Informal Notes on Reflectances

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Introduction

Lots of people—astronomers, geologists, agronomists, the paint industry, the terrestrial remote sensing community, the stealth aircraft folks—have studied how surfaces reflect light. Unfortunately, they often use different measures of "reflectance," and they all have their own terminology and notation even when they are measuring the same physical quantity. There are many opportunities for losing factors of π and cosines of angles, and it is sometimes nearly impossible to figure out *exactly* what is being discussed when you read a paper.

These *informal* notes were originally written in 1999 for use by investigators in the ONR-funded Coastal Benthic Optical Properties (CoBOP) research program, during which various measures of bottom reflectance were made. The notes are intended to give an overview of reflectance definitions, terminology, and notation as needed by oceanographers. . The notes are longer than I originally intended, but a bit too much discussion is better than not enough, if it helps people understand the various measures of reflectance. No attempt at literacy is made, and I include a few footnotes (just for your cultural enlightenment) and a sarcastic comment or two.

For the most part, I use the definitions and terminology given in Hapke (1993), which is the best introductory textbook I've found on reflectance, and in Nicodemus, et al. (1977; referenced below as NBS160). NBS160 is a National Bureau of Standards document that discusses the measurement of reflectance in great detail and is the authoritative document on the subject. However, I have changed some notation to correspond to what is commonly used in optical oceanography, e.g., as seen in *Light and Water* (Mobley, 1994). Appendix A compares the notation used in these books.

Terminology and Notation

For convenience, let the "surface" be a horizontal plane. This can be a physical surface such as the water-sand interface of a sandy bottom, or it can be simply a particular depth in the water column, say at 1 m above a sea grass bed. To conform to NBS160, I'll use subscript i to denote *incident* and r to denote *reflected*. In the oceanographic setting of a horizontal bottom, the light incident onto the surface is traveling downward, and the light reflected by the surface is traveling upward. Thus I'll sometimes use subscript d for *downward* (incident) and u for *upward* (reflected) when necessary to conform to common oceanographic usage.

In nature, light is usually incident onto a surface from all directions, and some of the incident light gets reflected by the surface into all directions. Therefore, *to completely understand the optical properties of a surface, we have to know how the surface reflects light going in any* *incident direction into any reflected direction.*

Figure 1 shows the geometry used to describe reflectance from a surface. A cartesian (x,y,z) coordinate system is chosen with the surface lying in the x-y plane and with the *z* axis is normal (upward in our case) to the surface, an element of which is shown in blue. There is a *collimated light source*, which provides the incident light, in direction (θ_i, ϕ_i); and there is a detector, which receives the reflected light, located at the viewing direction (θ_r, ϕ_r). Surface optical properties usually depend on the wavelength λ, so the complete description of the reflectance properties of a surface will be a function (the BRDF, defined below) of five variables: θ_i, ϕ_i, θ_r, ϕ_r, λ. To make our equations as simple as possible, we'll drop the λ, but keep in mind that everything discussed below depends on wavelength.

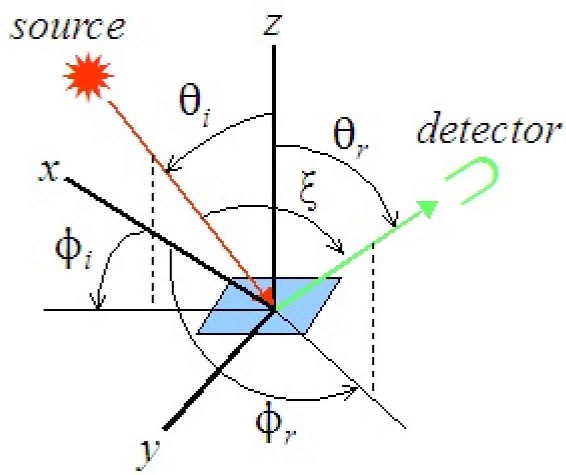

Fig. 1. Basic geometry for discussion of surface reflectance. The surface is in blue, the incident light is red, and the reflected light is green

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We can simplify things a bit by assuming that the surface is *azimuthally isotropic*, which means that its reflectance properties depend on the *difference* of ϕ_i and ϕ_r. (This is not the case for crops planted in rows or for ripples on a sandy bottom, for example.) The *specular direction* is the direction that a level mirror surface would reflect light: $(\theta_r, \phi_r) = (\theta_i, \phi_i + 180°)$. The *retroreflection direction* is the direction of exact backscatter: $(\theta_r, \phi_r) = (\theta_i, \phi_i)$. The angle ξ between the source and detector is called the phase angle[1]; it is computed from

$$\cos\xi \;=\; \cos\theta_i \cos\theta_r \;+\; \sin\theta_i \sin\theta_r \cos(\phi_i - \phi_r)\,. \qquad (1)$$

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[1] If the source is the sun and the detector is the moon and the earth is the surface, then the phase angle determines the phase of the moon as seen from the earth. This is the historical origin of the term "phase function" for the function that describes the angular pattern of scattered light; note that the scattering angle ψ is the complement of the phase angle: $\psi = 180 - \xi$.

Standards committees recommend using great care and precise language when talking about reflectance (which pretty much guarantees that almost no one will follow the recommendations, hence the confusion in the literature). For example, "reflectance" is supposed to be preceded by two adjectives: the first describes the source and the second the detector. Thus we have

the *directional-hemispherical reflectance*: tells how much light is reflected from a particular direction (θ_i, ϕ_i) into the hemisphere of all upward directions

the *hemispherical-directional reflectance*: tells how much light is reflected from all downward directions into a particular direction (θ_r, ϕ_r)

the *hemispherical-hemispherical* (or *bi-hemispherical*) *reflectance*: all downward directions into all upward directions. Note that the irradiance reflectance commonly used in optical oceanography, $R = E_u/E_d$, is a bi-hemispherical reflectance.

The Bi-directional Reflectance Distribution Function (BRDF)

We now define the *bi-directional* (i.e., *directional-directional*) *reflectance distribution function* (BRDF), which tells us *everything we need to know about how a surface reflects light*. The following discussion is based on NBS160, which treats these matters in great detail.

Conceptually, we like to think about a light beam traveling in a particular direction (θ_i, ϕ_i) being reflected into another particular direction (θ_r, ϕ_r). But since any source has some finite divergence, and any detector has some finite field of view, we can associate small solid angles $d\Omega_i$ and $d\Omega_r$ with the incident and reflected beams, respectively. The radiance of the incident beam is $L_i(\theta_i, \phi_i)$, and $L_r(\theta_r, \phi_r)$ is the reflected radiance. These quantities are shown in Fig. 2, which is a redrawn version of Fig. 1.

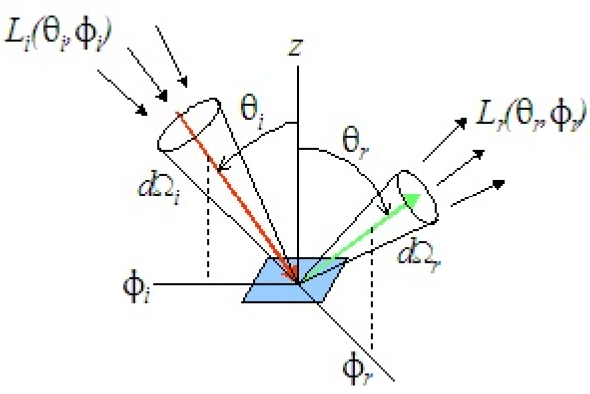

Fig. 2. Quantities used in the definition of the BRDF.

Our goal is to define a quantity that tells us how the reflective properties of the surface vary with incident and reflected directions. Therefore, consider a measurement in which we hold the direction of the detector in Fig. 2 constant while we vary the direction of the source. The BRDF is then defined as[2]

$$BRDF(\theta_i,\phi_i,\theta_r,\phi_r) \equiv \frac{dL_r(\theta_r,\phi_r)}{L_i(\theta_i,\phi_i)\cos\theta_i \, d\Omega_i(\theta_i,\phi_i)} \qquad [\mathrm{sr}^{-1}]. \qquad (2)$$

Note that if we change the only the magnitude of the incident radiance, the reflected radiance will change proportionately, and the BRDF will remain unchanged. However, if we change the direction of the incident or reflected beams while holding all else constant, the BRDF will in general change.

Equation (2) makes for an easy transition to the ivory towers of radiative transfer theory. Suppose we want to compute the total radiance heading upward in direction (θ_r,ϕ_r) owing to light incident onto the surface from all directions. We then rewrite (2) as

$$dL_r(\theta_r,\phi_r) = BRDF(\theta_i,\phi_i,\theta_r,\phi_r)\, L_i(\theta_i,\phi_i)\cos\theta_i \, d\Omega_i$$

and then integrate over all incident directions to get the total reflected radiance in direction (θ_r,ϕ_r):

$$\begin{aligned} L_r(\theta_r,\phi_r) &= \int_{2\pi_i} L_i(\theta_i,\phi_i)\, BRDF(\theta_i,\phi_i,\theta_r,\phi_r)\cos\theta_i \, d\Omega_i \\ &\equiv \int_{2\pi_i} L_i(\theta_i,\phi_i)\, r(\theta_i,\phi_i,\theta_r,\phi_r)\, d\Omega_i \, . \end{aligned} \qquad (3)$$

This last equation is exactly what is seen (with slightly different notation) in *Light and Water* Eq. (4.3), where $r(\theta_i, \phi_i, \theta_r, \phi_r)$ is called the *radiance reflectance function*. Clearly, $r(\theta_i, \phi_i, \theta_r, \phi_r) = BRDF(\theta_i, \phi_i, \theta_r, \phi_r)\cos\theta_i$, and the two functions are equivalent ways of describing a surface[3].

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[2] You will see Eq. (2) rewritten in various ways in the literature. For example, $L_i \cos\theta_i \, d\Omega_i$ is the irradiance of the incident collimated beam onto the horizontal surface, so NBS160 also writes Eq. (2) as dL_r/dE_i. However, Eq. (2) gives us all we need.

[3] Radiative transfer folks like to measure their irradiances on surfaces normal to the direction of light propagation, whereas people who live in the real world like to measure their irradiances on the surface of interest. The $\cos\theta_i$ factor just projects the incident beam irradiance

This is one of those places where it is easy to lose a cosine factor when comparing an observational paper and a theory paper. Also, some people like to add a factor of π to the numerator of Eq. (2). Finally, note that the BRDF is a reflectance per unit solid angle; it can have any non-negative value. As we'll see below, it's only when you integrate the BRDF over solid angle to get, for example, an irradiance reflectance that the result is bounded by one.

It is emphasized that the BRDF completely describes the net effect of everything that happens on or below the surface where it is measured. For example, if the BRDF is measured in the water column 1 m above a sea grass bed, then all the effects of the light interacting with the grass, sediments, and water below the 1 m surface are accounted for in this BRDF. Knowing the BRDF on this imaginary surface would, for example, allow Hydrolight to compute the radiance distribution in the region *above* the depth where the BRDF was measured[4]. *Predicting* or *computing* the BRDF of the grass and sediments is quite another story: to do that you have to understand all of the extremely complicated interactions of light with the grass and sediment particles.

By the way, there is an important reciprocity theorem about what happens if you interchange the positions if the source and detector. It states simply that

$$BRDF(\theta_i,\phi_i,\theta_r,\phi_r) = BRDF(\theta_r,\phi_r,\theta_i,\phi_i) \,. \qquad (4)$$

If you measure or conjure up a BRDF and it doesn't obey Eq. (4), then it's simply wrong.

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onto the horizontal surface.

[4] Indeed, this is how Hydrolight models infinitely deep, homogeneous water without actually solving the radiative transfer equation to extreme depth. The BRDF of an infinitely deep, homogeneous layer of water with known inherent optical properties can be found analytically (as you recall from Section 9.5 of *Light and Water*, which you have all read). Thus, when Hydrolight simulates infinitely deep water, it first computes the BRDF of the infinitely deep water below the maximum depth z_{max} you are interested in, and it then uses that BRDF at z_{max} just as though there were an actual physical bottom at z_{max}.

Examples of BRDFs

Many researchers in the oceanographic community are unfamiliar with BRDFs other than the one for Lambertian surfaces. Therefore it is worthwhile to take a look at a few BRDFs for the purpose of building intuition about what these things look like. In particular, we'll learn what features to look for in our own BRDFs, namely specular reflection and hot spots. We'll go from simple to complicated.

Example 1: The Lambertian BRDF

A *Lambertian surface* by definition reflects radiance equally into all directions[5]. Its BRDF is simply

$$BRDF_{\text{Lamb}}(\theta_i,\phi_i,\theta_r,\phi_r) \;=\; \frac{\rho}{\pi}\,, \tag{5}$$

where ρ is called the *reflectivity* of the surface. The reflectivity varies from zero for a completely absorbing ("black") surface, to one for a completely reflecting surface. There are no Lambertian surfaces in nature, but matte paper is a good approximation except at grazing angles (θ_i and θ_r near 90 degrees), where the surface begins to look "shiny."

In addition to their mathematical simplicity, Lambertian surfaces have an extremely important property. To see what it is, we compute the irradiance reflectance $R = E_u/E_d$ of a Lambertian surface (here we use *d* and *u* subscripts to identify the downwelling and upwelling irradiances, respectively, which is common usage in optical oceanography, but we'll keep the *i* and *r* subscripts in the integrals):

[5] There is a subtlety in this statement. For a given incident lighting, the number of photons reflected by *each point* of a Lambertian surface is proportional to $\cos\theta_r$, which is why Lambertian surfaces are sometimes called "cosine reflectors." However, if you view the surface with a radiance detector having a fixed field of view, the area of surface that you are viewing is proportional to $1/\cos\theta_r$. Thus the number of photons going into the detector is independent of θ_r, and the *reflected radiance* is independent of direction.

$$R = \frac{E_u}{E_d} = \frac{\iint_{2\pi_r} L_r(\theta_r,\phi_r)\,|\cos\theta_r|\,d\Omega_r}{\iint_{2\pi_i} L_i(\theta_i,\phi_i)\,|\cos\theta_i|\,d\Omega_i}$$

$$= \frac{\iint_{2\pi_r}\left[\iint_{2\pi_i} L_i(\theta_i,\phi_i)\,BRDF(\theta_i,\phi_i,\theta_r,\phi_r)\,|\cos\theta_i|\,d\Omega_i\right]|\cos\theta_r|\,d\Omega_r}{\iint_{2\pi_i} L_i(\theta_i,\phi_i)\,|\cos\theta_i|\,d\Omega_i} \qquad (6)$$

The first equation here is just the definitions of the plane irradiances in terms of the incident (downwelling) and reflected (upwelling) radiances. The differential of solid angle is $d\Omega = \sin\theta d\theta d\phi$. In going to the second equation, Eq. (3) has been used to write the upwelling radiance reflected from the surface in terms of the downwelling radiance onto the surface and the *BRDF* of the surface. *Equation (6) is completely general and is the fundamental equation for computing the irradiance reflectance of any surface, given the BRDF of the surface and the incident radiance onto the surface.*

Substituting the Lambertian BRDF of Eq. (5) into Eq. (6) and rearranging gives

$$R = \frac{\frac{\rho}{\pi}\left[\iint_{2\pi_i} L_i(\theta_i,\phi_i)\,|\cos\theta_i|\,d\Omega_i\right]\iint_{2\pi_r}|\cos\theta_r|\,d\Omega_r}{\iint_{2\pi_i} L_i(\theta_i,\phi_i)\,|\cos\theta_i|\,d\Omega_i} = \rho\,,$$

since the integrals over $2\pi_i$ cancel, and the integral over $2\pi_r$ equals π. *A Lambertian surface thus has the property that its irradiance reflectance R equals its reflectivity* ρ *and, furthermore, its irradiance reflectance R is independent of the incident radiance.* Both of these results are true *only* for Lambertian surfaces. For non-Lambertian surfaces, *R* generally depends both on the surface and on the incident lighting.

Example 2: A Physically-Based BRDF

BRDFs derived from a rigorous consideration of how light interacts with a surface or a scattering medium are extremely rare animals. Here is one such BRDF. Consider an infinitely deep layer of particles that scatter light isotropically and independently of each other. The albedo of single scattering of the bulk medium is $\omega_o = b/(a + b)$, where *a* and *b* are the absorption and scattering coefficients, respectively, of the medium. Then the BRDF at the surface of the infinitely deep layer of scattering particles is given by (Hapke, Eq. 8.47)

$$BRDF(\mu_i,\phi_i,\mu_r,\phi_r) = \frac{\omega_o}{4\pi}\frac{1}{\mu_i + \mu_r}\frac{1 + \mu_i}{1 + 2\gamma\mu_i}\frac{1 + \mu_r}{1 + 2\gamma\mu_r}. \qquad (7)$$

Here $\mu = \cos\theta$ and $\gamma = \sqrt{(1 - \omega_o)}$. This BRDF is plotted in Fig. 3 as a function of (θ_r, ϕ_r) for $(\theta_i, \phi_i) = (30°,0)$ and $\omega_o = 0.9$. In this and subsequent figures, we will let the source be in the $\phi_i = 0$ direction, in which case our geometry is specified by three angular variables: θ_i, θ_r, ϕ_r. The azimuthal viewing angle needs to be shown only for $0 \leq \phi_r \leq 180°$ because there is symmetry for $180° \leq \phi_r \leq 360°$. The two directions of particular interest, namely retroreflection and specular reflection, are labeled.

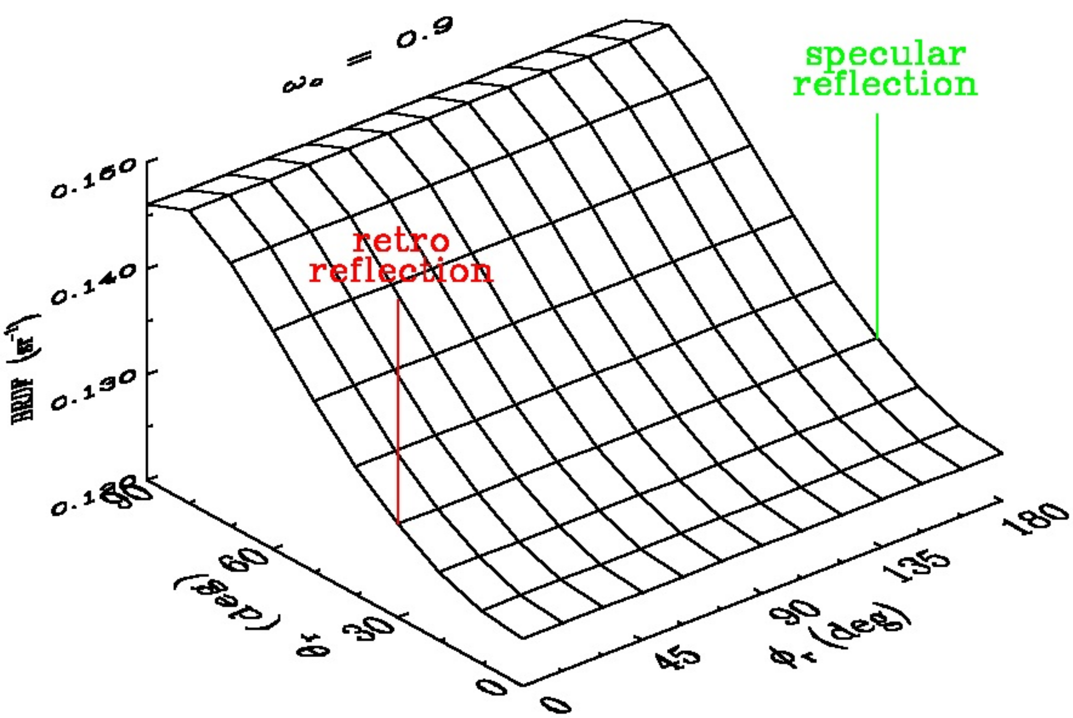

Fig. 3. The BRDF of Eq. (7) for $(\theta_i, \phi_i) = (30°,0)$ and $\omega_o = 0.9$. The red and green lines correspond to the directions of retroreflection and specular reflection.

This BRDF is called "physically based" because the optical or physical parameters of the medium (in this case, ω_o) appear explicitly in the formula for the BRDF. Such models are great for deducing the characteristics of the medium from BRDF measurements. In the present case, if you can justify assuming that the particles in your medium scatter light isotropically and independently, then you can adjust ω_o until Eq. (7) gives the best fit to your data. You have then learned something about the absorbing and scattering properties of the particles. The problem for us in CoBOP is that an assumption of isotropic and independent scattering is probably not justified for sediments whose particles are touching each other. Thus our sediments have more complicated BRDFs (Fig. 7, below). For the record, the derivation of Eq. (7) can be redone for particles that scatter anisotropically according to a one-term Henyey-Greenstein phase function. The resulting BRDF is more complicated, but also more realistic. See Hapke, Eq. (8.89) for the details.

Example 3: A BRDF with Specular Reflection

Many surfaces display at least some specular reflection. A good example is the sea surface itself. Figure 4 shows the BRDF of the sea surface and underlying water body as generated by Hydrolight. To generate this BRDF, Hydrolight was run with the sun at $(\theta_i, \phi_i) = (30°,0)$ in a black sky (to get a collimated incident irradiance). The surface was modeled as capillary waves for a 10 m s^{-1} wind speed, and the water was modeled using bio-optical models for case 1 water with a chlorophyll concentration of 2 mg m^{-3}. The wavelength is 450 nm. The reflected radiance used to compute the BRDF is the total of the water-leaving radiance and the surface-reflected sunlight (sun glitter).

The sun glitter appears in the BRDF as the large bump near the specular direction[6]. This BRDF does not show any noticeable retroreflection.

[6] The maximum of the BRDF is not exactly at the specular direction in this simulation, which is a consequence of the larger Fresnel reflectances (see Appendix B) for rays reflected through larger θ_r angles when $\phi_r = 180°$. The BRDF at $\phi_r = 180°$ drops off for $\theta_r > 50°$ because the number of reflected rays decreases faster than the Fresnel reflectance increases.

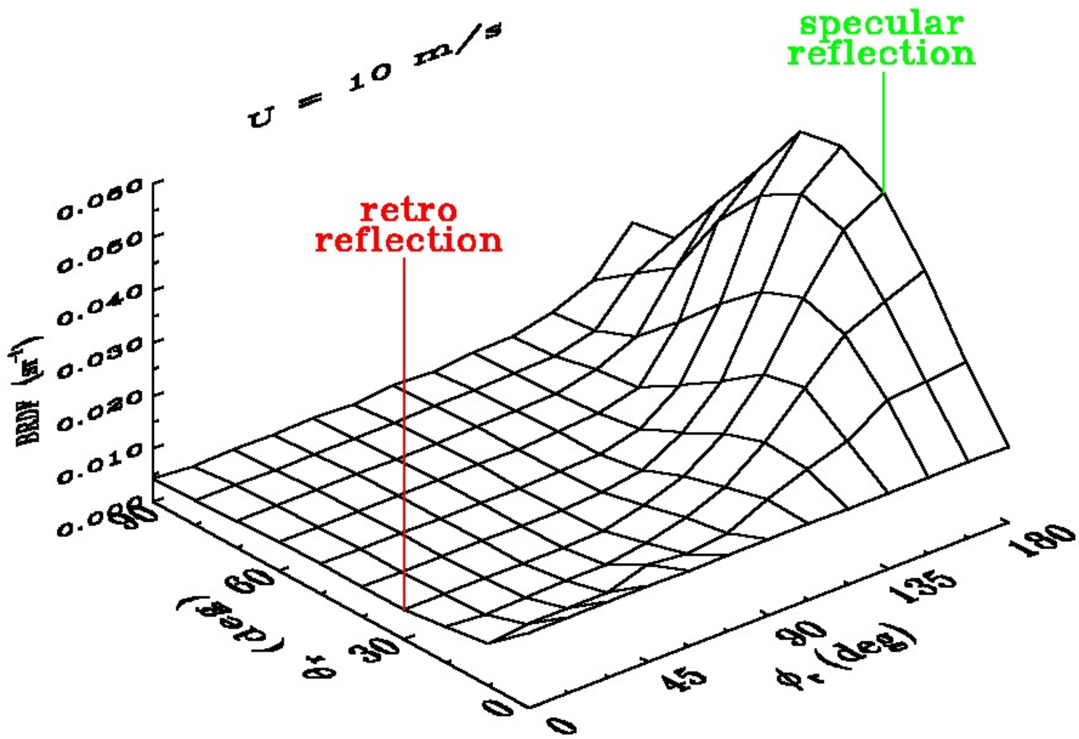

Fig. 4. The BRDF of the sea surface as computed by Hydrolight for a wind speed of 10 m s^{-1}.

This is also a physically-based BRDF in the sense that it was computed by a rigorous solution of the radiative transfer equation for a highly complicated system of sea surface and underlying water and phytoplankton. We just don't have a nice simple formula like Eq. (7) to show us how the inherent optical properties of the water determine the BRDF.

Example 4: A BRDF with a Hot Spot

Vegetation canopies and bare soils often show increased reflection near the retroreflection, or 180° backscatter, direction. This phenomenon[7] is usually called the "hot spot," "opposition

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[7] This isn't the same thing as "enhanced backscatter," which occurs within a fraction of a degree of the 180° backscatter direction, or the "glory" which you often see in a cloud around your airplane's shadow. The hot spot is caused by "shadow hiding" in the leaves or soil particles; enhanced backscatter is coherent backscatter from densely packed particles; and the glory arises from rays being refracted by spherical water droplets, somewhat like a rainbow.

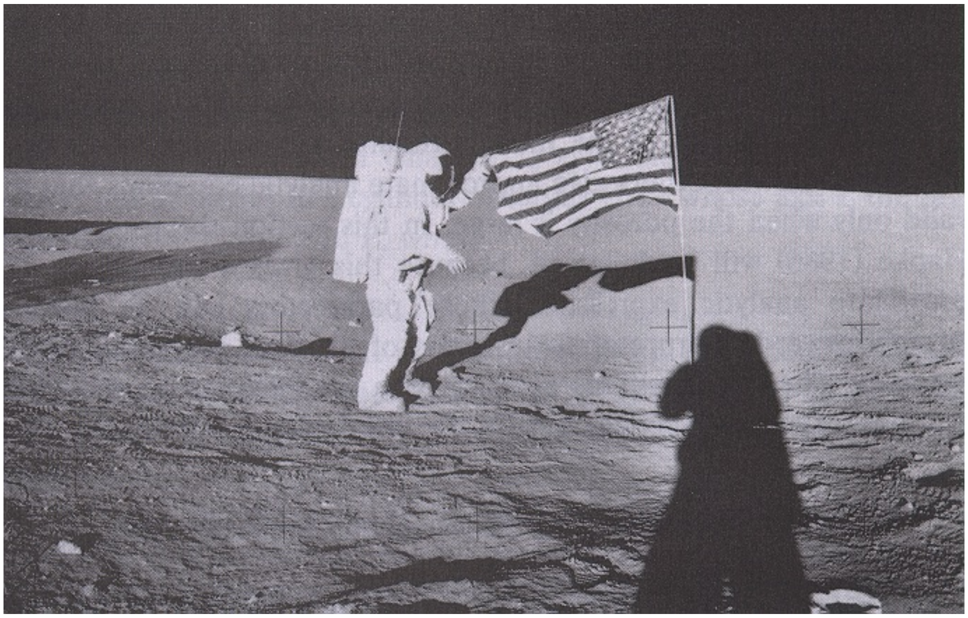

Fig. 5. Hot spot on the lunar surface. Note that the ground appears brighter around the shadow of the photographer's head. (This is a scanned image of a halftone picture and doesn't print well. You have to go to the moon to get the full effect.)

effect," or "Heiligenschein." Figure 5 shows an example from the surface of the moon. Note that the lunar surface is much brighter around the shadow of the photographer's head than it is at viewing directions to his side[8].

The terrestrial remote-sensing community has developed many *semi-analytic* BRDFs for vegetation canopies (a good summary is in Cabot and Dedieu, 1997). These BRDFs usually parameterize scattering within the medium via a very simple scattering phase function [generally the One-Term Henyey-Greenstein (OTHG) phase function]. Features like the hot spot are accounted for by tacking on *ad hoc* functions with more-or-less the right angular shape. The models generally have several free parameters whose values must be determined by a least-squares fit to a measured BRDF. These parameters may or may not be relatable to physical quantities.

[8] The artist Benvenuto Cellini once observed the Heiligenschein ("holy shine") around his own head but not around the heads of his companions. He took this to be a sign of his own genius.

One of the simplest such BRDFs is given by Rahman, et al. (1993):

$$BRDF(\theta_i,\phi_i,\theta_r,\phi_r) = \rho_o \frac{(\cos\theta_i \cos\theta_r)^{k-1}}{(\cos\theta_i + \cos\theta_r)^{1-k}} F(g) \left(1 + \frac{1-\rho_o}{1+G} \right) , \qquad (8)$$

where $F(g)$ is the OTHG phase function,

$$F(g) = \frac{1 - g^2}{\left[1 + g^2 - 2\, g \cos(\pi - \xi)\right]^{1.5}} ,$$

and G is

$$G = \left[\tan^2\theta_i + \tan^2\theta_r - 2 \tan\theta_i \tan\theta_r \cos(\phi_i - \phi_r)\right]^{1/2} .$$

This BRDF model has three parameters, ρ_o, k, and g, which must be determined by a best fit to the measured BRDF. Equation (8) was not derived from rigorous mathematical arguments applied to an underlying physical model of how the medium scatters light, as was Eq. (7). Equation (8) was pieced together using convenient mathematical functions chosen because they can give a reasonable reproduction of measured vegetation BRDFs. The OTHG phase function is being used to parameterize scattering within the vegetation or soil; the actual scattering is undoubtedly much more complicated. G is simply a function which has a minimum at $\theta_i = \theta_r$ and $\phi_i = \phi_r$; the factor $[1 + (1 - \rho_o)/(1 + G)]$ is then a maximum, which crudely approximates the hot spot. Figure 6 shows this BRDF for $(\theta_i, \phi_i) = (30°,0)$ and for parameter values of ρ_o = 0.133, $k = 0.851$, and $g = -0.114$, which were determined by fitting Eq. (8) to BRDF measurements of a wheat field. This vegetation BRDF shows an obvious hot spot in the retroreflection direction, roughly like the one seen in Fig. 5. There is no noticeable specular reflection.

Note that this BRDF becomes infinite for θ_i and θ_r both equal to 90°. This is a common feature of analytical BRDFs and it is perfectly OK. Remember, a BRDF basically gives a (unnormalized) probability of reflection per unit solid angle, and a probability density function can have any non-negative value. The only physical requirement is that when you integrate a BRDF to get a reflectance, as in Eq. (6), then the reflectance must be between 0 and 1. If you put the BRDF of Eq. (8) into Eq. (6), then the infinite BRDF at $\theta_i = \theta_r = 90°$ is being multiplied by $\cos\theta_i = 0$ and $\cos\theta_r = 0$, and the integral remains finite.

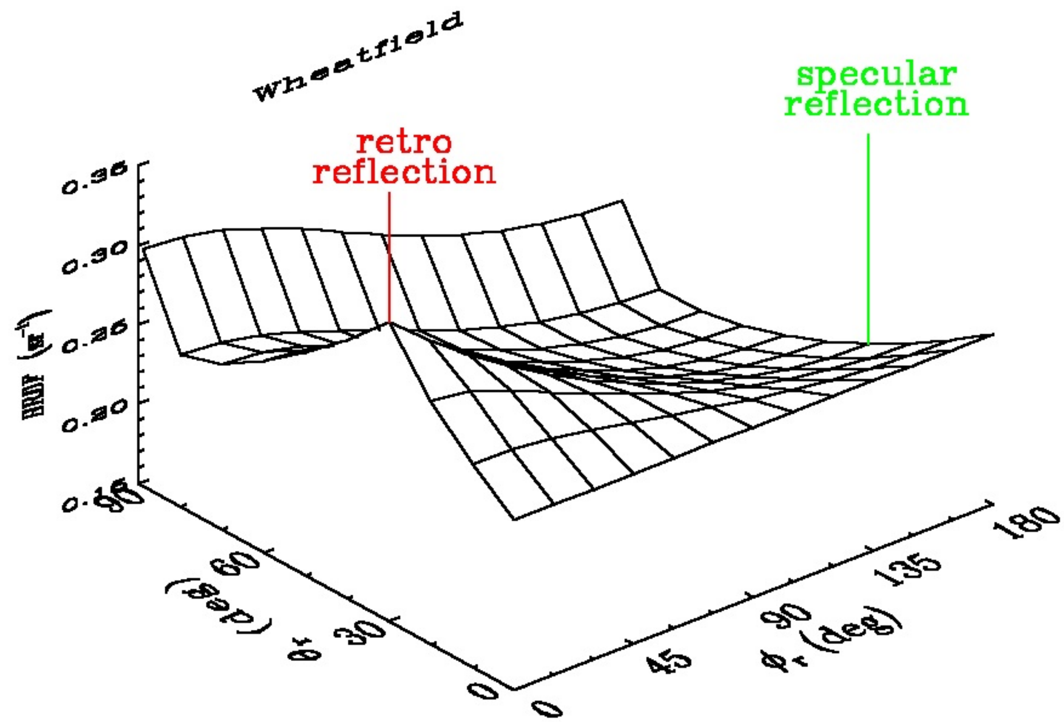

Fig. 6. The BRDF of Eq. (8) for parameter values typical of a wheat field.

Example 5: A Measured BRDF of Sand

Many surfaces display both specular reflection and hot spots to some degree. Figure 7 shows a BRDF measured by Ken Voss on a sample of ooid sand at the CoBOP Rainbow Gardens site. This figure has the light source at $(\theta_i, \phi_i) = (65°,0)$, which gives a BRDF with a noticeable hot spot and a bit of specular reflection. Incident angles nearer to the zenith direction (not shown here) look more Lambertian. Note that this figure shows the full 360° range of ϕ_r in order to show any azimuthal anisotropy of the sample; $\phi_r = \pm 180°$ are the same direction. The nominal wavelength is 568 nm.

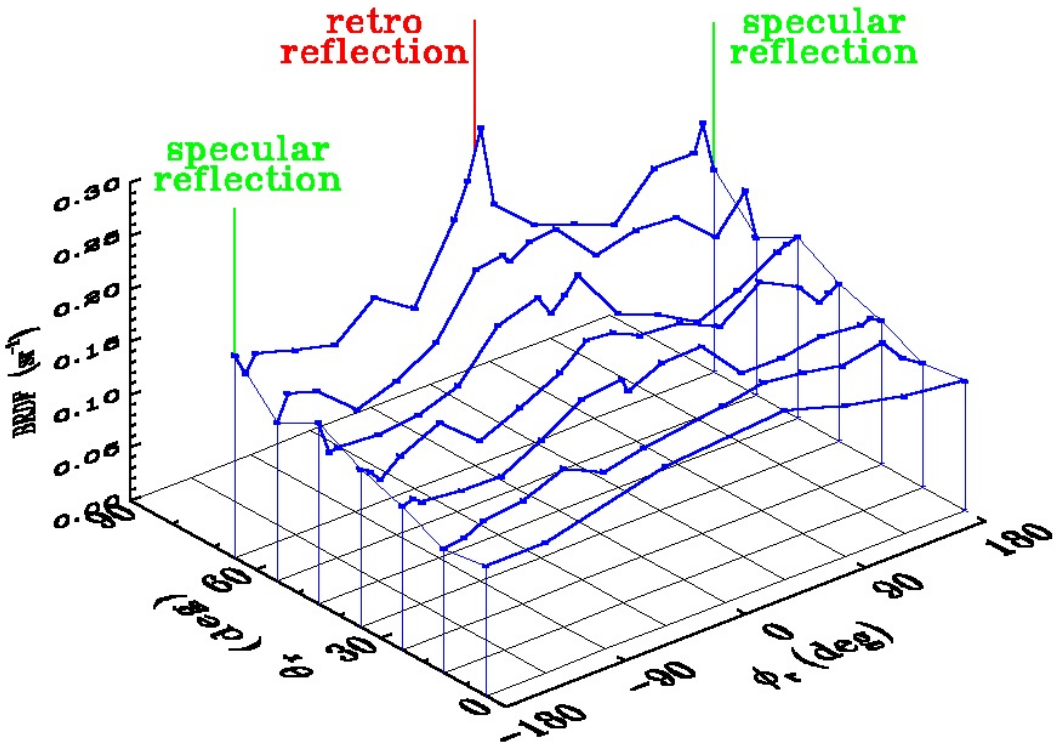

Fig. 7. BRDF of ooid sand measured at the CoBOP Rainbow Gardens site (data provided by Ken Voss).

Other Measures of Reflectance

If we could always measure or model the complete BRDF our radiative transfer problems would be solved. However, measuring even a partial BRDF for a few values of θ_i, ϕ_i, θ_r, ϕ_r, λ is a difficult and tedious task in the laboratory. As far as we know, no one has ever tried to measure a BRDF under water until CoBOP. Of course, everyone wants to have some easily made (compared to a BRDF) measure of surface reflectance that, with appropriate assumptions, can be used to model the optical properties of the surface. This leads us to various other reflectances and quantities derived from the BRDF.

Albedos

There are enough definitions of "albedo" to make you take early retirement. Hapke's book defines bolometric, Bond, geometric, hemispherical, normal, physical, plane, single-scattering, and spherical albedos, as well as an albedo factor. To add insult to injury, not a single one of these albedos corresponds to how "the" albedo is defined in *Light and Water* (page 193). Fortunately, we don't have to deal with all of these albedos in CoBOP, but we should clarify a point that can cause confusion when reading BRDF papers (which sometimes just say, "...and the albedos is..." without telling you which one they're talking about).

Optical oceanographers generally think of the albedo as being the ratio of the upwelling plane irradiance to the downwelling plane irradiance, for *whatever conditions of incident lighting you have in nature at the time of measurement.* (This is how the albedo is defined in *Light and Water*.) This is what you need to know to compute an energy balance in the real world. Thus the oceanographers' albedo is the same as the irradiance reflectance $R = E_u/E_d$, as computed via Eq. (6). The surface-reflectance people like to define their albedos and reflectances in terms of *isotropic illumination of the surface*, i.e., the incident radiance $L_i(\theta_i,\phi_i)$ seen in Eq. (6) is a constant independent of (θ_i,ϕ_i). For isotropic incident radiance, Eq. (6) reduces to just

$$A \equiv \frac{1}{\pi}\iint_{2\pi_r}\left[\iint_{2\pi_i} BRDF(\theta_i,\phi_i,\theta_r,\phi_r)\,|\cos\theta_i|\,d\Omega_i\right]|\cos\theta_r|\,d\Omega_r\,.$$

This quantity is called the Bond or spherical albedo, or the spherical or bi-hemispherical reflectance, or just "the" albedo, depending on the author's preference. Note that this A is not equal to $R = E_u/E_d$ unless the incident lighting is isotropic (which never occurs in nature) or unless the surface is Lambertian (which never occurs in nature). For a Lambertian surface, $A = \rho$.

The same convention of assuming isotropic illumination of the surface is often used when defining other reflectances, e.g., the hemispherical-directional reflectance. Note that the convention of using isotropic illumination when defining albedos and reflectances isn't necessarily bad: it removes a complicating factor—variable incident lighting—from the discussion of surface properties. It's just that we don't have that luxury in CoBOP, where we have to live with whatever incident radiance nature gives us.

The Irradiance Reflectance vs. The Bi-Hemispherical Reflectance

As already noted, the oceanographers' albedo or irradiance reflectance $R = E_u/E_d$ as given by Eq. (6) is a bi-hemispherical reflectance, but it is not "the" bi-hemispherical reflectance as defined in books such as Hapke, because our R uses the actual incident radiance distribution in Eq. (6), rather than an isotropic incident radiance.

The Remote-Sensing Reflectance

The remote-sensing reflectance,

$$R_{rs}(\theta_r,\phi_r) \equiv \frac{L_r(\theta_r,\phi_r)}{E_d} \qquad (\mathrm{sr}^{-1}),$$

has the same units as the BRDF, but they are not the same thing. Note in particular that R_{rs} uses the total (from all directions) downwelling irradiance E_d, whereas the incident irradiance in the definition of the BRDF is in a collimated beam. [Also, some people call this ratio the "remote-sensing reflectance" only if the measurement is being made just above the sea surface and L_r is the water-leaving radiance (the total upward radiance minus the surface-reflected sky radiance).]

The Reflectance Factor and the Radiance Factor

The *reflectance factor REFF* (also called the *reflectance coefficient*) is defined as the ratio of the BRDF of the surface to that of a perfectly diffuse surface under the same conditions of illumination and observation. "Perfectly diffuse" means a Lambertian surface with $\rho = 1$. Thus

$$REFF(\theta_i,\phi_i,\theta_r,\phi_r) \equiv \frac{BRDF(\theta_i,\phi_i,\theta_r,\phi_r)}{BRDF_{\mathrm{Lamb}}(\mathrm{with}\ \rho = 1)} = \pi\, BRDF(\theta_i,\phi_i,\theta_r,\phi_r).$$

The *radiance factor RADF* is defined as the reflectance factor for normal illumination, i.e., for $\theta_i = 0$. Thus

$$RADF(\theta_r,\phi_r) \equiv \pi\, BRDF(0,0,\theta_r,\phi_r).$$

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Appendix A: Comparison of Notation

For those of you who want to refer to Mobley (1997), Hapke (1993), or NBS160 along with these notes, here are the equivalents for several needed quantities.

Quantity	**These Notes**	**Mobley (1994)**	**Hapke (1993)**	**NBS160**
radiance	L	L	I	L
irradiance	E	E	J	E
single-scattering albedo	ω_o	ω_o	w	—
scattering angle	ψ	ψ	θ	—
mean cosine of scattering angle	g	g	ξ	—
phase angle	ξ	—	g	—
incident polar angle	θ_i	θ'	i	θ_i
reflected polar angle	θ_r	θ	e	θ_r
incident azimuthal angle	ϕ_i	ϕ'	set to 0	ϕ_i
reflected azimuthal angle	ϕ_r	ϕ	ψ	ϕ_r
solid angle	Ω	Ω	Ω	ω
BRDF	BRDF	$r/\cos\theta_i$	BRDF	f_r
reflectance	R	R	r	ρ

Appendix B: The Fresnel Reflectance as a BRDF

The Fresnel reflectance R_F is the reflectance of a perfectly smooth surface between two media of different indices of refraction. Formulas for R_F are given, for example, in Mobley (1994, page 157). Figure 8 gives R_F for unpolarized light incident onto either side of an air-water surface; the water has a real index of refraction (relative to the air) of $n = 1.34$.

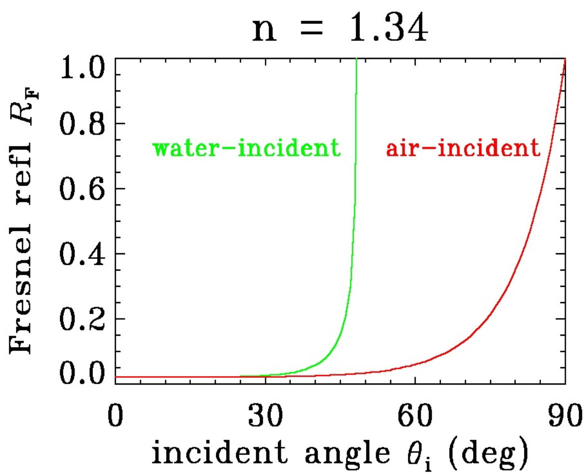

Fig. 8. Fresnel reflectance of an air-water surface for unpolarized incident light.

R_F can be combined with Dirac delta functions to create a BRDF. Consider the BRDF

$$BRDF(\theta_i,\phi_i,\theta_r,\phi_r) = 2\, R_F\, \delta(\sin^2\theta_r - \sin^2\theta_i)\, \delta(\phi_r - \phi_i \pm \pi)\,. \qquad (9)$$

Inserting Eq. (9) into Eq. (3) gives

$$\begin{aligned} L_r(\theta_r,\phi_r) &= 2\, R_F \int_{2\pi_i} L_i(\theta_i,\phi_i)\, \delta(\sin^2\theta_r - \sin^2\theta_i)\, \delta(\phi_r - \phi_i \pm \pi) \cos\theta_i \sin\theta_i\, d\theta_i\, d\phi_i \\ &= R_F\, L_i(\theta_i = \theta_r,\ \phi_i = \phi_r \pm \pi)\,. \end{aligned}$$

This last equation is the form usually seen in the definition of the Fresnel reflectance as being the ratio of reflected to incident radiances for angles related by the law of reflection.

Appendix C: Using the BRDF as a Probability Distribution

Equation (3) shows how the BRDF is used in Hydrolight, which is always working with *radiances*. In a Monte Carlo simulation, you are tracking many individual *photon packets* as they interact with the medium and its boundary surfaces. In this case, the BRDF must be used as a probability distribution function (pdf) to determine the direction and weight of the reflected photon packet whenever a photon packet hits the bottom. This is a tricky business the first time you encounter it, and journal articles never give you the necessary details (which, after you spend days or weeks figuring them all out, are then pronounced to be "obvious"). With luck, you'll never need to know this stuff, so feel free to quit reading now. But, if you ever have to develop a Monte Carlo code that includes bottom reflectance, you'll thank me for putting this on paper. So, without further ado:

Given: A photon packet with weight w_i is incident onto the bottom in direction (θ_i, ϕ_i). The BRDF of the bottom is known.

Needed: The weight w_r and direction (θ_r, ϕ_r) of the reflected photon packet.

Since the input direction (θ_i, ϕ_i) is known, the BRDF$(\theta_i, \phi_i, \theta_r, \phi_r)$ can be viewed as an (unnormalized) bivariate pdf for the reflected angles θ_r and ϕ_r. Note that, in general, these angles are correlated. Proceed as follows:

1) Compute the directional-hemispherical reflectance for the given (θ_i, ϕ_i):

$$\begin{aligned}\rho^{dh}(\theta_i,\phi_i) &= \int_{2\pi_i} BRDF(\theta_i,\phi_i,\theta_r,\phi_r)\cos\theta_r\, d\Omega(\theta_r,\phi_r) \\ &= \int_0^{2\pi}\int_0^{\pi} BRDF(\theta_i,\phi_i,\theta_r,\phi_r)\cos\theta_r \sin\theta_r\, d\theta_r\, d\phi_r\,.\end{aligned} \tag{C1}$$

2) The reflected packet weight is $w_r = \rho^{dh}(\theta_i, \phi_i)\, w_i$.

3) Compute the cumulative distribution function (cdf) for ϕ_r by

$$CDF_\phi(\phi_r) = \frac{1}{\rho^{dh}(\theta_i,\phi_i)} \int_0^{\phi_r}\int_0^{\pi/2} BRDF(\theta_i,\phi_i,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi\,. \tag{C2}$$

Note that the directional-hemispherical reflectance is being used to convert the BRDF into a normalized bivariate pdf for θ_r and ϕ_r. We are then "integrating out" the θ_r dependence to leave a pdf for ϕ_r, which is then being used to construct the cdf for ϕ_r.

4) Draw a random number $\Re$ from a uniform [0,1] distribution. Solve the equation

$$\Re = CDF_\phi(\phi_r) \tag{C3}$$

for ϕ_r. This is the randomly determined azimuthal angle of the reflected photon packet.

5) Compute the cdf for angle θ_r from

$$CDF_\theta(\theta_r) = \frac{\int_0^{\theta_r} BRDF(\theta_i,\phi_i,\theta,\phi_r)\cos\theta\,\sin\theta\,d\theta}{\int_0^{\pi/2} BRDF(\theta_i,\phi_i,\theta,\phi_r)\cos\theta\,\sin\theta\,d\theta}. \tag{C4}$$

Note that the angle ϕ_r determined in step 4 is used in the BRDF in Eq. (C4) when evaluating the θ intetrals. This is how the correlation between θ_r and ϕ_r. is accounted for in the determination of the reflection angles.

6) Draw a new random number $\Re$ from a uniform [0,1] distribution and solve the equation

$$\Re = CDF_\theta(\theta_r) \tag{C5}$$

for θ_r. This is the randomly determined polar angle of the reflected photon packet. You can now send the new photon packet on its way.

For all but the simplest BRDFs, Eqs. (C1) to (C5) all must be evaluated numerically for each photon packet, which can be an enormous computer cost when millions of photon packets are being traced.

A Simple Example.

The Minnaert BRDF[9] is

$$BRDF_{\mathrm{Minn}}(\theta_i,\phi_i,\theta_r,\phi_r) \;=\; \frac{\rho}{\pi}\left(\cos\theta_i\cos\theta_r\right)^k .$$

Note that for $k = 0$ this reduces to the Lambertian BRDF. Equations (C1) to (C5) can be evaluated analytically for the Minnaert BRDF. Equation (C1) evaluates to

$$\rho^{\mathrm{dh}} \;=\; \frac{2\rho}{k+2}\cos^k\theta_i ,$$

which reduces to $\rho^{\mathrm{dh}} = \rho$ for a Lambertian surface. Equation (C2) gives just

$$CDF_\phi(\phi_r) \;=\; \frac{\phi_r}{2\pi} .$$

Plugging this into Eq. (C3) and solving for ϕ_r gives

$$\phi_r \;=\; 2\pi\Re .$$

Thus the azimuthal angle is uniformly distributed over 2π radians. The cdf for θ_r as given by (C4) is

$$CDF_\theta(\theta_r) \;=\; 1 - \cos^{k+2}\theta_r .$$

Equation (C5) then gives

$$\theta_r \;=\; \cos^{-1}\left(\sqrt[k+2]{\Re}\right),$$

---

[9] This BRDF was conjured up to explain the curious fact that the full moon appears almost uniformly bright from the center to the edge of the lunar disk. If the lunar dust were a Lambertian reflector, the full moon would appear bright at the center and darker at the edge. However, the Minnaert BRDF agrees with observation over only a limited range of angles.

after noting that 1 - $\Re$ has the same distribution as $\Re$. For a Lambertian surface, the randomly generated θ_r angles are distributed as $\cos^{-1}(\sqrt{\Re})$, which certainly isn't intuitive. However, this distribution is precisely what is necessary to make the number of reflected photons *per unit solid angle* proportional to $\cos\theta_r$, as mentioned in footnote 5.