(2-\frac{u^2}{2}\right)^2$$

Using the following notation we then obtain the new diffraction component of the phase function with obliquity factor included:

$$p_{diff}(\mu,n,\theta)=\frac{3}{16\pi}\frac{\left(2-\frac{u^2}{2}\right)^2}{(1-\delta)^2\delta^v}$$
$$\left(\left[v(1-\delta)-(1-\delta^v)\right]+\frac{4}{3u^2}\left[\delta(1-\delta^v)-v(1-\delta)\right]\right)$$

$$v=\frac{3-\mu}{2}\quad,\quad \delta=\frac{u^2}{3(\mathrm{n}-1)^2}\quad,\quad u=2\sin(\theta/2)$$

The maximum normalization error for n=1.1 is 6.1% at m=4.4. The error in the most likely value for ocean waters n=1.09 and m=3.65 is 3%. More precise results could be obtained but at the cost of considerable added complexity not warranted at this stage.
The new formulation leads to lower values of the diffraction term in the back hemisphere and thus helps the reflection terms to dominate. At 90 degrees the new diffraction term is 25% of the old.

5. EVALUATION OF THE REFRACTED TERM

We also need to modify the refraction formula from Van de Hulst[5] to better account for the large angle refraction. This can be done by using the formulas found in Chen[6].

$$p_{scat}^{refr}(\theta) = \frac{N_{0-refr}}{\left(n - \cos\left(\frac{\theta}{2}\right)\right)^2} \left[\frac{1}{\left[\left(2\sin\left(\frac{\theta}{2}\right)\right)^2 + 4\left(\mathrm{n} - \cos\left(\frac{\theta}{2}\right)\right)^2\right]^2} \right]$$

$$N_{0-refr} = \frac{2(n-1)^4 \left(n+1^3\right)}{\pi\left[(n+1)(3n-1) + \left(3n^2+1\right)(n-1)\, \ln\left(\frac{n(n-1)}{\left(n^2+1\right)}\right)\right]}$$

These new expressions were derived using the full Generalized Eikonal Approximation and are much more accurate than the ones given by Van de Hulst[5]. They account much better for the large angle scattering contribution. They lead to a much lower value of the refraction term in the back hemisphere which thus allows the reflection terms to dominate. For example at an index of 1.1 and 90 degrees, the new form is 1/20 of the old Van de Hulst form.

6. EVALUATION OF THE REFLECTED TERM

The evaluation of most of the reflected term is not modified in the present work. The results are the same as those found in Jonasz-Fournier[3] and in Fournier[4] with the exception of the $\omega_{2,back}$ term for which we have found a new exact formulation.

$$\begin{aligned}\omega_{2,back} = -\frac{16}{15(\,n^2-1)^6 (n^2+1)^7} \Big\{ & n^4 \left(n^2+1\right)(n\text{-}1)\big(79\,n^{17} - 131\,n^{16} + 270\,n^{15} - 430\,n^{14} \\ & + 3094\,n^{13} - 2170\,n^{12} + 2626\,n^{11} - 3010\,n^{10} + 9784\,n^9 - 3460\,n^8 + 3010\,n^7 \\ & - 2626\,n^6 + 4674\,n^5 - 590\,n^4 + 430\,n^3 - 270\,n^2 + 225\,n + 15\,\big) \\ & + 120\,n^6 \left(\,3n^{16} + 44\,n^{12} + 98\,n^8 + 44\,n^4 + 3\,\right) \ln\left(\frac{\sqrt{n^2\text{-}1}}{n\text{-}1}\right) \\ & + 15\,n^4\left(n^4+1\right)\left(\,n^{16} + 60\,n^{12} + 262\,n^8 + 60\,n^4 + 1\,\right) \ln\left(\frac{n\text{-}1}{n^2(n+1)}\right)\Big\}\end{aligned}$$

7. EVALUATION OF THE BACKSCATTER RATIO

Using the expressions outlined above and a model of the expected particle structure it is possible to compute the ratio of the radiation scattered in the back hemisphere to the total radiation scattered in all directions. This is a key parameter in both the analysis of LIDAR returns and underwater radiative transfer theory. To do this we use the model of a shelled particle with an effective index of refraction[4]. The results for the backscatter ratio as a function of shell index and roughness factor are shown in figure 2. The Junge particle size distribution exponent was the mean oceanic value[3] of 3.6.

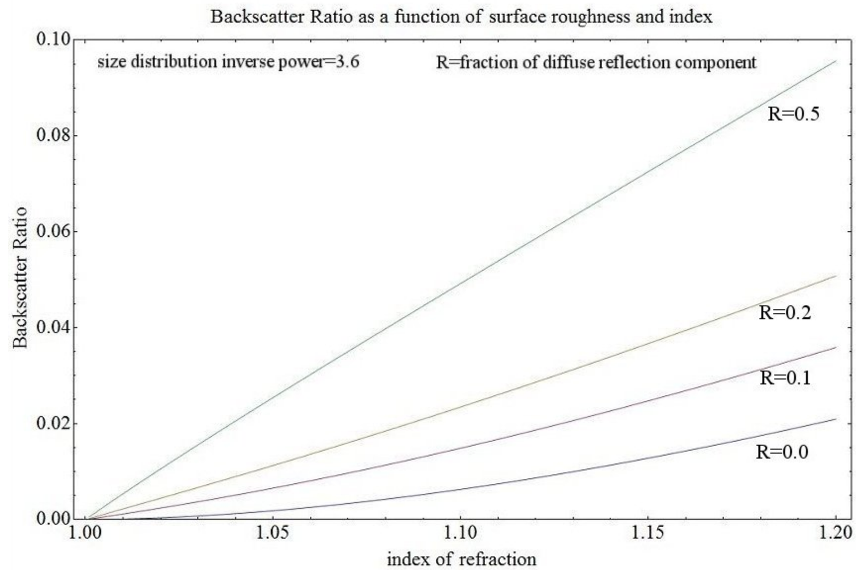

Fig 3. Modeled Backscatter ratio for a hard shelled particle as a function of surface index of refraction and surface roughness factor. The results show how much the effect of surface roughness is significant in the backscattering hemisphere and it would not be modeled by exact scattering codes which assume and require smooth surfaces.

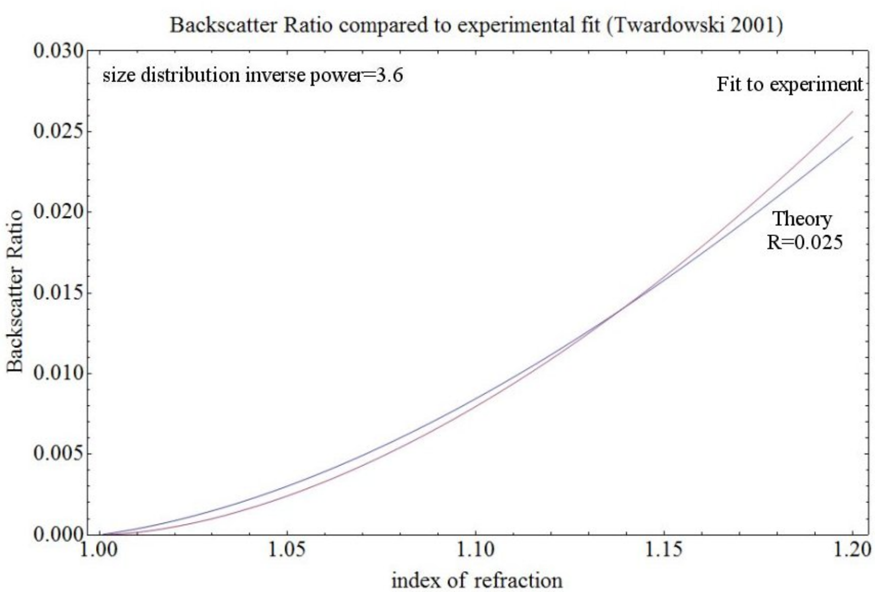

Fig 4. Backscatter ratio for a hard shelled particle as a function of surface index of refraction. The Junge particle size distribution power law exponent was the oceanic mean value of 3.6 and the best fit to the experimental results was found for a surface roughness of 0.025.

An extensive study experimental and theoretical study of the relationship between mean index of refraction and the backscattering ratio was carried out by Twardowki et al[7]. Their carefully researched empirical formula is compared to our model of a shelled particle where we have used an inverse power of 3.6, the mean value for several hundred fits of the Fournier Forand phase function to experimentally measured oceanic phase functions[3]. The close agreement for this very sensitive parameter is a clear indication that the present model could be used to significantly improve the accuracy of both LIDAR return calculations and oceanic radiative transfer models.

8. CONCLUSIONS

A new physics based analytic phase function has been derived and tested against experimental results for the backscattering ratio as a function of index of refraction. It was shown that it is sufficiently precise to allow an accurate determination of the backscatter ratio based on a simple hard shell model and an estimate of the mean index of refraction and the surface roughness of the scattering particles. Since the phase function is in this physical parameter form one can now bring to bear all the prior oceanographic knowledge one has about the nature of the scattering particles present in the different types of waters to bracket the range of the optical parameters when analyzing the LIDAR returns. This approach has the potential to significantly improve the real-time analysis of LIDAR signals to extract the optical parameters of the water column between the optical system on autonomous underwater vehicle and the scene of interest. Given these parameters and the predicted phase function, it will possible to determine the range at which the autonomous vehicle must maintain itself to obtain images sufficient quality to satisfy the requirements of its mission and feed this information into its motion control algorithms. This unattended optimization of optical imaging systems operating on board AUV's will completely change the field of remote underwater optical imaging. The presence of a tether to a surface ship is by far the most costly and range constraining condition which renders all but the more localized or shallow imaging operations unfeasible or unaffordable. A successful demonstration of fully autonomous underwater E/O imaging would mean completely opening up the oceans to low cost large-scale visual survey and inspection operations.

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