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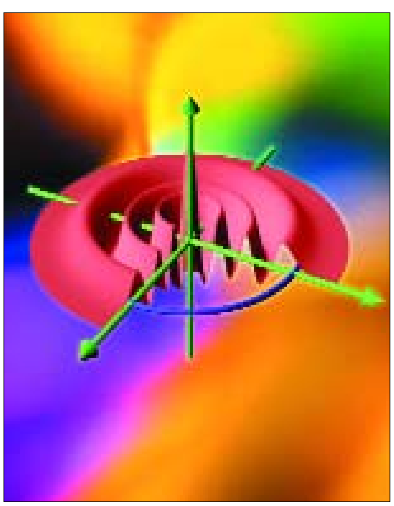

About the cover

Artist's rendition of a Wigner function for six photons (see Mack and Schleich, p. 28). This issue of *OPN Trends* was conceived to bring together different views regarding a question asked over the course of centuries: What is the nature of light? Despite significant progress in our understanding, it remains an open question.

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Advertising Sales
Beth Bay
212-529-4029
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The nature of light: what is a photon?

Chandrasekhar Roychoudhuri and Rajarshi Roy
Guest Editors

This issue of *OPN Trends* was conceived to bring together different views regarding a question that was first posed in ancient times but remains unanswered today. What, indeed, is “the nature of light”? Many of us still feel perplexed when asked by a student to answer the seemingly simple question, “What is a photon?”

OCIS codes: 270.0270, 260.0260.

It is staggering to consider the degree to which civilization has evolved in the approximately 90 years since Niels Bohr’s ad hoc quantization of atoms based on experimentally measured line spectra The changes that have occurred, including the growth of our knowledge of the micro and macro material worlds and the emergence of new technologies, have progressed far beyond the imagination of people living at the turn of the 20th century. In the 21st century, the pace of development will accelerate as a result of the rapid evolution of photonics. Yet the underlying science of the field is still Maxwell’s classical electromagnetism, not the field-quantized photon. We can certainly expect new photon-led breakthroughs in which the quantized nature of photons is intrinsically important, e.g., quantum encryption. The issue is important both in the scientific and in the technology driven socio-economic contexts.

Writing a semi-popular article on the nature of the photon is a difficult task. We are very thankful that a number of renowned scientists have accepted the challenge and written five superb articles for all of us to enjoy. Each article in this issue of *OPN Trends* presents a somewhat different selection of facts and illuminates historical events with interesting comments. As for the photon itself, we find here a variety of approaches that place it in different contexts. There are descriptions of the photon based on experiments that have used progressively refined probes to measure the interaction of light with matter. We also find descriptions of theoretical advances that have required an ever increasing understanding of the role of light in the conceptual framework of the physical universe as we view it today.

We invite our readers to embark on an exciting adventure. Before reading these articles, jot down what you think are the most pertinent facts you have learned about photons. Then read the articles in this issue in any order that appeals you. How deeply you engage yourself in this task depends on you. We ourselves are certain that we will revisit the articles and read them again for many years. At any stage of your reading, write down what you think of the photon as a result of what you have read here and what you have learned from other sources, for the photon is not an object that can be pinned down like a material object, say, a beautiful butterfly in a collection. The photon tells us, “I am who I am!” in no uncertain terms and invites us to get better acquainted with it. The chronicle will surely amuse and amaze you, for you will realize that any description of the photon, at any time—even when made by the most learned expert—is but a glimpse of a reality that holds wonders beyond the grasp of any human. At least, that is how it appears to some of us today.

Chandrasekhar Roychoudhuri, University of Connecticut, and Rajarshi Roy, University of Maryland, College Park, are the guest editors of this issue of OPN Trends.

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We dedicate this special issue to Professor Willis Lamb on the occasion of his 90th birthday. The “Lamb shift” triggered the development of the field of quantum electrodynamics and Professor Lamb has wrestled with the photon longer and more creatively than almost anyone alive today.

Chandrasekhar Roychoudhuri (chandra@phys.uconn.edu) is with the Photonics Lab, Physics Department, University of Connecticut, Storrs, Conn. Rajarshi Roy (rroy@glue.umd.edu) is with the Institute for Physical Science and Technology, University of Maryland, College Park, Md.

Light reconsidered

Arthur Zajonc
Physics Department, Amherst College

I therefore take the liberty of proposing for this hypothetical new atom, which is not light but plays an essential part in every process of radiation, the name photon.[1] Gilbert N. Lewis, 1926

OCIS codes: 270.0270.

Light is an obvious feature of everyday life, and yet light's true nature has eluded us for centuries. Near the end of his life Albert Einstein wrote, "All the fifty years of conscious brooding have brought me no closer to the answer to the question: What are light quanta? Of course today every rascal thinks he knows the answer, but he is deluding himself." We are today in the same state of "learned ignorance" with respect to light as was Einstein.

In 1926 when the chemist Gilbert Lewis suggested the name "photon," the concept of the light quantum was already a quarter of a century old. First introduced by Max Planck in December of 1900 in order to explain the spectral distribution of blackbody radiation, the idea of concentrated atoms of light was suggested by Einstein in his 1905 paper to explain the photoelectric effect. Four years later on September 21, 1909 at Salzburg, Einstein delivered a paper to the Division of Physics of German Scientists and Physicians on the same subject. Its title gives a good sense of its content: "On the development of our views concerning the nature and constitution of radiation."[2]

Einstein reminded his audience how great had been their collective confidence in the wave theory and the luminiferous ether just a few years earlier. Now they were confronted with extensive experimental evidence that suggested a particulate aspect to light and the rejection of the ether outright. What had seemed so compelling was now to be cast aside for a new if as yet unarticulated view of light. In his Salzburg lecture he maintained "that a profound change in our views on the nature and constitution of light is imperative," and "that the next stage in the development of theoretical physics will bring us a theory of light that can be understood as a kind of fusion of the wave and emission theories of light." At that time Einstein personally favored an atomistic view of light in which electromagnetic fields of light were "associated with singular points just like the occurrence of electrostatic fields according to the electron theory." Surrounding these electromagnetic points he imagined fields of force that superposed to give the electromagnetic wave of Maxwell's classical theory. The conception of the photon held by many if not most working physicists today is, I suspect, not too different from that suggested by Einstein in 1909.

Others in the audience at Einstein's talk had other views of light. Among those who heard Einstein's presentation was Max Planck himself. In his recorded remarks following Einstein's lecture we see him resisting Einstein's hypothesis of atomistic light quanta propagating through space. If Einstein were correct, Planck asked, how could one account for interference when the length over which one detected interference was many thousands of wavelengths? How could a quantum of light interfere with itself over such great distances if it were a point object? Instead of quantized electromagnetic fields Planck maintained that "one should attempt to transfer the whole problem of the quantum theory to the area of *interaction* between matter and radiation energy." That is, only the exchange of energy between the atoms of the radiating source and the classical electromagnetic field is quantized. The exchange takes place in units of Planck's constant times the frequency, but the fields remain continuous and classical. In essence, Planck was holding out for a semi-classical theory in which only the atoms and their interactions were quantized while the free fields remained classical. This view has had a long and honorable history, extending all the way to the end of the 20th century. Even today we often use a semi-classical approach to handle many of the problems of quantum optics, including Einstein's photoelectric effect.[3]

The debate between Einstein and Planck as to the nature of light was but a single incident in the four thousand year inquiry concerning the nature of light.[4] For the ancient Egyptian light was the activity of their god Ra seeing. When Ra's eye (the Sun) was open, it was day. When it was closed, night fell. The dominant view in ancient Greece focused likewise on vision, but now the vision of human beings instead of the gods. The Greeks and most of their successors maintained that inside the eye a pure ocular fire radiated a luminous stream out into the world. This was the most important factor in sight. Only with the rise of Arab optics do we find strong arguments advanced against the extromissive theory of light expounded by the Greeks. For example around 1000 A.D. Ibn al-Haytham (Alhazen in the West) used his invention of the *camera obscura* to advocate for a view of light in which rays streamed from luminous sources traveling in straight lines to the screen or the eye.

By the time of the scientific revolution the debate as to the physical nature of light had divided into the two familiar camps of waves and particles. In broad strokes Galileo and Newton maintained a corpuscular view of light, while Huygens, Young and Euler advocated a wave view. The evidence supporting these views is well known.

The elusive single photon

One might imagine that with the more recent developments of modern physics the debate would finally be settled and a clear view of the nature of light attained. Quantum electrodynamics (QED) is commonly treated as the most successful physical theory ever invented, capable of predicting the effects of the interaction between charged particles and electromagnetic radiation with unprecedented precision. While this is certainly true, what view of the photon does the theory advance? And how far does it succeed in fusing wave and particle ideas? In 1927 Dirac, one of the inventors of QED, wrote confidently of the new theory that, "There is thus a complete harmony between the wave and quantum descriptions of the interaction."[5] While in some sense quantum field theories do move beyond wave particle duality, the nature of light and the photon remains elusive. In order to support this I would like to focus on certain fundamental features of our understanding of photons and the philosophical issues associated with quantum field theory.[6]

In QED the photon is introduced as the unit of excitation associated with a quantized mode of the radiation field. As such it is associated with a plane wave of precise momentum, energy and polarization. Because of Bohr's principle of complementarity we know that a state of definite momentum and energy must be completely indefinite in space and time. This points to the first difficulty in conceiving of the photon. If it is a particle, then in what sense does it have a location? This problem is only deepened by the puzzling fact that, unlike other observables in quantum theory, there is no Hermetian operator that straightforwardly corresponds to position for photons. Thus while we can formulate a well-defined quantum-mechanical concept of position for electrons, protons and the like, we lack a parallel concept for the photon and similar particles with integer spin. The simple concept of spatio-temporal location must therefore be treated quite carefully for photons.

We are also accustomed to identifying an object by a unique set of attributes. My height, weight, shoe size, etc. uniquely identify me. Each of these has a well-defined value. Their aggregate is a full description of me. By contrast the single photon can, in some sense, take on multiple directions, energies and polarizations. Single-photon spatial interference and quantum beats require superpositions of these quantum descriptors for single photons. Dirac's refrain "photons interfere with themselves" while not universally true is a reminder of the importance of superposition. Thus the single photon should *not* be thought of as like a simple plane wave having a unique direction, frequency or polarization. Such states are rare special cases. Rather the superposition state for single photons is the common situation. Upon detection, of course, light appears as if discrete and indivisible possessing well-defined attributes. In transit things are quite otherwise.

Nor is the single photon state itself easy to produce. The anti-correlation experiments of Grangier, Roger and Aspect provide convincing evidence that with suitable care one can prepare single-photon states of light.[7] When sent to a beam splitter such photon states display the type of statistical correlations we would expect of particles. In particular the single photons appear to go one way or the other. Yet such single-photon states can interfere with themselves, even when run in "delayed choice."[8]

More than one photon

If we consider multiple photons the conceptual puzzles multiply as well. As spin one particles, photons obey Bose-Einstein statistics. The repercussions of this fact are very significant both for our conception of the photon and for technology. In fact Planck's law for the distribution of blackbody radiation makes use of Bose-Einstein statistics. Let us compare the statistics suited to two conventional objects with that of photons. Consider two marbles that are only distinguished by their colors: red (R) and green (G). Classically, four distinct combinations exist: RR, GG, RG and GR. In writing this we presume that although identical except for color, the marbles are, in fact, distinct because they are located at different places. At least since Aristotle we have held that two objects cannot occupy exactly the same location at the same time and therefore the two marbles, possessing distinct locations, are two distinct objects.

Photons by contrast are defined by the three quantum numbers associated with momentum, energy and polarization; position and time do not enter into consideration. This means that if two photons possess the same three values for these quantum numbers they are indistinguishable from one another. Location in space and in time is no longer a means for theoretically distinguishing photons as elementary particles. In addition, as bosons, any number of photons can occupy the same state, which is unlike the situation for electrons and other fermions. Photons do not obey the Pauli Exclusion Principle. This fact is at the foundation of laser theory because laser operation requires many photons to occupy a single mode of the radiation field.

To see how Bose-Einstein statistics differ from classical statistics consider the following example. If instead of marbles we imagine we have two photons in our possession which are distinguished by one of their attributes, things are quite different. For consistency with the previous example I label the two values of the photon attribute R and G. As required by Bose-Einstein statistics, the states available to the two photons are those that are symmetric states under exchange: RR, GG and $\frac{1}{2}$(RG + GR). The states RG and GR are non-symmetric, while the combination $\frac{1}{2}$(RG – GR) is anti-symmetric. These latter states are not suitable for photons. All things being equal we expect equal occupation for the three symmetric states with 1/3 as the probability for finding a pair of photons in each of the three states, instead of $\frac{1}{4}$ for the case of two marbles. This shows that it makes no sense to continue to think of photons as if they were "really" in classical states like RG and GR.

Experimentally we can realize the above situation by sending two photons onto a beam splitter. From a classical perspective there are four possibilities. They are sketched out in Fig. 1. We can label them RR for two right-going photons, UR for up and right, RU for right and up, and UU for the two photons going up. The quantum amplitudes for the UR and RU have opposite signs due to the reflections which the photons undergo in Fig. 1c, which leads to destructive interference between these two amplitudes. The signal for one photon in each direction therefore vanishes. Surprisingly both photons are always found together. Another way of thinking about the experiment is in terms of the bosonic character of photons. Instead of thinking of the photons as having individual identities we should really think of there being three ways of pairing the two photons: two up (UU), two right (RR) and the symmetric combination (1/2(UR + RU)). All things being equal, we would expect the experiment to show an even distribution between the three options, 1/3 for each. But the experiment does not show this; why not? The answer is found in the opposite signs associated with UR and RU due to reflections. As a consequence the proper way to write the state for combination of b and c is $^1/_2$(UR – RU). But this is antisymmetric and therefore forbidden for photons which must have a symmetric state.

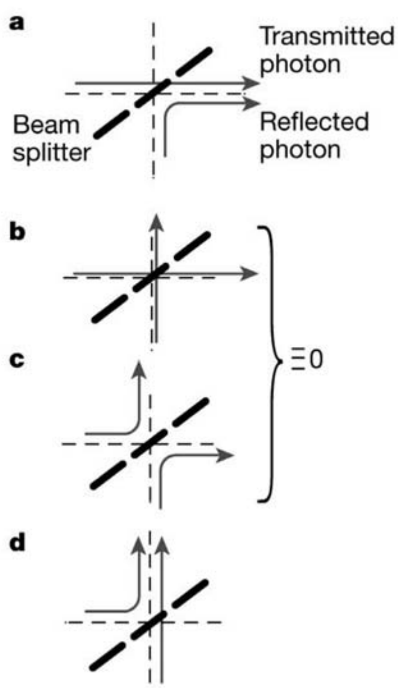

Fig. 1. Copyright permission granted by *Nature.*[9]

From this example we can see how Bose statistics confounds our conception of the identity of individual photons and rather treats them as aggregates with certain symmetry properties. These features are reflected in the treatment of photons in the formal mathematical language of Fock space. In this representation we only specify how many quanta are to be found in each mode. All indexing of individual particles disappears.

Photons and relativity

In his provocatively titled paper "Particles do not Exist," Paul Davies advances several profound difficulties for any conventional particle conception of the photon, or for that matter for particles in general as they appear in relativistic quantum field theory.[10] One of our deepest tendencies is to reify the features that appear in our theories. Relativity confounds this habit of mind, and many of the apparent paradoxes of relativity arise because of our erroneous expectations due to this attitude. Every undergraduate is confused when, having mastered the electromagnetic theory of Maxwell he or she learns about Einstein's treatment of the electrodynamics of moving bodies. The foundation of Einstein's revolutionary 1905 paper was his recognition that the values the electric and magnetic fields take on are always relative to the observer. That is, two observers in relative motion to one another will record on their measuring instruments different values of E and B for the same event. They will, therefore, give different causal accounts for the event. We habitually reify the electromagnetic field so that particular values of E and B are imagined as truly extant in space independent of any observer. In relativity we learn that in order for the laws of electromagnetism to be true in different inertial frames the values of the electric and magnetic fields (among other things) must change for different inertial frames. Matters only become more subtle when we move to accelerating frames.

Davies gives special attention to the problems that arise for the photon and other quanta in relativistic quantum field theory. For example, our concept of reality has, at its root, the notion that either an object exists or it does not. If the very existence of a thing is ambiguous, in what sense is it real? Exactly this is challenged by quantum field theory. In particular the quantum vacuum is the state in which no photons are present in any of the modes of the radiation field. However the vacuum only remains empty of particles for inertial observers. If instead we posit an observer in a uniformly accelerated frame of reference, then what was a vacuum state becomes a thermal bath of photons for the accelerated observer. And what is true for accelerated observers is similarly true for regions of space-time curved by gravity. Davies uses these and other problems to argue for a vigorous Copenhagen interpretation of quantum mechanics that abandons the idea of a "particle as a really existing thing skipping between measuring devices."

To my mind, Einstein was right to caution us concerning light. Our understanding of it has increased enormously in the 100 years since Planck, but I suspect light will continue to confound us, while simultaneously luring us to inquire ceaselessly into its nature.

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What is a photon?

Rodney Loudon
University of Essex, Colchester, UK

The concept of the photon is introduced by discussion of the process of electromagnetic field quantization within a closed cavity or in an open optical system. The nature of a single-photon state is clarified by consideration of its behaviour at an optical beam splitter. The importance of linear superposition or entangled states in the distinctions between quantum-mechanical photon states and classical excitations of the electromagnetic field is emphasized. These concepts and the ideas of wave–particle duality are illustrated by discussions of the effects of single-photon inputs to Brown–Twiss and Mach–Zehnder interferometers. Both the theoretical predictions and the confirming experimental observations are covered. The defining property of the single photon in terms of its ability to trigger one, and only one, photodetection event is discussed.

OCIS codes: 270.0270.

The development of theories of the nature of light has a long history, whose main events are well reviewed by Lamb[1]. The history includes strands of argument in favor of either a particle or a wave view of light. The realm of *classical optics* includes all of the phenomena that can be understood and interpreted on the basis of classical wave and particle theories. The conflicting views of the particle or wave essence of light were reconciled by the establishment of the quantum theory, with its introduction of the idea that all excitations simultaneously have both particle-like and wave-like properties. The demonstration of this dual behavior in the real world of experimental physics is, like so many basic quantum-mechanical phenomena, most readily achieved in optics. The fundamental properties of the photon, particularly the discrimination of its particle-like and wave-like properties, are most clearly illustrated by observations based on the use of beam splitters. The realm of *quantum optics* includes all of the phenomena that are not embraced by classical optics and require the quantum theory for their understanding and interpretation. The aim of the present article is to try to clarify the nature of the photon by considerations of electromagnetic fields in optical cavities or in propagation through free space.

Single photons and beam splitters

A careful description of the nature of the photon begins with the electromagnetic field inside a *closed* optical resonator, or perfectly-reflecting cavity. This is the system usually assumed in textbook derivations of Planck's radiation law[2]. The field excitations in the cavity are limited to an infinite discrete set of spatial modes determined by the boundary conditions at the cavity walls. The allowed standing-wave spatial variations of the electromagnetic field in the cavity are identical in the classical and quantum theories. However, the time dependence of each mode is governed by the equation of motion of a harmonic oscillator, whose solutions take different forms in the classical and quantum theories. Unlike its classical counterpart, a quantum harmonic oscillator of angular frequency ω can only be excited by energies that are integer multiples of $\hbar\omega$. The integer n thus denotes the number of energy quanta excited in the oscillator. For application to the electromagnetic field, a single spatial mode whose associated harmonic oscillator is in its n^{th} excited state unambiguously contains n photons, each of energy $\hbar\omega$. Each photon has a spatial distribution within the cavity that is proportional to the square modulus of the complex field amplitude of the mode function. For the simple, if unrealistic, example of a one-dimensional cavity bounded by perfectly reflecting mirrors, the spatial modes are standing waves and the photon may be found at any position in the cavity except the nodes. The single-mode photons are said to be *delocalized*.

These ideas can be extended to *open* optical systems, where there is no identifiable cavity but where the experimental apparatus has a finite extent determined by the sources, the transverse cross sections of the light beams, and the detectors. The discrete standing-wave modes of the closed cavity are replaced by discrete travelling-wave modes that propagate from sources to detectors. The simplest system to consider is the optical beam splitter, which indeed is the central component in many of the experiments that study the quantum nature of light. Fig. 1 shows a representation of a lossless beam splitter, with two input arms denoted 1 and 2 and two output arms denoted 3 and 4. An experiment to distinguish the classical and quantum natures of light consists of a source that emits light in one of the input arms and which is directed by the beam splitter to detectors in the two output arms. The relevant spatial modes of the system in this example include a joint excitation of the selected input arm and both output arms.

The operators $\hat{a}_i$ in Fig. 1 are the *photon destruction operators* for the harmonic oscillators associated with the two input $(i = 1,2)$ and two output $(i = 3,4)$ arms. These destruction operators essentially represent the amplitudes of the quantum electromagnetic fields in the four arms of the beam splitter, analogous to the complex classical field amplitudes. The real electric-field operators of the four arms are proportional to the sum of $\hat{a}_i \exp(-i\omega t)$ and the Hermitean conjugate operators $\hat{a}_i^\dagger \exp(i\omega t)$. The proportionality factor includes Planck's constant $\hbar$, the angular frequency ω, and the permittivity of free space ε_0, but its detailed form does not concern us here. For the sake of brevity, we refer to $\hat{a}_i$ as the *field* in arm i. The operator $\hat{a}_i^\dagger$ is the p*hoton creation operator* for arm i and it has the effect of generating a single-photon state $|1\rangle_i$ in arm

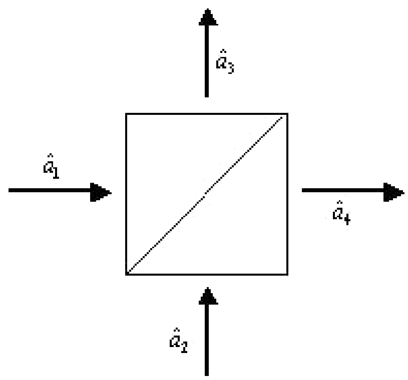

Fig. 1. Schematic representation of an optical beam splitter showing the notation for the field operators in the two input and two output arms. In practice the beam-splitter cube is often replaced by a partially reflecting plate at 45° or a pair of optical fibers in contact along a fused section.

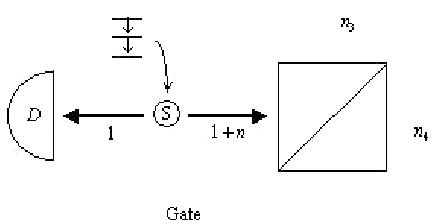

Fig. 2. Brown–Twiss interferometer using a single-photon input obtained from cascade emission with an electronic gate.

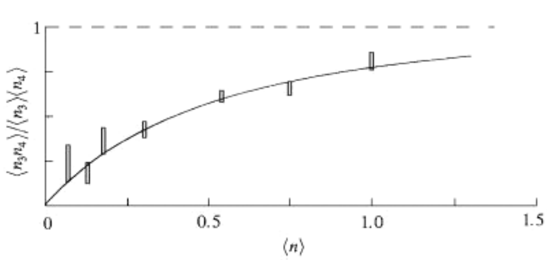

Fig. 3. Normalized output correlation as a function of the average additional photon number $\langle n\rangle$, as measured in the experiment represented in Fig. 2. (After ref. 9).

i, according to

$$\hat{a}_i^\dagger |0\rangle = |1\rangle_i . \tag{1}$$

Here $|0\rangle$ is the *vacuum state* of the entire input–output system, which is defined as the state with no photons excited in any of the four arms.

The relations of the output to the input fields at a symmetric beam splitter have forms equivalent to those of classical theory,

$$\hat{a}_3 = R\hat{a}_1 + T\hat{a}_2 \quad \text{and} \quad \hat{a}_4 = T\hat{a}_1 + R\hat{a}_2, \tag{2}$$

where R and T are the reflection and transmission coefficients of the beam splitter. These coefficients are generally complex numbers that describe the amplitudes and phases of the reflected and transmitted light relative to those of the incident light. They are determined by the boundary conditions for the electromagnetic fields at the partially transmitting and partially reflecting interface within the beam splitter. The boundary conditions are the same for classical fields and for the quantum-mechanical field operators $\hat{a}_i$. It follows that the coefficients satisfy the standard relations[3]

$$|R|^2 + |T|^2 = 1 \quad \text{and} \quad RT^* + TR^* = 0. \tag{3}$$

It can be shown[2] that these beam-splitter relations ensure the conservation of optical energy from the input to the output arms, in both the classical and quantum forms of beam-splitter theory.

The essential property of the beam splitter is its ability to convert an input photon state into a *linear superposition* of output states. This is a basic quantum-mechanical manipulation that is less easily achieved and studied in other physical systems. Suppose that there is one photon in input arm 1 and no photon in input arm 2. The beam splitter converts this joint input state to the output state determined by the simple calculation

$$\begin{aligned} |1\rangle_1 |0\rangle_2 = \hat{a}_1^\dagger |0\rangle &= \left(R\hat{a}_3^\dagger + T\hat{a}_4^\dagger\right)|0\rangle \\ &= T\,|1\rangle_3 |0\rangle_4 + R\,|0\rangle_3 |1\rangle_4 , \end{aligned} \tag{4}$$

where $|0\rangle$ is again the vacuum state of the entire system. The expression for $\hat{a}_1^\dagger$ in terms of output arm operators is obtained from the Hermitean conjugates of the relations in eqn (2) with the use of eqn (3). In words, the state on the right is a superposition of the state with one photon in arm 3 and nothing in arm 4, with probability amplitude T, and the state with one photon in arm 4 and nothing in arm 3, with amplitude R. This conversion of the input state to a linear superposition of the two possible output states is the basic quantum-mechanical process performed by the beam splitter. In terms of travelling-wave modes, this example combines the input-arm excitation on the left of eqn (4) with the output-arm excitation on the right of eqn (4) to form a joint single-photon excitation of a mode of the complete beam-splitter system.

Note that the relevant spatial mode of the beam splitter, with light incident in arm 1 and outputs in arms 3 and 4, is the same in the classical and quantum theories. What is quantized in the latter theory is the energy content of the electromagnetic field in its distribution over the complete spatial extent of the mode. In the classical theory, an incident light beam of intensity I_1 excites the two outputs with intensities $|T|^2 I_1$ and $|R|^2 I_1$, in contrast to the excitation of the quantum state shown on the right of eqn (4) by a single incident photon. A state of this form, with the property that each contribution to the superposition is a product of states of different subsystems (output arms), is said to be *entangled*. Entangled states form the basis of many of the applications of quantum technology in information transfer and processing[4].

Brown-Twiss interferometer

The experiment described in essence by eqn (4) above is performed in practice by the use of a kind of interferometer first constructed by Brown and Twiss in the 1950s. They were not able to use a single-photon input but their apparatus was essentially that illustrated in Fig. 1 with light from a mercury arc incident in arm 1. Their interest was in measurements of the angular diameters of stars by interference of the intensities of starlight[5] rather than the interference of field amplitudes used in traditional classical interferometers. The techniques they developed work well with the random multiphoton light emitted by arcs or stars.

However, for the study of the quantum entanglement represented by the state on the right of eqn (4), it is first necessary to obtain a single-photon input state, and herein lies the main difficulty of the experiment. It is true, of course, that most sources emit light in single-photon processes but the sources generally contain large numbers of emitters whose emissions occur at random times, such that the experimenter cannot reliably isolate a single photon. Even when an ordinary light beam is heavily attenuated, statistical analysis shows that single-photon effects cannot be detected by the apparatus in Fig. 1. It is necessary to find a way of identifying the presence of one and only one photon. The earliest reliable methods of single-photon generation depended on optical processes that generate photons in pairs. Thus, for example, the nonlinear optical process of parametric down conversion[6] replaces a single incident photon by a pair of photons whose frequencies sum to that of the incident photon to ensure energy conservation. Again, two-photon cascade emission is a process in which an excited atom decays in two steps, first to an intermediate energy level and then to the ground state, emitting two photons in succession with a delay determined by the lifetime of the intermediate state[7]. If one of the photons of the pair produced by these processes is detected, it is known that the other photon of the pair must be present more-or-less simultaneously. For a two-photon source sufficiently weak that the time separation between one emitted pair and the next is longer than the resolution time of the measurement, this second photon can be used as the input to a single-photon experiment. More versatile single-photon light sources are now available[8].

The arrangement of the key single-photon beam-splitter experiment[9] is represented in Fig. 2. Here, the two photons came from cascade emission in an atomic Na light source S and one of them activated photodetector D. This first detection opened an electronic gate that activated the recording of the responses of two detectors in output arms 3 and 4 of the Brown–Twiss beam splitter. The gate was closed again after a period of time sufficient for the photodetection. The experiment was repeated many times and the results were processed to determine the average values of the mean photocounts $\langle n_3 \rangle$ and $\langle n_4 \rangle$ in the two arms and the average value $\langle n_3 n_4 \rangle$ of their correlation product. It is convenient to work with the normalized correlation $\langle n_3 n_4 \rangle / \langle n_3 \rangle \langle n_4 \rangle$, which is independent of the detector efficiencies and beam splitter reflection and transmission coefficients. In view of the physical significance of the entangled state in (4), the single-photon input should lead to a single photon either in arm 3 or arm 4 but never a photon in both output arms. The correlation $\langle n_3 n_4 \rangle$ should therefore ideally vanish.

However, in the real world of practical experiments, a purely single-photon input is difficult to achieve. In addition to the twin of the photon that opens the gate, n additional 'rogue' photons may enter the Brown–Twiss interferometer during the period that the gate is open, as represented in Fig. 2. These rogue photons are emitted randomly by other atoms in the cascade light source and their presence allows two or more photons to pass through the beam splitter during the detection period. Fig. 3 shows experimental results for the normalized correlation, with its dependence on the average number $\langle n \rangle$ of additional photons that enter the interferometer for different gate periods. The continuous curve shows the calculated value of the correlation in the presence of the additional rogue photons. It is seen that both experiment and theory agree on the tendency of the correlation to zero as $\langle n \rangle$ becomes very small, in confirmation of the quantum expectation of the particle-like property of the output photon exciting only one of the output arms.

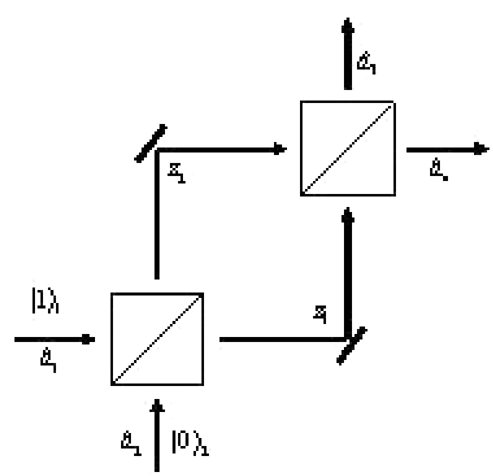

Fig. 4. Representation of a Mach–Zehnder interferometer showing the notation for input and output field operators and the internal path lengths.

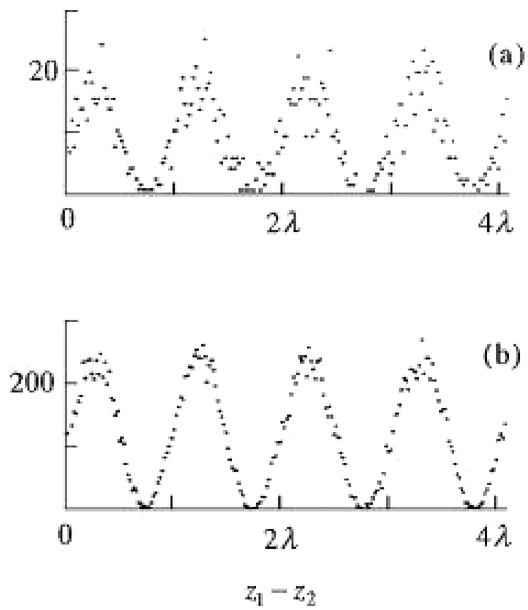

Fig. 5. Mach–Zehnder fringes formed from series of single-photon measurements as a function of the path difference expressed in terms of the wavelength. The vertical axis shows the number of photodetections in arm 4 for (a) 1 sec and (b) 15 sec integration times per point. The latter fringes have 98% visibility. (After ref. 9).

Mach–Zehnder interferometer

The excitation of one photon in a single travelling-wave mode is also frequently considered in the discussion of the quantum theory of the traditional classical amplitude-interference experiments, for example Young's slits or the Michelson and Mach–Zehnder interferometers. Each classical or quantum spatial mode in these systems includes input light waves, *both* paths through the interior of the interferometer, and output waves appropriate to the geometry of the apparatus. A one-photon excitation in such a mode again carries an energy quantum $\hbar\omega$ distributed over the entire interferometer, including both internal paths. Despite the absence of any localization of the photon, the theory provides expressions for the distributions of light in the two output arms, equivalent to a determination of the interference fringes.

The arrangement of a Mach–Zehnder interferometer with a single-photon input is represented in Fig. 4. The two beam splitters are assumed to be symmetric and identical, with the properties given in eqn (3). The complete interferometer can be regarded as a composite beam splitter, whose two output fields are related to the two input fields by

$$\hat{a}_3 = R_{\mathrm{MZ}}\hat{a}_1 + T_{\mathrm{MZ}}\hat{a}_2 \quad \text{and} \quad \hat{a}_4 = T_{\mathrm{MZ}}\hat{a}_1 + R'_{\mathrm{MZ}}\hat{a}_2, \quad (5)$$

similar to eqn (2) but with different reflection coefficients in the two relations. Without going into the details of the calculation[2], we quote the quantum result for the average number of photons in output arm 4 when the experiment is repeated many times with the same internal path lengths z_1 and z_2,

$$\langle n_4\rangle = \left|T_{\mathrm{MZ}}\right|^2 = \left|RT\left(\mathrm{e}^{i\omega z_1/c} + \mathrm{e}^{i\omega z_2/c}\right)\right|^2$$
$$= 4\left|R\right|^2\left|T\right|^2\cos^2\left[\omega\left(z_1 - z_2\right)/2c\right]. \quad (6)$$

The fringe pattern is contained in the trigonometric factor, which has the same dependence on frequency and relative path length as found in the classical theory. Fig. 5 shows the fringe pattern measured with the same techniques as used for the Brown–Twiss experiment of Figs. 2 and 3. The average photon count $\langle n_4\rangle$ in output arm 4 was determined[9] by repeated measurements for each relative path length. The two parts of Fig. 5 show the improvements in fringe definition gained by a fifteenfold increase in the number of measurements for each setting.

The existence of the fringes seems to confirm the wave-like property of the photon and we need to consider how this behavior is consistent with the particle-like properties that show up in the Brown–Twiss interferometer. For the Mach–Zehnder interferometer, each incident photon must propagate through the apparatus in such a way that the probability of its leaving the interferometer by arm 4 is proportional to the calculated mean photon number in eqn (6). This is achieved only if each photon excites both internal paths of the interferometer, so that the input state at the second beam splitter is determined by the complete interferometer geometry. This geometry is inherent in the entangled state in the output arms of the first beam splitter from eqn (4), with the output labels 3 and 4 replaced by internal path labels, and in the propagation phase factors for the two internal paths shown in T_{MZ} in eqn (6). The photon in the Mach–Zehnder interferometer should thus be viewed as a composite excitation of the appropriate input arm, internal paths and output arms, equivalent to the spatial field distribution produced by illumination of the input by a classical light beam. The interference fringes are thus a property not so much of the photon itself as of the spatial mode that it excites.

The internal state of the interferometer excited by a single photon is the same as that investigated by the Brown–Twiss experiment. There is, however, no way of performing both kinds of interference experiment simultaneously. If a detector is placed in one of the output arms of the first beam splitter to detect photons in the corresponding internal path, then it is not possible to avoid obscuring that path, with consequent destruction of the interference fringes. A succession of suggestions for more and more ingenious experiments has failed to provide any method for simultaneous fringe and path observations. A complete determination of the one leads to a total loss of resolution of the other, while a partial determination of the one leads to an accompanying partial loss of resolution of the other[10].

Detection of photon pulses

The discussion so far is based on the idea of the photon as an excitation of a single traveling-wave mode of the complete optical system considered. Such an excitation is independent of the time and it has a nonzero probability over the whole system, apart from isolated interference nodes. This picture of delocalized photons gives reasonably correct results for the interference experiments treated but it does not provide an accurate representation of the physical processes in real experiments. The typical light source acts by spontaneous emission and this is the case even for the two-photon emitters outlined above. The timing of an emission is often determined by the random statistics of the source but, once initiated, it occurs over a limited time span Δt and the light is localized in the form of a *pulse* or *wavepacket*. The light never has a precisely defined angular frequency and ω is distributed over a range of values $\Delta\omega$ determined by the nature of the emitter, for example by the radiative lifetime for atoms or by the geometry of the several beams involved in a nonlinear-optical process. The minimum values of pulse duration and frequency spread are related by Fourier transform theory such that their product $\Delta t\Delta\omega$ must have a value at least of order unity.

The improved picture of the photon thus envisages the excitation of a pulse that is somewhat localized in time and involves several traveling-wave modes of the optical system. These modes are exactly the same as the collection of those used in single-mode theory and they are again the same as the spatial modes of classical theory. Their frequency separation is often small compared to the wavepacket frequency spread $\Delta\omega$, and it is convenient to treat their frequency ω as a continuous variable. The theories of optical interference experiments based on these single-photon continuous-mode wavepackets are more complicated than the single-mode theories but they provide more realistic descriptions of the measurements. For example, the frequency spread of the wavepacket leads to a blurring of fringe patterns and its limited time span may lead to a lack of simultaneity in the arrival of pulses by different paths, with a destruction of interference effects that depend on their overlap.

The good news is that the single-mode interference effects outlined above survive the change to a wavepacket description of the photon for optimal values of the pulse parameters. The discussions of the physical significances of the Brown–Twiss and Mach–Zehnder interference experiments in terms of particle-like and wave-like properties thus remain valid. However, some of the concepts of single-mode theory need modification. Thus, the single-mode photon creation operator $\hat{a}^\dagger$ is replaced by the *photon wavepacket creation operator*

$$\hat{a}_i^\dagger = \int \mathrm{d}\omega\xi(\omega)\hat{a}\dagger(\omega), \qquad (7)$$

where $\xi(\omega)$ is the spectral amplitude of the wavepacket and $\hat{a}^\dagger(\omega)$ is the continuous-mode creation operator. The integration over frequencies replaces the idea of a single energy quantum $\hbar\omega$ in a discrete mode by an average quantum $\hbar\omega_0$, where ω_0 is an average frequency of the wavepacket spectrum $|\xi(\omega)|^2$.

The main change in the description of experiments, however, lies in the theory of the optical detection process[2]. For the detection of photons by a phototube, the theory must allow for its switch-on time and its subsequent switch-off time; the difference between the two times is the *integration time*. The more accurate theory includes the need for the pulse to arrive during an integration time in order for the photon to be detected. More importantly, it shows that the single-photon excitation created by the operator defined in eqn (7) can at most trigger a single detection event. Such a detection only occurs with certainty, even for a 100% efficient detector, in conditions where the integration time covers essentially all of the times for which the wavepacket has significant intensity at the detector. Of course, this feature of the theory merely reproduces some obvious properties of the passage of a photon wavepacket from a source to a detector but it is nevertheless gratifying to have a realistic representation of a practical experiment. Real phototubes miss some fraction of the incident wavepackets, but the effects of detector efficiencies of less than 100% are readily included in the theory[2].

So what is a photon?

The question posed by this special issue has a variety of answers, which hopefully converge to a coherent picture of this somewhat elusive object. The present article presents a series of three physical systems in which the spatial distribution of the photon excitation progresses from a single discrete standing-wave mode in a closed cavity to a single discrete traveling-wave mode of an open optical system to a traveling pulse or wavepacket. The first two excitations are spread over the complete optical system but the wavepacket is localized in time and contains a range of frequencies. All of these spatial distributions of the excitation are the same in the classical and quantum theories. What distinguishes the quantum theory from the classical is the limitation of the energy content of the discrete-mode systems to integer multiples of the $\hbar\omega$ quantum. The physically more realistic wavepacket excitation also carries a basic energy quantum $\hbar\omega_0$, but ω_0 is now an average of the frequencies contained in its spectrum. The single-photon wavepacket has the distinguishing feature of causing at most a single photodetection and then only when the detector is in the right place at the right time.

It cannot be emphasized too strongly that the spatial modes of the optical system, classical and quantum, include the combinations of all routes through the apparatus that are excited by the light sources. In the wavepacket picture, a single photon excites this complete spatial distribution, however complicated, and what is measured by a detector is determined both by its position within the complete system and by the time dependence of the excitation. The examples outlined here show how particle-like and wave-like aspects of the photon may appear in suitable experiments, without any conflict between the two.

Acknowledgment

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What is a photon?

David Finkelstein
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332

Modern developments in the physicist's concept of nature have expanded our understanding of light and the photon in ever more startling directions. We take up expansions associated with the established physical constants $c, \hbar, G$, and two proposed "transquantum" constants $\hbar', \hbar''$.

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From the point of view of experience, "What is a photon?" is not the best first question. We never experience a photon as it "is." For example, we never see a photon in the sense that we see an apple, by scattering diffuse light off it and forming an image of it on our retina. What we experience is what photons do. A better first question is "What do photons do?" After we answer this we can define what photons are, if we still wish to, by what they do.

Under low resolution the transport of energy, momentum and angular momentum by electromagnetic radiation often passes for continuous but under sufficient resolution it breaks down into discrete jumps, quanta. Radiation is not the only way that the electromagnetic field exerts forces; there are also Coulomb forces, say, but only the radiation is quantized. Even our eyes, when adapted sufficiently to the dark, see any sufficiently dim light as a succession of scintillations. What photons do is couple electric charges and electric or magnetic multipoles by discrete irreducible processes of photon emission and absorption connected by continuous processes of propagation. All electromagnetic radiation resolves into a flock of flying photons, each carrying its own energy, momentum and angular momentum.

Francis Bacon and Isaac Newton were already certain that light was granular in the 17th century but hardly anyone anticipated the radical conceptual expansions in the physics of light that happened in the 20th century. Now a simple extrapolation tells us to expect more such expansions.

These expansions have one basic thing in common: Each revealed that the resultant of a sequence of certain processes depends unexpectedly on their order. Processes are said to *commute* when their resultant does *not* depend on their order, so what astounded us each time was a non-commutativity. Each such discovery was made without connection to the others, and the phenomenon of non-commutativity was called several things, like non-integrability, inexactness, anholonomy, curvature, or paradox (of two twins, or two slits). These aliases must not disguise this underlying commonality. Moreover the prior commutative theories are unstable relative to their non-commutative successors in the sense that an arbitrarily small change in the commutative commutation relations can change the theory drastically,[9] but not in the non-commutative relations.

Each of these surprising non-commutativities is proportional to its own small new fundamental constant. The expansion constants and non-commutativities most relevant to the photon so far have been k (Boltzmann's constant, for the kinetic theory of heat) c (light speed, for special relativity), G (gravitational constant, for general relativity), h (Planck's constant, for quantum theory), e (the electron charge, for the gauge theory of electromagnetism), g (the strong coupling constant) and W (the mass of the W particle, for the electroweak unification). These constants are like flags. If we find a c in an equation, for instance, we know we are in the land of special relativity. The historic non-commutativities introduced by these expansions so far include those of reversible thermodynamic processes (for k), boosts (changes in the velocity of the observer, for c), filtration or selection processes (for h), and space-time displacements (of different kinds of test-particles for G, e, and g).

Each expansion has its inverse process, a *contraction* that reduces the fundamental constant to 0, recovering an older, less accurate theory in which the processes commute.[6] Contraction is a well-defined mathematical process. Expansion is the historical creative process, not a mathematically well-posed problem. When these constants are taken to 0, the theories "contract" to their more familiar forms; but in truth the constants are not 0, and the expanded theory is more basic than the familiar one, and is a better starting point for further exploration.

Einstein was the magus of these expansions, instrumental in raising the flags of k, c, G and h. No one comes close to his record. In particular he brought the photon back from the grave to which Robert Young's diffraction studies had consigned it, though he never accommodated to the h expansion.

Each expansion establishes a reciprocity between mutually coupled concepts that was lacking before it, such as that between space and time in special relativity. Each thereby dethroned a false absolute, an unmoved mover, what Frances Bacon called an "idol," usually an "idol of the theater." Each made physics more relativistic, more processual, less mechanical.

There is a deeper commonality to these expansions. Like earthquakes and landslides, they stabilize the region where they occur, specifically against small changes in the expansion constant itself.

Each expansion also furthered the unity of physics in the sense that it replaced a complicated kind of symmetry (or group) by a simple one.

Shifting our conceptual basis from the familiar idol-ridden theory to the strange expanded theory has generally led to new and deeper understanding. The Standard Model, in particular, gives the best account of the photon we have today, combining expansions of quantum theory, special relativity, and gauge theory, and it shows signs of impending expansions as drastic as those of the past. Here we describe the photon as we know it today and speculate about the photon of tomorrow.

1. $\boxed{c}$ The expansion constant c of special relativity, the speed of light, also measures how far the photon flouts Euclid's geometry and Galileo's relativity. In the theory of space-time that immediately preceded the c expansion, associated with the relativity theory of Galileo, reality is a collection of objects or fields distributed over space at each time, with the curious codicil that different observers in uniform relative motion agree about simultaneity — having the same time coordinate — but not about colocality — having the same space coordinates. One could imagine history as a one-dimensional stack of three-dimensional slices. If V is a boost vector, giving the velocity of one observer O' relative to another O, then in Galileo relativity: $x' = x - Vt$ but $t' = t$. The transformation $x' = x - Vt$ couples time into space but the transformation $t' = t$ does not couple space into time. O and O' slice history the same way but stack the slices differently.

Special relativity boosts couple time into space and space back into time, restoring reciprocity between space and time. The very constancy of c implies this reciprocity. Relatively moving observers may move different amounts during the flight of a photon and so may disagree on the distance Δx covered by a photon, by an amount depending on Δt. In order to agree on the speed $c = \Delta x/\Delta t$, they must therefore disagree on the duration Δt as well, and by the same factor. They slice history differently.

We could overlook this fundamental reciprocity for so many millennia because the amount by which space couples into time has a coefficient $1/c^2$ that is small on the human scale of the second, meter, and kilogram. When $c \to \infty$ we recover the old relativity of Galileo.

The c non-commutativity is that between two boosts B, B' in different directions. In Galileo relativity $BB' = B'B$; one simply adds the velocity vectors v and v' of B and B' to compute the resultant boost velocity $v + v' = v' + v$ of BB' or $B'B$. In special relativity BB' and $B'B$ differ by a rotation in the plane of the two boosts, called Thomas precession, again with a coefficient $1/c^2$.

The reciprocity between time and space led to a parallel one between energy and momentum, and to the identification of mass and energy. The photon has both. The energy and momentum of a particle are related to the rest-mass m_0 in special relativity by $E^2 - c^2p^2 = (m_0c^2)^2$. The parameter m_0 is 0 for the photon, for which $E = cp$. When we say that the photon "has mass 0," we speak elliptically. We mean that it has rest-mass 0. Its mass is actually E/c^2.

Some say that a photon is a bundle of energy. This statement is not meaningful enough to be wrong. In physics, energy is one property of a system among many others. Photons have energy as they have spin and momentum and cannot *be* energy any more than they can be spin or momentum. In the late 1800's some thinkers declared that all matter is made of one philosophical stuff that they identified with energy, without much empirical basis. The theory is dead but its words linger on.

When we speak of a reactor converting mass into energy, we again speak elliptically and archaically. Strictly speaking, we can no more heat our house by converting mass into energy than by converting Centigrade into Fahrenheit. Since the c expansion, mass *is* energy. They are the same conserved stuff, mass-energy, in different units. Neither ox-carts nor nuclear reactors convert mass into energy. Both convert rest mass-energy into kinetic mass-energy.

2. $\boxed{G}$ In special relativity the light rays through the origin of space-time form a three-dimensional cone in four dimensions, called the light cone, whose equation is $c^2t^2 - x^2 - y^2 - z^2 = 0$. Space-time is supposed to be filled with such light cones, one at every point, all parallel, telling light where it can go. This is a reciprocity failure of special relativity: Light cones influence light, light does not influence light cones. The light-cone field is an idol of special relativity.

In this case general relativity repaired reciprocity. An acceleration $\mathbf{a}$ of an observer is equivalent to a gravitational field $\mathbf{g} = -\mathbf{a}$ in its local effects. Even in the presence of gravitation, special relativity still describes correctly the infinitesimal neighborhood of each space-time point. Since an acceleration clearly distorts the field of light cones, and gravity is locally equivalent to acceleration, Einstein identified gravity with such a distortion. In his G expansion, which is general relativity, the light-cone field is as much a dynamical variable as the electromagnetic field, and the two fields influence each other reciprocally, to an extent proportional to Newton's gravitational constant G.

The light-cone directions dx at one point x can be defined by the vanishing of the norm $d\tau^2 = \sum_{\mu\nu} g_{\mu\nu}(x)dx^\mu dx^\nu = 0$; since Einstein, one leaves such summation signs implicit. General relativity represents gravity in each frame by the coefficient matrix $g_{..}$, which now varies with the space-time point. To have the light cones uniquely determine the matrix g, one may posit $\det g = 1$. The light cones guide photons and planets, which react back on the light cones through their energy and momentum. Newton's theory of gravity survives as the linear term in a series expansion of Einstein's theory of gravity in powers of G under certain physical restrictions.

The startling non-commutativity introduced by the G expansion is space-time curvature. If T, T' are infinitesimal translations along two orthogonal coordinate axes then in special relativity $TT' = T'T$ and in a gravitational field $TT' \neq T'T$. The differences $TT' - T'T$ define curvature. The Einstein gravitational equations describe how the flux of momentum-energy — with coefficient G — curves space-time. When $G \to 0$ we recover the flat space-time of special relativity.

Photons are the main probes in two of the three classic tests of general relativity, which provided an example of a successful gauge theory that ultimately inspired the gauge revolution of the Standard Model. The next expansion that went into the Standard Model is the h expansion.

3. $\boxed{\hbar}$ Before quantum mechanics, the theory of a physical system split neatly into two phases. *Kinematics* tells about all the complete descriptions of the system or of reality, called states. *Dynamics* tells about how states change in dynamical processes. Operationally speaking, kinematics concerns filtration processes, which select systems of one kind, and dynamics concerns propagation processes, which change systems of one kind into another. Filtration processes represent predicates about the system. Such "acts of election" seem empirically to commute, Boole noted in 1847, as he was laying the foundations of his laws of thought.[4] But dynamical processes represent actions on the system and need not commute.

In h-land, quantum theory, filtrations no longer commute. This is what we mean operationally when we say that observation changes the system observed.

Such non-commuting filtrations were first used practically by Norse navigators who located the cloud-hidden sun by sighting clouds through beam-splitting crystals of Iceland spar. This phenomenon, like oil-slick colors and partial specular reflection, was not easy for Newton's granular theory of light. Newton speculated that some kind of invisible transverse guide wave accompanied light corpuscles and controlled these phenomena, but he still argued for his particle theory of light, declaring that light did not "bend into the shadow," or diffract, as waves would. Then Thomas Young exhibited light diffraction in 1804, and buried the particle theory of light.

Nevertheless Étienne-Louis Malus still applied Newton's photon theory to polarization studies in 1805. Malus was truer than Newton to Newton's own experimental philosophy and anticipated modern quantum practice. He did not speculate about invisible guide waves but concerned himself with experimental predictions, specifically the transition probability P — the probability that a photon passing the first filter will pass the second. For linear polarizers with polarizing axes along the unit vectors **a** and **b** normal to the light ray, $P = |\mathbf{a} \cdot \mathbf{b}|^2$, the Malus law. Malus may have deduced his law as much from plausible principles of symmetry and conservation as from experiment.

Write $f' < f$ to mean that all f' photons pass f but not conversely, a relation schematized in Figure 1.

A filtration process f is called *sharp* (homogeneous, pure) if it has no proper refinement $f' < f$.

In mechanics one assumed implicitly that if 1 and 2 are two sharp filtration processes, then the transition probability for a particle from 1 to pass 2 is either 0 (when 1 and 2 filter for different kinds of particle) or 1 (when they filter for the same kind); briefly put, that all sharp filtrations are

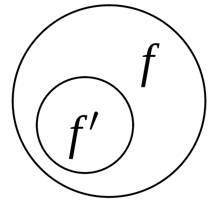

Fig. 1. If no such f' exists, f is sharp.

non-dispersive. (Von Neumann 1934 spoke of pure ensembles rather than sharp filtrations; the upshot is the same.) The successive performance of filtration operations, represented by P_2P_1, to be read from right to left, is a kind of AND combination of predicates and their projectors, though the resultant of two filtrations may not be a filtration.

The Malus law, applied to two sharp filtrations in succession implies that even sharp filtrations are dispersive, and that photon filtrations do not commute, confirming Boole's uncanny premonition. Since we do not directly perceive polarization, we need three polarizing filters to verify that two do not commute. Let the polarization directions of P_1 and P_2 be obliquely oriented, neither parallel nor orthogonal. Compare experiments $P_1P_2P_1$ and $P_1P_1P_2 = P_1P_2$. Empirically, and in accord with the Malus law, all photons from P_1P_2 pass through P_1 but not all from P_2P_1 pass through P_1. Therefore empirically $P_1P_2P_1 \neq P_1P_1P_2$, and so $P_2P_1 \neq P_1P_2$.

This non-commutativity revises the logic that we use for photons.

If we generalize **a** and **b** to vectors of many components, representing general ideal filtration processes, Malus' Law becomes the fundamental Born statistical principle of quantum physics today. The guide wave concept of Newton has evolved into the much less object-like wave-function concept of quantum theory. The traditional boundary between commutative kinematical processes of information and non-commutative dynamical processes of transformation has broken down.

One reasons today about photons, and quantum systems in general, with a special quantum logic and quantum probability theory. One represents quantum filtrations and many other processes by matrices, and expresses quantum logic with matrix addition and multiplication; hence the old name "matrix mechanics."’

We can represent any photon source by a standard perfectly white source ∘ followed by suitable processes, and any photon counter by a standard perfect counter • preceded by suitable processes. This puts experiments into a convenient standard form

$$\bullet \leftarrow P_n \leftarrow \ldots \leftarrow P_1 \leftarrow \circ \tag{1}$$

of a succession of physical processes between a source and a target.

Quantum theory represents all these intermediate processes by square matrices, related to experiment by the generalized Malus-Born law: For unit incident flux from ∘, the counting rate P at • for this experiment is determined by the matrix product

$$T = T_n \ldots T_1 \tag{2}$$

and its Hermitian conjugate T^* (the complex-conjugate transpose of T) as the trace

$$P = \frac{\mathrm{Tr}\, T^* T}{\mathrm{Tr}\, 1}\,. \tag{3}$$

This is the unconditioned probability for transmission. A photon that stops in the first filter contributes 0 to the count at the counter but 1 to the count at the source. The vectors **a** and **b** of the Malus law are column vectors on which these quantum matrices act.

The physical properties of the quantum process determine the algebraic properties of its quantum matrix. For example a filtration operation P for photon polarizations becomes a 2×2 projection matrix or projector, one obeying $P^2 = P = P^\dagger$.

Heisenberg introduced quantum non-commutativity through the (non-) commutation relation

$$xp - px = i\hbar, \tag{4}$$

for the observables of momentum p and position x, not for filtrations. ($\hbar \equiv h/2\pi$ is a standard abbreviation.) But all observables are linear combinations of projectors, even in classical thought, and all projectors are functions of observables, polynomials in the finite-dimensional cases. So Heisenberg's non-commutativity of observables is equivalent to the non-commutativity of filtration processes, and so leads to a quantum logic.

The negation of the predicate P is $1 - P$ for quantum logic as for Boole logic. Quantum logic reduces to the Boole logic for diagonal filtration matrices, with elements 0 or 1. Then Boole rules. The classical logic also works well for quantum experiments with many degrees of freedom. Two directions chosen at random in a space of huge dimensionality are almost certainly almost orthogonal, and then Boole's laws almost apply. Only in low-dimensional playgrounds like photon polarization do we easily experience quantum logic.

Quantum theory represents the passage of time in an isolated system by a unitary matrix $U = U^{-1\dagger}$ obeying Heisenberg's Equation, the first-order differential equation $i\hbar dU/dt = HU$. It does not give a complete description of what evolves, but only describes the process. H is called the Hamiltonian operator and historically was at first constructed from the Hamiltonian of a classical theory. U appears as a block in (1) and a factor in (2) for every time-lapse t between operations.

$U(t)$ transforms any vector $\psi(0)$ to a vector $\psi(t)$ that obeys the Schrödinger Equation $i\hbar\, d\psi/dt = H\psi$ during the transformation U. A quantum vector ψ is not a dynamical variable or a complete description of the system but represents an irreversible operation of filtration, and so the Schrödinger Equation does not describe the change of a dynamical variable. The Heisenberg Equation does that. The Schrödinger Equation describes a coordinate-transformation that solves the Heisenberg Equation. The pre-quantum correspondent of the Heisenberg Equation is the Hamiltonian equation of motion, giving the rate of change of all observables. In the correspondence between quantum and pre-quantum concepts as $\hbar \to 0$, the Heisenberg Equation is the quantum equation of motion. The pre-quantum correspondent of the Schrödinger Equation is the Hamilton-Jacobi Equation, which is an equation for a coordinate transformation that solves the equation of motion, and is not the equation of motion.

As has widely been noted, starting with the treatises of Von Neumann and Dirac on the fundamental principles of quantum theory, the input wave-function for a transition describes a sharp input filtration *process*, not a system variable. Common usage nevertheless calls the input wave-function of an experiment the "state of the photon."

There are indeed systems whose states are observable wave-functions. They are called waves. But a quantum wave-function is not the state of some wave. Calling it the "quantum state" is a relic of early failed attempts at a wave theory of the atom. The "state-vector" is not the kind of thing that can be a system observable in quantum theory. Each observable is a fixed operator or matrix.

The state terminology, misleading as it is, may be too widespread and deep-rooted to up-date. After all, we still speak of "sunrise" five centuries after Copernicus. One must read creatively and let context determine the meaning of the word "state." In spectroscopy it usually refers to a sharp input or output operation.

It is problematical to attribute absolute values even to true observables in quantum theory. Consider a photon in the middle of an experiment that begins with a process of linear polarization along the x axis and ends with a right-handed circular polarization around the z axis, given that the photon passes both polarizers. Is it polarized along the x axis or y axis? If we reason naively forwards from the first filter, the polarization between the two filters is certainly along the x axis, since the photon passed the first filter. If we reason naively backwards from the last filter, the intermediate photon polarization must be circular and right-handed, since it is going to pass the last filter; it has probabiltiy $1/2$ of being along the x axis. If we peek — measure the photon polarization in the middle of the experiment — we only answer a different question, concerning an experiment that ends with our new measurement. Measurements on a photon irreducibly and unpredictably change the photon, to an extent measured by h, so the question of the value between measurements has no immediate experimental meaning.

Common usage conventionally assign the input properties to the photon. Assigning the output properties would work as well. Either choice breaks the time symmetry of quantum theory unnecessarily. The most operational procedure is to assign a property to the photon not absolutely but only relative to an experimenter who ascertains the property, specifying in particular whether the experimenter is at the input or output end of the optical bench. Quantum logic thus requires us to put some of our pre-quantum convictions about reality on holiday, but they can all come back to work when h can be neglected.

The photon concept emerges from the combination of the Maxwell equations with the Heisenberg non-commutativity (4). Pre-quantum physicists recognized that by a Fourier analysis into waves $\sim e^{ik\cdot x}$ one can present the free electromagnetic field in a box as a collection of infinitely many linear harmonic oscillators, each with its own canonical coordinate q, canonical momentum p, and Hamiltonian

$$H = \frac{1}{2}(p^2 + \omega^2 q^2). \tag{5}$$

When the coefficient of p is scaled to unity in this way, the coefficient of q^2 is the square of the natural frequency ω of the oscillator. The Fourier analysis associates a definite wavevector k with each oscillator. The energy spectrum of each oscillator is the set of roots E of the equation $HX = EX$ with arbitrary non-zero “eigenoperator” X.

The energy spectrum is most elegantly found by the ladder method. One seeks a linear combination a of q and p that obeys $Ha = a(H - E_1)$. This means that a lowers E (and therefore H) by steps of E_1 in the sense that if $HX = EX$ then $H(aX) = (E - E_1)(aX)$, unless $aX = 0$. Such an a, if it exists, is called a ladder operator, therefore. It is easy to see that a ladder operator exists for the harmonic oscillator, namely $a = 2^{-1/2}(p - iq)$, with energy step $E_1 = \hbar\omega$. One scales a so that H takes the form

$$H = \hbar\omega(n + \frac{1}{2}), \tag{6}$$

$n = a^\dagger a$, and a lowers n by steps of 1: $na = a(n-1)$. Then n counts “excitation quanta” of the harmonic oscillator, each contributing an energy $E_1 = \hbar\omega$ to the total energy, and a momentum $\hbar k$ to the total momentum. The excitation quanta of the electromagnetic field oscillators are photons. The operator a is called an annihilation operator or annihilator for the photon because it lowers the photon count by 1. By the same token its adjoint is a photon creator.

The term $1/2$ in H contributes a zero-point energy that is usually arbitrarily discarded, primarily because any non-zero vacuum energy would violate Lorentz invariance and so disagree somewhat with experiment. One cannot deduce that the vacuum energy is zero from the present dynamical theory, and astrophysicists are now fairly sure that it is not zero.

A similar process leads to the excitation quanta of the field oscillators of other fields. Today one accounts for all allegedly fundamental quanta as excitation quanta of suitably designed field oscillators.

Now we can say what a photon is. Consider first what an apple is. When I move it from one side of the table to the other, or turn it over, it is still the same apple. So the apple is not its state, not what we know about the apple. Statistical mathematicians formulate the concept of a constant object with varying properties by identifying the object — sometimes called a random variable — not with one state but with the space of all its possible states. This works just as well for quantum objects as for random objects, once we replace states by more appropriate actions on the quantum object. The object is defined, for example, by the processes it can undergo. For example, the sharp filtration processes for one photon, relative to a given observer, form a collection with one structural element, the transition probability between two such processes. For many purposes we can identify a photon with this collection of processes.

The filtration processes mentioned are usually represented by lines through the origin in a Hilbert space. If we are willing to start from a Hilbert space, we can define a photon by its Hilbert space; not by one wave-function, which just says one way to produce a photon, but the collection of them all. This gives preference to input over output and spoils symmetry a bit. One restores time symmetry by using the algebra of operators rather than the Hilbert space to define the photon. In words, the photon is the creature on which those operations can act.

From the current viewpoint the concept of photon is not as fundamental as that of electromagnetic field. Not all electromagnetic interactions are photon-mediated. There are also static forces, like the Coulomb force. Different observers may split electromagnetic interactions into radiation and static forces differently. Gauge theory leads us to quantum fields, and photons arise as quantum excitations of one of these fields.

Quantum theory has a non-Boolean logic in much the sense that general relativity has a non-Euclidean geometry: it renounces an ancient commutativity. A Boolean logic has non-dispersive predicates called states, common to all observers; a quantum logic does not. Attempting to fit the quantum non-commutativity of predicates into a classical picture of an object with absolute states is like attempting to fit special relativity into a space-time with absolute time. Possibly we can do it but probably we shouldn't. If we accept that the expanded logic contracts to the familiar one when $\hbar to 0$, we can go on to the next expansion.

4. $\boxed{\hbar'\hbar''}$ In this section I describe a possible future expansion suggested by Segal[9] that might give a simpler and more finite structure to the photon and other quanta. There are clear indications, both experimental and structural, that quantum theory is still too commutative. Experiment indicates limits to the applicability of the concept of time both in the very small and the very large, ignored by present quantum theory. The theoretical assumption that all feasible operations commute with the imaginary i makes i a prototypical idol. The canonical commutation relations are unstable.

To unseat this idol and stabilize this instability, one first rewrites the defining relations for a photon oscillator in terms of antisymmetric operators $\widehat{q} := iq, \widehat{p} = -ip$:

$$\begin{aligned} \widehat{q}\widehat{p} - \widehat{p}\widehat{q} &= \hbar i, \\ i\widehat{q} - \widehat{q}i &= 0, \\ i\widehat{p} - \widehat{p}i &= 0. \end{aligned} \tag{7}$$

One stabilizing variation, for example, is

$$\begin{aligned} \widehat{q}\widehat{p} - \widehat{p}\widehat{q} &= \hbar i, \\ i\widehat{q} - \widehat{q}i &= \hbar'\widehat{p}, \end{aligned}$$

$$i\widehat{p} - \widehat{p}i \quad = \quad -\bar{h}''\widehat{q} \tag{8}$$

with Segal constants $\bar{h}', \bar{h}'' > 0$ supplementing the Planck quantum constant $\bar{h}$.[9] No matter how small the Segal constants are, if they have the given sign the expanded oscillator commutation relations can be rescaled to angular momentum relations[2]

$$\begin{aligned}
\widehat{L}_x\widehat{L}_y - \widehat{L}_y\widehat{L}_x &= \widehat{L}_z, \\
\widehat{L}_y\widehat{L}_z - \widehat{L}_z\widehat{L}_y &= \widehat{L}_x, \\
\widehat{L}_z\widehat{L}_x - \widehat{L}_x\widehat{L}_z &= \widehat{L}_y.
\end{aligned} \tag{9}$$

by a scaling

$$\begin{aligned}
\check{q} &= Q\widehat{L}_1, \\
\check{p} &= P\widehat{L}_2, \\
\check{i} &= J\widehat{L}_3,
\end{aligned} \tag{10}$$

with

$$\begin{aligned}
J &= \sqrt{\bar{h}'\bar{h}''} = \frac{1}{l}, \\
Q &= \sqrt{\bar{h}\bar{h}'}, \\
P &= \sqrt{\bar{h}\bar{h}''}.
\end{aligned} \tag{11}$$

As customary we have designated the maximum eigenvalue of $|\widehat{L}_z|$ by l. This theory is now stabilized by its curvature against further small changes in $\bar{h}, \bar{h}', \bar{h}''$; just as a small change in curvature turns any straight line into a circle but leaves almost all circles circular; and just as quantum theory is stable against small changes in $\bar{h}$.

To be sure, when $\bar{h}', \bar{h}'' \to 0$ we recover the quantum theory. As in all such expansions of physical theory, the quantum theory with c-number i is a case of probability zero in an ensemble of more likely expanded theories with operator i's. The canonical commutation relations might be right, but that would be a miracle of probability 0. Data always have some error bars, so an exactly zero commutator is never based entirely on experiment and usually incorporates faith in some prior absolute: here i. Renouncing that absolute makes room for a more stable kind of theory, based more firmly on experiment and at least as consistent with the existing data. Which one of these possibilities is in better agreement with experiment than the canonical theory can only be learned from experiment.

The most economical way to stabilize the Heisenberg relations is to close them on themselves as we have done here. A more general stabilization might also couple each oscillator to others. In the past the stabilizations that worked have usually been economical but not always.

These transquantum relations describe a rotator, not an oscillator. What we have thought were harmonic oscillators are more likely to be quantum rotators. It has been recognized for some time that oscillators can be approximated by rotators and conversely.[1,2,7] In particular, photons too are infinitely more likely to be quanta of a kind of rotation than of oscillation. If so, they can still have exact ladder operators, but their ladders now have a top as well as a bottom, with $2l+1$ rungs for rotational transquantum number l.

In the most intense lasers, there can be as many as 10^{13} photons in one mode at one time.[8] Then $2l \geq 10^{13}$ and $h'h'' \leq 10^{-26}$ in order of magnitude.

When we expand the commutation relations for time and energy in this way, the two new transquantum constants that appear indeed limit the applicability of these concepts both in the small and the large. They make the photon advance step by quantum step. We will probably never be able to visualize a photon but we might soon be able to choreograph one; to describe the process rather than the object.

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The concept of the photon—revisited

Ashok Muthukrishnan,[1] **Marlan O. Scully,**[1,2] **and M. Suhail Zubairy**[1,3]
[1]*Institute for Quantum Studies and Department of Physics, Texas A&M University, College Station, TX 77843*
[2]*Departments of Chemistry and Aerospace and Mechanical Engineering, Princeton University, Princeton, NJ 08544*
[3]*Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan*

The photon concept is one of the most debated issues in the history of physical science. Some thirty years ago, we published an article in Physics Today entitled "The Concept of the Photon,"[1] in which we described the "photon" as a classical electromagnetic field plus the fluctuations associated with the vacuum. However, subsequent developments required us to envision the photon as an intrinsically quantum mechanical entity, whose basic physics is much deeper than can be explained by the simple 'classical wave plus vacuum fluctuations' picture. These ideas and the extensions of our conceptual understanding are discussed in detail in our recent quantum optics book.[2] In this article we revisit the photon concept based on examples from these sources and more.

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The "photon" is a quintessentially twentieth-century concept, intimately tied to the birth of quantum mechanics and quantum electrodynamics. However, the root of the idea may be said to be much older, as old as the historical debate on the nature of light itself – whether it is a wave or a particle – one that has witnessed a seesaw of ideology from antiquity to present. The transition from classical to quantum descriptions of light presents yet another dichotomy, one where the necessity of quantizing the electromagnetic field (over and above a quantization of matter) has been challenged. The resolution lies in uncovering key behavior of quantum light fields that are beyond the domain of the classical, such as vacuum fluctuations and quantum entanglement, which necessitate a quantum theory of radiation.[2–5] Nevertheless, a precise grasp of the "photon" concept is not an easy task, to quote Albert Einstein:

> "These days, every Tom, Dick and Harry thinks he knows what a photon is, but he is wrong."

We ought to proceed with diligence and caution. In the words of Willis Lamb:[6]

> "What do we do next? We can, and should, use the Quantum Theory of Radiation. Fermi showed how to do this for the case of Lippmann fringes. The idea is simple, but the details are somewhat messy. A good notation and lots of practice makes it easier. Begin by deciding how much of the universe needs to be brought into the discussion. Decide what normal modes are needed for an adequate treatment. Decide how to model the light sources and work out how they drive the system."

We proceed to elucidate the photon concept by specific experiments (real and gedanken) which demonstrate the need for and shed light on the meaning of the "photon." Specifically, we will start by briefly reviewing the history of the wave-particle debate and then giving seven of our favorite examples, each clarifying some key aspect of the quantum nature of light. The two facets of the photon that we focus on are vacuum fluctuations (as in our earlier article[1]), and aspects of many-particle correlations (as in our recent book[2]). Examples of the first are spontaneous emission, Lamb shift, and the scattering of atoms off the vacuum field at the entrance to a micromaser. Examples of the second facet include quantum beats, quantum eraser, and photon correlation microscopy. Finally, in the example of two-site downconversion interferometry, the essence of both facets is combined and elucidated.

In the final portions of the article, we return to the basic questions concerning the nature of light in the context of the wave-particle debate: <u>What</u> is a photon and <u>where</u> is it? To the first question, we answer in the words of Roy Glauber:

> "A photon is what a photodetector detects."

To the second question (on the locality of the photon), the answer becomes: "A photon is *where* the photodetector detects it." In principle, the detector could be a microscopic object such as an atom. Guided by this point of view, we address the much debated issue of the existence of a photon wave function $\psi(\mathbf{r},t)$.[2,7,8] Arguments to the contrary notwithstanding, we show that the concept of the photon wave function arises naturally from the quantum theory of photodetection (see Ref. [2], ch. 1). A wealth of insight is gained about the interference and entanglement properties of light by studying such one-photon, and related two-photon, 'wave functions'.[2]

Light – wave or particle?

The nature of light is a very old issue in the history of science. For the ancient Greeks and Arabs, the debate centered on the connection between light and vision. The tactile theory, which held that our vision was initiated by our eyes reaching out to "touch" or feel something at a distance, gradually lost ground to the emission theory, which postulated that vision resulted from illuminated objects emitting energy that was sensed by our eyes. This paradigm shift is mainly due to the eleventh-century Arab scientist Abu Ali Hasan Ibn Al-Haitham (or 'Alhazen') who laid the groundwork for classical optics through investigations into the refraction and dispersion properties of light. Later Renaissance thinkers in Europe envisioned light as a stream of particles, perhaps supported by the ether, an invisible medium thought to permeate empty space and all transparent materials.

In the seventeenth century, Pierre de Fermat introduced the *principle of least time* to account for the phenomenon of refraction. Equivalently, his principle states that a ray of light takes the path that minimizes the optical path length between two points in space:

$$\delta \int_{\mathbf{r}_0}^{\mathbf{r}} n\, ds = 0, \tag{1}$$

where $n = c/v$ is the (spatially varying) refractive index that determines the velocity of the light particle, and δ denotes a variation over all paths connecting $\mathbf{r}_0$ and $\mathbf{r}$. Fermat's principle is the foundation for geometrical optics, a theory based on the view that light is a particle that travels along well-defined geometrical rays. The idea of light as particle (or 'corpuscle') was of course adopted by Isaac Newton, who bequeathed the weight of his scientific legacy, including the bearing of his laws of mechanics, on the nature of light.

Christian Huygens on the other hand, a contemporary of Newton, was a strong advocate of the wave theory of light. He formulated a principle (that now bears his name) which describes wave propagation as the interference of secondary wavelets arising from point sources on the existing wavefront. It took the mathematical genius of Augustin Fresnel, 150 years later, to realize the consequences of this discovery, including a rigorous development of the theory of wave diffraction. Light does not form sharp, geometrical shadows that are characteristic of a particle, but bends around obstacles and apertures.

The revival of the wave theory in the early nineteenth century was initiated by Thomas Young. In 1800, appearing before the Royal Society of London, Young spoke for an analogy between light and sound, and declared later that a two-slit interference experiment would conclusively demonstrate the wave nature of light (see Figure 1). It is hard for the modern reader to visualize how counter-intuitive this suggestion was at the time. The idea that a screen uniformly illuminated by a single aperture could develop dark fringes with the introduction of a second aperture – that the addition of *more* light could result in *less* illumination – was hard for Young's audience to digest.

Likewise, Fresnel's diffraction theory was received with skepticism by the judges on the 1819 prize committee in Paris. In particular, the esteemed Pierre Simon de Laplace was very skeptical of the wave theory. His protégé, Siméon-Denis Poisson, highlighted the seemingly absurd fact that the theory implied a bright spot at the center of the shadow of an illuminated opaque disc, now known as Poisson's spot. The resistance to switch from a particle description to a wave description for light by these pre-eminent scientists of the early nineteenth century gives an indication of the great disparity between these two conceptions. It was a precursor of the struggle to come a hundred years later with the advent of quantum mechanics.

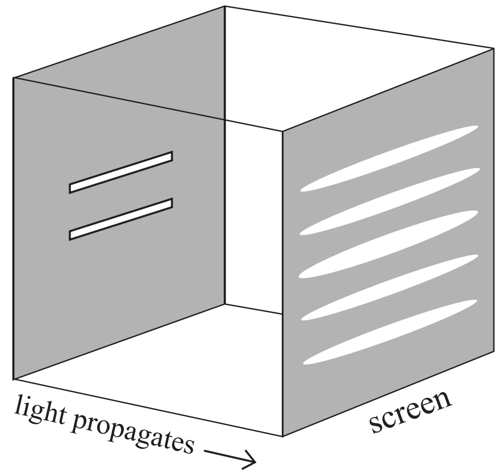

Fig. 1. *Young's two-slit experiment* – Light incident on two slits in a box propagates along two pathways to a given point on the screen, displaying constructive and destructive interference. When a single photon is incident on the slits, it is detected with highest probability at the interference peaks, but never at the interference nodes.

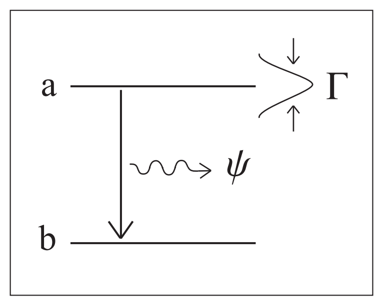

Fig. 2. *Spontaneous emission* – Two-level atom, with upper-level linewidth Γ spontaneously emits a photon. Fluctuations in the vacuum field cause the electron in the excited state to decay to the ground state in a characteristic time Γ^{-1}.

The wave theory really came into its own in the late nineteenth century in the work of James Clerk Maxwell. His four equations, known to all students of undergraduate physics, is the first self-contained theory of radiation. Receiving experimental confirmation by Heinrich Hertz, the Maxwell theory unified the disparate phenomena of electricity and magnetism, and gave physical meaning to the transverse polarizations of light waves. The far-reaching success of the theory explains the hubris of late nineteenth century physicists, many of whom believed that there were really only two "clouds" on the horizon of physics at the dawn of the twentieth century. Interestingly enough, both of these involved light.

The first cloud, namely the null result of the Michelson-Morley experiment, led to special relativity, which is the epitome of classical mechanics, and the logical capstone of classical physics. The second cloud, the Rayleigh-Jeans ultraviolet (UV) catastrophe and the nature of blackbody radiation, led to the advent of quantum mechanics, which of course was a radical change in physical thought. While both of these problems involved the radiation field, neither (initially) involved the concept of a photon. That is, neither Albert Einstein and Hendrik Lorentz in the first instance, nor Max Planck in the second, called upon the particulate nature of light for the explanation of the observed phenomena. Relativity is strictly classical, and Planck quantized the energies of the oscillators in the walls of his cavity, not the field.[9]

The revival of the particle theory of light, and the beginning of the modern concept of the photon, was due to Einstein. In his 1905 paper on the photoelectric effect,[10] the emission of electrons from a metallic surface irradiated by UV rays, Einstein postulated that light comes in discrete bundles, or quanta of energy, borrowing Planck's five-year old hypothesis: $E = \hbar\nu$, where ν is the circular frequency and $\hbar$ is Planck's constant divided by 2π. This re-introduced the particulate nature of light into physical discourse, not as localization in space in the manner of Newton's corpuscles, but as discreteness in energy. But irony upon irony, it is a historical curiosity that Einstein got the idea for the photon from the physics of the photoelectric effect. In fact, it can be shown that the essence of the photoelectric effect does not require the quantization of the radiation field,[11] a misconception perpetuated by the mills of textbooks, to wit, the following quote from a mid-century text:[12]

> "Einstein's photoelectric equation played an enormous part in the development of the modern quantum theory. But in spite of its generality and of the many successful applications that have been made of it in physical theories, the equation:
>
> $$\hbar\nu = E + \Phi \tag{2}$$
>
> is, as we shall see presently, based on a concept of radiation – the concept of 'light quanta' – completely at variance with the most fundamental concepts of the classical electromagnetic theory of radiation."

We will revisit the photoelectric effect in the next section and place it properly in the context of radiation theory.

Both the Planck hypothesis and the Einstein interpretation follow from considerations of how energy is *exchanged* between radiation and matter. Instead of an electromagnetic wave continuously driving the amplitude of a classical oscillator, we have the discrete picture of light of the right frequency absorbed or emitted by a quantum oscillator, such as an atom in the walls of the cavity, or on a metallic surface. This seemingly intimate connection between energy quantization and the interaction of radiation with matter motivated the original coining of the word "photon" by Gilbert Lewis in 1926:[13]

> "It would seem inappropriate to speak of one of these hypothetical entities as a particle of light, a corpuscle of light, a light quantum, or light quant, if we are to assume that it spends only a minute fraction of its existence as a carrier of radiant energy, while the rest of the time it remains as an important structural element within the atom... I therefore take the liberty of proposing for this hypothetical new atom, which is not light but plays an essential part in every process of radiation, the name *photon*."

Energy quantization is the essence of the old quantum theory of the atom proposed by Niels Bohr. The electron is said to occupy discrete orbitals with energies E_i and E_j, with transitions between them caused by a photon of the right frequency: $\nu = (E_i - E_j)/\hbar$. An ingenious interpretation of this quantization in terms of matter waves was given by Louis de Broglie, who argued by analogy with standing waves in a cavity, that the wavelength of the electron in each Bohr orbital is quantized – an integer number of wavelengths would have to fit in a circular orbit of the right radius. This paved the way for Erwin Schrödinger to introduce his famous wave equation for matter waves, the basis for (non-relativistic) quantum mechanics of material systems.

Quantum mechanics provides us with a new perspective on the wave-particle debate, vis á vis Young's two-slit experiment (Figure 1). In the paradigm of quantum interference, we add the probability amplitudes associated with different pathways through an interferometer. Light (or matter) is neither wave nor particle, but an intermediate entity that obeys the superposition principle. When a single photon goes through the slits, it registers as a point-like event on the screen (measured, say, by a CCD array). An accumulation of such events over repeated trials builds up a probabilistic fringe pattern that is characteristic of classical wave interference. However, if we arrange to acquire information about which slit the photon went through, the interference nulls disappear. Thus, from the standpoint of complementarity, both wave and particle perspectives have equal validity. We will return to this issue later in the article.

The semiclassical view

The interaction of radiation and matter is key to understanding the nature of light and the concept of a photon. In the semiclassical view, light is treated classically and only matter is quantized. In other words, both are treated on an *equal* footing: a wave theory of light (the Maxwell equations) is combined self-consistently with a wave theory of matter (the Schrödinger equation). This yields a remarkably accurate description of a large class of phenomena, including the photoelectric effect, stimulated emission and absorption, saturation effects and nonlinear spectroscopy, pulse propagation phenomena, "photon" echoes, etc. Many properties of laser light, such as frequency selectivity, phase coherence, and directionality, can be explained within this framework.[14]

The workhorse of semiclassical theory is the two-level atom, specifically the problem of its interaction with a sinusoidal light wave.[15] In reality, real atoms have lots of levels, but the two-level approximation amounts to isolating a particular transition that is nearly resonant with the field frequency ν. That is, the energy separation of the levels is assumed to be $E_a - E_b = \hbar\omega \approx \hbar\nu$. Such a comparison of the atomic energy difference with the field frequency is in the spirit of the Bohr model, but note that this already implies a discreteness in light energy, $\Delta E = \hbar\nu$. That a semiclassical analysis is able to bring out this discreteness – in the form of *resonance* – is a qualitative dividend of this approach.

Schrödinger's equation describes the dynamics of the atom, but how about the dynamics of the radiation field? In the semiclassical approach, one assumes that the atomic electron cloud $\psi^*\psi$, which is polarized by the incident field, acts like an oscillating *charge* density, producing an ensemble dipole moment that re-radiates a classical Maxwell field. The effects of radiation reaction, i.e., the back action of the emitted field on the atom, are taken into account by requiring the coupled Maxwell-Schrödinger equations to be self-consistent with respect to the *total* field. That is, the field that the atoms see should be consistent with the field radiated. In this way, semiclassical theory becomes a self-contained description of the dynamics of a quantum mechanical atom interacting with a classical field. As we have noted above, its successes far outweigh our expectations.

Let us apply the semiclassical analysis to the photoelectric effect, which provided the original impetus for the quantization of light. There are three observed features of this effect that need accounting. First, when light shines on a photoemissive surface, electrons are ejected with a kinetic energy E equal to $\hbar$ times the frequency ν of the incident light less some work function Φ, as in Eq. (2). Second, it is observed that the rate of electron ejection is proportional to the square of the incident electric field E_0. Third, and more subtle, there is not necessarily a time delay between the instant the field is turned on, and the time when the photoelectron is ejected, contrary to classical expectations.

All three observations can be nominally accounted for by applying the semiclassical theory to lowest order in perturbation of the atom-field interaction $V(t) = -eE_0r$.[11] This furnishes a Fermi Golden Rule for the probability of transition of the electron from the ground state g of the atom to the kth excited state in the continuum:

$$P_k = \left[2\pi(e|r_{kg}|E_0/2\hbar)^2 t\right] \delta[\nu - (E_k - E_g)/\hbar], \qquad (3)$$

where er_{kg} is the dipole matrix element between the initial and final states. The δ-function (which has units of time) arises from considering the frequency response of the surface, and assuming that t is at least as long as several optical cycles: $\nu t \gg 1$. Now, writing energy $E_k - E_g$ as $E + \Phi$, we see that the δ-function immediately implies Eq. (2). The second fact is also clearly contained in Eq. (3) since P_k is proportional to E_0^2. The third fact of photoelectric detection, the finite time delay, is explained in the sense that P_k is linearly proportional to t, and there is a finite probability of the atom being excited even at infinitesimally small times.

Thus, the experimental aspects of the photoelectric effect are completely understandable from a semiclassical point of view. Where we depart from a classical intuition for light is a subtle issue connected with the third fact, namely that there is negligible time delay between the incidence of light and the photoelectron emission. While this is understandable from an *atomic* point of view – the electron has finite probability of being excited even at very short times – the argument breaks down when we consider the implications for the field. That is, if we persist in thinking about the field classically, energy is not conserved. Over a time interval t, a classical field E_0 brings in a flux of energy $\varepsilon_0 E_0^2 At$ to bear on the atom, where A is the atomic cross-section. For short enough times t, this energy is negligible compared to $\hbar\nu$, the energy that the electron supposedly absorbs (instantaneously) when it becomes excited. We just do not have the authority, within the Maxwell formalism, to affect a similar *quantum jump* for the field energy.

For this and other reasons (see next section), it behooves us to supplement the epistemology of the Maxwell theory with a quantized view of the electromagnetic field that fully accounts for the probabilistic nature of light and its inherent fluctuations. This is exactly what Paul Dirac did in the year 1927, when the photon concept was, for the first time, placed on a logical foundation, and the quantum theory of radiation was born.[16] This was followed in the 1940s by the remarkably successful theory of quantum electrodynamics (QED) – the quantum theory of interaction of light and matter – that achieved unparalleled numerical accuracy in predicting experimental observations. Nevertheless, a short twenty years later, we would come back full circle in the saga of semiclassical theory, with Ed Jaynes questioning the need for a quantum theory of radiation at the 1966 conference on Coherence and Quantum Optics at Rochester, New York.

"Physics goes forward on the shoulders of doubters, not believers, and I doubt that QED is necessary," declared Jaynes. In his view, semiclassical theory – or 'neoclassical' theory, with the addition of a radiation reaction field acting back on the atom – was sufficient to explain the Lamb shift, thought by most to be the best vindication yet of Dirac's field quantization and QED theory (see below). Another conference attendee, Peter Franken, challenged Jaynes to a bet. One of us (MOS) present at the conference recalls Franken's words: "You are a reasonably rich man. So am I, and I say put your money where your face is!" He wagered $100 over whether the Lamb shift could or could not be calculated without QED. Jaynes took the bet that he could, and Willis Lamb agreed to be the judge.

In the 1960s and 70s, Jaynes and his collaborators reported partial success in predicting the Lamb shift using neoclassical theory.[17] They were able to make a qualitative connection between the shift and the physics of radiation reaction – in the absence of field quantization or vacuum fluctuations – but failed to produce an accurate numerical prediction which could be checked against experiment. For this reason, at the 1978 conference in Rochester, Lamb decided to yield the bet to Franken. An account of the arguments for and against this decision was summarized by Jaynes in his paper at the conference.[18] In the end, QED had survived the challenge of semiclassical theory, and vacuum fluctuations were indeed “very real things” to be reckoned with.

Seven examples

Our first three examples below illustrate the reality of vacuum fluctuations in the electromagnetic field as manifested in the physics of the atom. The “photon” acquires a stochastic meaning in this context. One speaks of a classical electromagnetic field with fluctuations due to the vacuum. To be sure, one cannot “see” these fluctuations with a photodetector, but they make their presence felt, for example, in the way the atomic electrons are “jiggled” by these random vacuum forces.

1. Spontaneous emission

In the phenomenon of spontaneous emission,[19] an atom in the excited state decays to the ground state and spontaneously emits a photon (see Figure 2). This “spontaneous” emission is in a sense stimulated emission, where the stimulating field is a vacuum fluctuation. If an atom is placed in the excited state and the field is classical, the atom will never develop a dipole moment and will never radiate. In this sense, semiclassical theory does not account for spontaneous emission. However, when vacuum fluctuations are included, we can think conceptually of the atom as being stimulated to emit radiation by the fluctuating field, and the back action of the emitted light will drive the atom further to the ground state, yielding decay of the excited state. It is in this way that we understand spontaneous emission as being due to vacuum fluctuations.

2. Lamb shift

Perhaps the greatest triumph of field quantization is the explanation of the Lamb shift[20] between, for example, the $2s_{1/2}$ and $2p_{1/2}$ levels in a hydrogenic atom. Relativistic quantum mechanics predicts that these levels should be degenerate, in contradiction to the experimentally observed frequency splitting of about 1 GHz. We can understand the shift intuitively[21] by picturing the electron forced to fluctuate about its first-quantized position in the atom due to random kicks from the surrounding, fluctuating vacuum field (see Figure 3). Its average displacement $\langle \Delta \mathbf{r} \rangle$ is zero, but the squared displacement $\langle \Delta \mathbf{r} \rangle^2$ is slightly nonzero, with the result that the electron “senses” a slightly different Coulomb pull from the positively charged nucleus than it normally would. The effect is more prominent nearer the nucleus where the Coulomb potential falls off more steeply, thus the s orbital is affected more than the p orbital. This is manifested as the Lamb shift between the levels.

3. Micromaser – scattering off the vacuum

A micromaser consists of a single atom interacting with a single-mode quantized field in a high-Q cavity.[22] An interesting new perspective on vacuum fluctuations is given by the recent example of an excited atom scattering off an effective potential barrier created by a vacuum field in the cavity (see Figure 4).[23] When the atomic center-of-mass motion is quantized, and the atoms are travelling slow enough (their kinetic energy is smaller than the atom-field interaction energy), it is shown that they can undergo reflection from the cavity, even when it is initially empty, i.e. there are no photons. The reflection of the atom takes place due to the discontinuous change in the strength of the coupling with vacuum fluctuations at the input to the cavity. This kind of reflection off an edge discontinuity is common in wave mechanics. What is interesting in this instance is that the reflection is due to an abrupt change in coupling with the vacuum between the inside and the outside of the cavity. It is then fair to view this physics as another manifestation of the effect of vacuum fluctuations, this time affecting the center-of-mass dynamics of the atom.

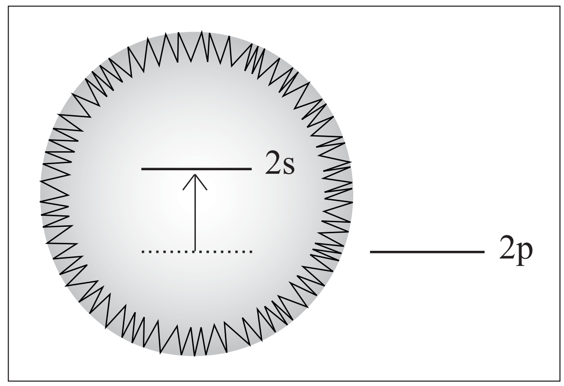

Fig. 3. *Lamb shift* – Schematic illustration of the Lamb shift of the hydrogenic $2s_{1/2}$ state relative to the $2p_{1/2}$ state. Intuitive understanding of the shift as due to random jostling of the electron in the $2s$ orbital by zero-point fluctuations in the vacuum field.

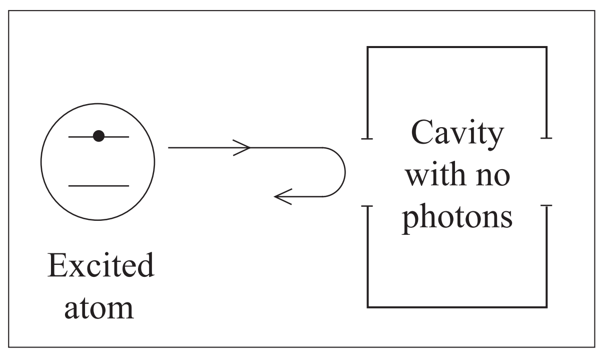

Fig. 4. *Scattering off the vacuum* – An excited atom approaching an empty cavity can be reflected for slow enough velocities. The vacuum cavity field serves as an effective potential barrier for the center-of-mass wave function of the atom.

Our next three examples involve the concept of multi-particle entanglement, which is a distinguishing feature of the quantized electromagnetic field. Historically, inter-particle correlations have played a key role in fundamental tests of quantum mechanics, such as the EPR paradox, Bell inequalities and quantum eraser. These examples illustrate the real-

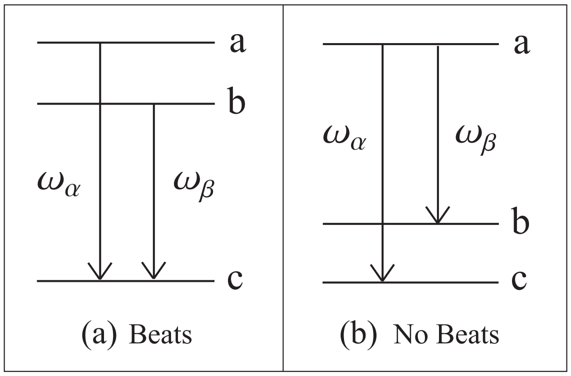

Fig. 5. *Quantum beats* – a) When a single atom decays from either of two upper levels to a common lower level, the two transition frequencies produce a beat note $\omega_\alpha - \omega_\beta$ in the emitted photon. b) No beats are present when the lower levels are distinct, since the final state of the atom provides distinguishing information on the decay route taken by the photon.

ity of quantum correlations in multi-photon physics. In recent years, entangled photons have been key to applications in quantum information and computing, giving rise to new technologies such as photon correlation microscopy (see below).

4. Quantum beats

In general, beats arise whenever two or more frequencies of a wave are simultaneously present. When an atom in the excited state undergoes decay along two transition pathways, the light produced in the process is expected to register a beat note at the difference frequency, $\omega_\alpha - \omega_\beta$, in addition to the individual transition frequencies ω_α and ω_β. However, when a single atom decays, beats are present only when the two final states of the atom are identical (see Figure 5). When the final states are distinct, quantum theory predicts an absence of beats.[24] This is so because the two decay channels end in different atomic states [$|b\rangle$ or $|c\rangle$ in Figure 5(b)]. We now have which-path information since we need only consult the atom to see which photon (α or β) was emitted – i.e. the entanglement between the atom and the quantized field destroys the interference. Classical electrodynamics, vis á vis semiclassical theory, cannot explain the "missing" beats.

5. Quantum eraser and complementarity

In the quantum eraser,[25] the which-path information about the interfering particle is *erased* by manipulating the second, entangled particle. Complementarity is enforced not by the uncertainty principle (through a measurement process), but by a quantum correlation between particles.[26] This notion can be realized in the context of two-photon interferometry.[27–29] Consider the setup shown in Figure 6, where one of two atoms $i = 1,2$ emits two photons ϕ_i and γ_i. Interference is observed in ϕ only when the spatial origin of γ cannot be discerned, i.e., when detector D_1 or D_2 clicks. Erasure occurs when the γ photon is reflected (rather than transmitted) at beamsplitter BS1 or BS2, which in the experiment occurs *after* the ϕ photon has been detected. Thus, quantum entanglement between the photons enables a realization of 'delayed choice',[30] which cannot be simulated by classical optics.

6. Photon correlation microscopy

Novel interference phenomena arise from second-order correlations of entangled photons, such as arise from the spontaneous cascade decay of a three-level atom (where the emitted photons are correlated in frequency and time of emission).[2] When two such atoms are spatially separated and one of them undergoes decay, a two-photon correlation measurement enables high-resolution *spectral* microscopy on the atomic level structure.[31] It can be shown that the resolution of the upper two levels a and b in each atom is limited only by the linewidth Γ_a, and not by Γ_a and Γ_b together (as is usually the case). This phenomenon relies on the path and frequency entanglement between the two photons arising from spatially separated cascade sources.

A further consequence of the two-atom geometry is the enhancement in *spatial* resolution that occurs because the photons are entangled in path – that is, the photon pair arises from one atom or the other, and their joint paths interfere. Coincident detection of the two photons (each of wavelength λ) shows a fringe resolution that is enhanced by a factor of two as compared to the classical Rayleigh limit, $\lambda/2$. This enables applications in high-resolution lithography.[32,33] The fringe doubling is due to the fact that the two photons propagate along the same path, and their sum frequency, 2ω, characterizes their joint detection probability. Path entanglement cannot be simulated by (co-propagating) classical light pulses.

7. Two-site downconversion interferometry

In what follows, we consider a two-particle interferometry experiment that allows us to elucidate both facets of the photon considered above – vacuum fluctuations and quantum entanglement. The thought experiment we have in mind is based on an actual one that was carried out using parametric downconversion.[34] Consider the setup shown in Figure 7, where two atoms $i = 1,2$ are fixed in position and one of them emits two photons, labeled ϕ_i and γ_i, giving rise to a two-photon state that is a superposition of emissions from each atom:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\phi_1\rangle|\gamma_1\rangle + |\phi_2\rangle|\gamma_2\rangle\right). \quad (4)$$

This is an entangled state in the sense that an emission of ϕ_i is always accompanied by an emission of γ_i, for $i = 1$ or 2. Let us suppose that we are interested in interference of the ϕ photon only, as measured by varying the path lengths of ϕ_1 and ϕ_2 to detector D_ϕ. The γ photon serves as a marker that potentially records which atom emitted the ϕ photon. It is found that by inserting (or removing) a beamstop in the path of γ_1, the interference fringes can be made to vanish (or re-appear) at D_ϕ, even when D_γ is not actually observed.

It is interesting to explain this phenomenon using stochastic electrodynamics[35] (as was done with the Lamb shift). Let us replace the two photons ϕ and γ with *classical* light fields

$E_i^{\phi}(\mathbf{r},t)$ and $E_i^{\gamma}(\mathbf{r},t)$, generated respectively by dipole transitions *a*-*b* and *b*-*c* in each atom *i*. If the atoms are initially in a superposition of states *a* and *c*, then zero-point fluctuations in the field mode γ will introduce population into level *b* (from *a*), with a random phase $\varphi_{\gamma,i}$. The first-order interference in the field mode ϕ will now depend on an ensemble average over the vacuum-induced two-atom phase difference: $\langle E_1^{\phi} E_2^{\phi}\rangle \propto \langle \exp[-i(\varphi_{\gamma,1} - \varphi_{\gamma,2})]\rangle$. This quantity goes to zero if the two phases are statistically independent, which is the case when the beamstop is in place between the two atoms. Thus, we have here a connection between vacuum fluctuation physics (which is responsible for spontaneous emission of photons), and two-particle correlation physics (which is the key to quantum erasure).

The quantum field theory view

A quantum theory of radiation[2–5] is indispensable to understanding the novel properties of light mentioned above. Central to the theory is the idea of field quantization, which develops the formal analogy with the quantum mechanics of the harmonic oscillator. The position q and momentum p of an oscillating particle satisfy the commutation relation $[\hat{q},\hat{p}] = \hat{q}\hat{p} - \hat{p}\hat{q} = i\hbar$. In the case of the radiation field, q and p represent the electric (E) and magnetic (B) fields of the light in a given wave-vector and polarization mode k. Thus, the quantum electromagnetic field consists of an *infinite* product of such generalized harmonic oscillators, one for each mode of the field. A Heisenberg-type uncertainty relation applies to these quantized Maxwell fields:

$$\Delta E \Delta B \geq \hbar/2 \times \text{constant}. \tag{5}$$

Such field fluctuations are an intrinsic feature of the quantized theory. The uncertainty relation can also be formulated in terms of the in-phase ($\mathscr{E}_p$) and in-quadrature ($\mathscr{E}_q$) components of the electric field, where $E(t) = \mathscr{E}_p \cos\nu t + \mathscr{E}_q \sin\nu t$.

To introduce the notion of a photon, it is convenient to recast the above quantization of the field in terms of a Fourier decomposition, or in terms of the normal modes of a field in a cavity. These correspond to the positive frequency (going like $e^{-i\nu t}$) and negative frequency (going like $e^{i\nu t}$) parts of the electric field respectively (summed over all modes k):

$$\begin{aligned} E(\mathbf{r},t) &= E^+(\mathbf{r},t) + E^-(\mathbf{r},t) \\ &= \sum_k [\alpha_k \mathscr{E}_k(\mathbf{r}) \exp(-i\nu_k t) \\ &\quad + \alpha_k^* \mathscr{E}_k^*(\mathbf{r}) \exp(i\nu_k t)]. \end{aligned} \tag{6}$$

Here α_k is the amplitude of oscillation, and $\mathscr{E}_k(\mathbf{r})$ is a mode function like $\exp(i\mathbf{k}\cdot\mathbf{r})$ for travelling waves in free space and $\sin(\mathbf{k}\cdot\mathbf{r})$ for standing waves in a box. We consider the oscillator amplitudes α_k and α_k^*, corresponding to harmonic motion, to be quantized by replacing $\alpha_k \to \hat{a}_k$ and $\alpha_k^* \to \hat{a}_k^\dagger$. By analogy to the quantum mechanics of the harmonic oscillator, the application of $\hat{a}$ produces a field state with one *less* quantum of energy, and the application of $\hat{a}^\dagger$ produces a field state with one *more* quantum of energy. This naturally leads to discrete energies for the radiation field in each mode: $n_k = 0,1,2,$ etc.

Both wave and particle perspectives are present in the quantum view – the former in the picture of a stochastic electromagnetic field, and the latter in the language of particle creation and annihilation. Combining these points of view, one can think of the "photon" as a discrete excitation of a set of modes $\{k\}$ of the electromagnetic field in some cavity, where the mode operators satisfy the boson commutation relation: $[\hat{a}_k, \hat{a}_k^\dagger] = 1$. Questions such as how to define the cavity, and what normal modes to use, cannot be answered once and for all, but depend on the particular physical setup in the laboratory (see quote by Willis Lamb at the beginning). Guided by this operational philosophy, we revisit the wave-particle debate on the nature of light in the guise of the following questions.

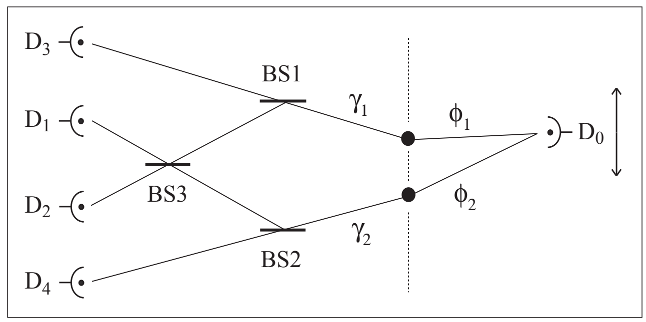

Fig. 6. *Quantum eraser* – One of two atoms (solid circles) emits two photons ϕ_i and γ_i. Interference is observed in ϕ by scanning detector D_0. Beamsplitters BS1-BS3 direct γ to four detectors. A click in detectors D_3 or D_4 provides which-path information on γ, preventing interference in ϕ. A click in detectors D_1 or D_2 erases which-path information and restores interference in ϕ. Figure adapted from Ref. [29].

What is a photon, and where is it?

In other words, in what manner (and to what extent) can we regard the photon as a true 'particle' that is localized in space? When first introduced, the photon was conceived of as a particulate carrier of discrete light energy, $E = \hbar\nu$, a conception guided by considerations of the interaction between radiation and matter. From semiclassical arguments, we saw how this discreteness was related to finite energy spacings in the atom. Here, we pursue this line of reasoning further to inquire whether a fully quantized theory of matter-radiation interaction can lend a characteristic of *spatial discreteness* to the photon when it interacts with a finite-sized atom. This line of thinking derives from the quantum theory of photodetection[36] (which, incidentally, also relies on the photoelectric effect).

Closely related to the issue of photon localization is the (much debated) question of the existence of a photon wave function $\psi(\mathbf{r},t)$,[2,7,8] analogous to that of an electron or neutrino (cf. Figure 8). The connection is that if such a wave function exists, then we can interpret $|\psi|^2 dV$ as the probability of finding the photon in an infinitesimal volume element

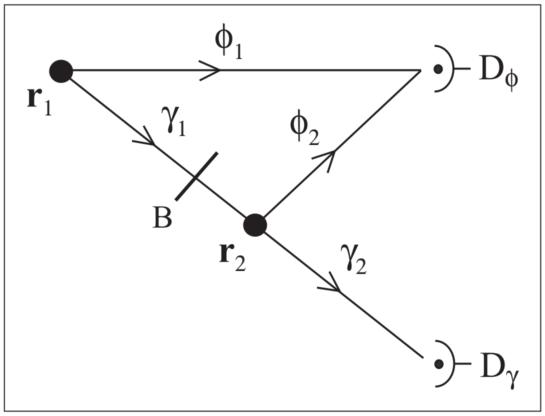

Fig. 7. *Two-site downconversion interferometry* – Two atoms are located at $\mathbf{r}_1$ and $\mathbf{r}_2$, one of which emits two photons, labeled ϕ_i and γ_i. Detectors D_ϕ and D_γ measure the respective photons. Inserting the beamstop B in the path of γ_1 allows us to infer (potentially, by checking D_γ) which atom emitted the ϕ photon. This potential which-path information is sufficient to prevent the interference of ϕ_1 and ϕ_2 possibilities at D_ϕ. Setup models the experiment of Ref. [34].

dV in space, and pursue the localization of the entire photon to an arbitrarily small volume constrained only by the uncertainty principle. Moreover, a 'first-quantized theory' of the electromagnetic field would be interesting from the point of view of discussing various quantum effects that result from wave interference and entanglement. It would also allow us to treat the mechanics of the photon on par with that of massive particles, such as electrons and atoms, and enable a unified treatment of matter-radiation interaction that supersedes the semiclassical theory in rigor, but still avoids the language of field quantization.

Concerning the issue of 'where' the photon is, one is reminded of an often asked question in introductory quantum mechanics: "How can a single particle go through both slits in a Young-type experiment?"

Richard Feynman answers this by saying "nobody knows, and its best if you try not to think about it." This is good advice if you have a picture of a single photon as a particle. On the other hand if you think of the photon as nothing more nor less than a single quantum excitation of the appropriate normal mode, then things are not so mysterious, and in some sense intuitively obvious.

What we have in mind (referring to Figure 1) is to consider a large box having simple normal modes and to put two holes in the box associated with the Young slits. If light is incident on the slits, we will have on the far wall of the box an interference pattern characteristic of classical wave interference, which we can describe as a superposition of normal modes. Now we quantize these normal modes and find that a photodetector on the far wall will indeed respond to the single quantum excitation of a set of normal modes which are localized at the peaks of the interference pattern, and will not respond when placed at the nodes. In this sense, the issue is a non sequitur. The photon is common to the box and has no independent identity in going through one hole or the other.

But to continue this discussion, let us ask what it is that the photodetector responds to. As we will clarify below, this is essentially what has come to be called the photon wave function.[2] Historical arguments have tended to disfavor the existence of such a quantity. For example, in his book on quantum mechanics,[37] Hendrik Kramers asks whether "it is possible to consider the Maxwell equations to be a kind of Schrödinger equation for light particles." His bias against this view is based on the disparity in mathematical form of the two types of equations (specifically, the number of time derivatives in each). The former admits *real* solutions ($\sin\nu t$ and $\cos\nu t$) for the electric and magnetic waves, while the latter is restricted to *complex* wave functions ($e^{i\nu t}$ or $e^{-i\nu t}$, but not both). Another argument is mentioned by David Bohm in his quantum theory book,[38] where he argues that there is no quantity for light equivalent to the electron probability density $P(x) = |\psi(x)|^2$:

> There is, strictly speaking, no function that represents the probability of finding a light quantum at a given point. If we choose a region large compared with a wavelength, we obtain approximately
>
> $$P(x) \cong \frac{\mathscr{E}^2(x) + \mathscr{H}^2(x)}{8\pi h\nu(x)},$$
>
> but if this region is defined too well, $\nu(x)$ has no meaning.

Bohm goes on to argue that the continuity equation, which relates the probability density and current density of an electron, cannot be written for light. That is, a precise statement of the conservation of probability cannot be made for the photon. In what follows, we will see that we can partially overcome the objections raised by Kramers and Bohm.

Let us develop the analogy with the electron a bit further. Recall that the wave function of an electron in the coordinate representation is given by $\psi(\mathbf{r},t) = \langle \mathbf{r}|\psi\rangle$, where $|\mathbf{r}\rangle$ is the position state corresponding to the exact localization of the electron at the point $\mathbf{r}$ in space. Now the question is, can we write something like this for the photon? The answer is, strictly speaking, "no," because there is no $|\mathbf{r}\rangle$ state for the photon, or more accurately, there is no particle creation operator that creates a photon at an exact point in space. Loosely speaking, even if there were, $\langle \mathbf{r}'|\mathbf{r}\rangle \neq \delta(\mathbf{r}-\mathbf{r}')$ on the scale of a photon wavelength. Nevertheless, we can still define the *detection* of a photon to a precision limited only by the size of the atom (or detector) absorbing it, which can in principle be much smaller than the wavelength. This gives precise, operational meaning to the notion of "localizing" a photon in space.

If we detect the photon by an absorption process, then the interaction coupling the field and the detector is described by the annihilation operator $\hat{E}^+(\mathbf{r},t)$, defined in Eq. (6). According to Fermi's Golden Rule, the matrix element of this operator between the initial and final states of the field determines

	Photon	Neutrino
Eikonal physics	Ray optics (Fermat): $\delta \int n\, ds = 0$	Classical mechanics (Hamilton): $\delta \int L\, dt = 0$
"Wave" mechanics	Maxwell equations: $\dot{\mathbf{\Psi}} = -\frac{i}{\hbar}\begin{bmatrix} 0 & -c\mathbf{s}\cdot\mathbf{p} \\ c\mathbf{s}\cdot\mathbf{p} & 0 \end{bmatrix}\mathbf{\Psi}$	Dirac equations: $\dot{\mathbf{\Phi}} = -\frac{i}{\hbar}\begin{bmatrix} 0 & -c\sigma\cdot\mathbf{p} \\ c\sigma\cdot\mathbf{p} & 0 \end{bmatrix}\mathbf{\Phi}$
Quantum field theory	$\hat{E}^{+}(\mathbf{r},t) = \sum_{\mathbf{k}} \hat{a}_{\mathbf{k}}(t)\,\mathcal{E}_{\mathbf{k}}(\mathbf{r})$ $\lvert\dot{\psi}\rangle = -\frac{i}{\hbar}\mathcal{H}_{\gamma}\lvert\psi\rangle$	$\hat{\phi}(\mathbf{r},t) = \sum_{\mathbf{p}} \hat{c}_{\mathbf{p}}(t)\,\phi_{\mathbf{p}}(\mathbf{r})$ $\lvert\dot{\phi}\rangle = -\frac{i}{\hbar}\mathcal{H}_{\nu}\lvert\phi\rangle$

Fig. 8. Comparison of physical theories of a photon and a neutrino. Eikonal physics describes both in particle terms, showing the parallel between Fermat's principle in optics and Hamilton's principle in classical mechanics (L is the Lagrangian). The Maxwell equations can be formulated in terms of photon wave functions, in the same form that the Dirac equations describe the relativistic wave mechanics of the neutrino. Here, $\mathbf{\Psi}$ is a six-vector representing the wave functions associated with the electric and magnetic fields, $\mathbf{p} = (\hbar/i)\nabla$ as usual, and $\mathbf{s} = (s_x, s_y, s_z)$ are a set of 3×3 matrices that take the place of the Pauli matrices σ_x, σ_y and σ_z. See Ref [2] for details. Finally, quantum field theory gives a unified description of both the photon and the neutrino in terms of quantized field operators.

the transition probability. If there is only one photon initially in the state $|\psi\rangle$, then the relevant final state is the vacuum state $|0\rangle$. The probability density of detecting this photon at position $\mathbf{r}$ and time t is thus proportional to[2]

$$G^{(1)}_{\psi} = |\langle 0|\hat{E}^{+}(\mathbf{r},t)|\psi\rangle|^2 = \kappa\,|\psi_{\mathscr{E}}(\mathbf{r},t)|^2. \qquad (7)$$

Here, κ is a dimensional constant such that $|\psi_{\mathscr{E}}|^2$ has units of inverse volume. The quantity $\psi_{\mathscr{E}}(\mathbf{r},t)$ may thus be regarded as a kind of 'electric-field wave function' for the photon, with $\{\langle 0|\hat{E}^{+}(\mathbf{r},t)\}^{\dagger} = \hat{E}^{-}(\mathbf{r},t)|0\rangle$ playing the role of the position state $|\mathbf{r}\rangle$. That is, by summing over infinitely many wave vectors in Eq. (6), and appealing to Fourier's theorem, $\hat{E}^{-}(\mathbf{r},t)$ can be interpreted as an operator that *creates* the photon at the position $\mathbf{r}$ out of the vacuum. Of course, we have to be careful not to take this interpretation too precisely.

It is interesting to calculate $\psi_{\mathscr{E}}(\mathbf{r},t)$ for the photon spontaneously emitted by an atom when it decays. Consider a two-level atom located at $\mathbf{r}_0$, initially excited in level a and decaying at a rate Γ to level b, as shown in Figure 2. The emitted field state $|\psi\rangle$ is a superposition of one-photon states $|1_k\rangle$, summed over all modes k, written as

$$|\psi\rangle = \sum_k \frac{g_{ab,k}\, e^{-i\mathbf{k}\cdot\mathbf{r}_0}}{(\nu_k - \omega) + i\Gamma/2}\,|1_k\rangle, \qquad (8)$$

where ω is the atomic frequency, and $g_{ab,k}$ is a coupling constant that depends on the dipole moment between levels a and b. The spectrum of the emitted field is approximately Lorentzian, which corresponds in the time domain to an exponential decay of the excited atom. Calculating $\psi_{\mathscr{E}}(\mathbf{r},t)$ for this state, we obtain

$$\psi_{\mathscr{E}}(\mathbf{r},t) = K\,\frac{\sin\eta}{r}\;\theta(t - r/c)\,\exp[-i(\omega + i\Gamma/2)(t - r/c)], \qquad (9)$$

where K is a normalization constant, $r = |\mathbf{r} - \mathbf{r}_0|$ is the radial distance from the atom, and η is the azimuthal angle with respect to the atomic dipole moment. The step function $\theta(t - r/c)$ is an indication that nothing will be detected until the light from the atom reaches the detector, travelling at the speed c. Once the detector starts seeing the pulse, the probability of detection $|\psi_{\mathscr{E}}|^2$ decays exponentially in time at the rate Γ. The spatial profile of the pulse mimics the radiation pattern of a classical dipole.

To what extent can we interpret Eq. (9) as a kind of wave function for the emitted photon? It certainly has close parallels with the Maxwell theory, since it agrees with what we would write down for the electric field in the far zone of a damped, radiating dipole. We can go even further, and introduce *vector* wave functions $\mathbf{\Psi}_{\mathscr{E}}$ and $\mathbf{\Psi}_{\mathscr{H}}$ corresponding to the electric and magnetic field vectors $\mathbf{E}$ and $\mathbf{H}$ respectively, and show that these satisfy the Maxwell equations (see Figure 8). This formalism provides the so-called "missing link" between classical Maxwell electrodynamics and quantum field theory.[7] But we have to be careful in how far we carry the analogy with mechanics. For example, there is no real position operator $\hat{\mathbf{r}}$ for the photon in the wave-mechanical limit, as there is for a first-quantized electron. Nevertheless, the wave function $\psi_{\mathscr{E}}(\mathbf{r},t)$ does overcome the main objection of Kramers (since it is complex) and partially overcomes that of Bohm (photodetection events are indeed localized to distances smaller than a wavelength).

The real payoff of introducing a photon wave function comes when we generalize this quantity to two or more photons. A 'two-photon wave function' $\Psi_{\mathscr{E}}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)$ may be introduced along similar lines as above, and used to treat problems in second-order interferometry (see Ref [2], chap. 21). Entanglement between the two photons results in an inseparability of the wave function: $\Psi_{\mathscr{E}}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2) \neq \phi_{\mathscr{E}}(\mathbf{r}_1,t_1)\,\gamma_{\mathscr{E}}(\mathbf{r}_2,t_2)$, as in the example of the two-photon state in Eq. (4). The novel interference effects associated with such states may be explained in terms of this formalism.

Thus, the photon wave function concept is useful in comparing the interference of classical and quantum light, and allows us to home in on the key distinction between the two paradigms. In particular, through association with photodetection amplitudes, multi-photon wave functions incorporate the phenomenology of quantum-correlated measurement, which is key to explaining the physics of entangled light.

Conclusion

What is a photon? In this article, we have strived to address this concept in unambiguous terms, while remaining true to its wonderfully multi-faceted nature. The story of our quest to understand the character of light is a long one indeed, and parallels much of the progress of physical theory. Dual conceptions of light, as wave and particle, have co-existed since antiquity. Quantum mechanics officially sanctions this duality, and puts both concepts on an equal footing (to wit, the quantum eraser). The quantum theory of light introduces vacuum fluctuations into the radiation field, and endows field states with quantum, many-particle correlations. Each of these developments provides us with fresh insight on the photon question, and allows us to hone our perspective on the wave-particle debate.

The particulate nature of the photon is evident in its tendency to be absorbed and emitted by matter in discrete units, leading to quantization of light energy. In the spatial domain, the localization of photons by a photodetector makes it possible to define a 'wave function' for the photon, which affords a 'first-quantized' view of the electromagnetic field by analogy to the quantum mechanics of material particles. Quantum interference and entanglement are exemplified by one-photon and two-photon wave functions, which facilitate comparisons to (and clarify departures from) classical wave optics. Moreover, this interpretive formalism provides a bridge between the two ancient, antithetical conceptions of light – its locality as a particle, and its functionality as a wave.

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A photon viewed from Wigner phase space

Holger Mack and Wolfgang P. Schleich
Abteilung für Quantenphysik at the Universität Ulm, Germany

I don't know anything about photons, but I know one when I see one
Roy J. Glauber

We present a brief history of the photon and summarize the canonical procedure to quantize the radiation field. Our answer to the question "what is a photon?" springs from the Wigner representation of quantum mechanics as applied to a single photon number state.

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1. Introduction

For centuries light in its various manifestations has been a pace maker for physics. We are reminded of the wave-particle controversy of classical light between Thomas Young and Isaac Newton. We also recall the decisive role of the null aether experiment of Albert A. Michelson in the birth of special relativity. Many more examples could be listed. However, three phenomena that opened the quantum era stand out most clearly. (i) Black-body radiation paved the way for quantum mechanics. (ii) The level shift in the fine structure of hydrogen, that is the Lamb shift marks the beginning of quantum electrodynamics, and (iii) the almost thirty years lasting debate[1] between Albert Einstein and Niels Bohr on the double-slit experiment could open the path way to quantum information processing in our still young millenium.

The photon as a continuous source of inspiration and its illusiveness has repeatedly been emphasized[2] by John A. Wheeler: Catchy phrases such as "the photon — a smoky dragon", "no elementary quantum phenomenon is a phenomenon until it is a recorded phenomenon", and "it from bit" were coined by him to express in a vivid way the seemingly acausal behavior of the photon in the delayed-choice experiment, the special role of the observer in quantum mechanics, and the concept of a participatory universe due to the measurement process, respectively.

The opposite view, one free from any mystery has been strongly advocated[3] by Willis E. Lamb. According to him the word "photon" should be striken from the dictionary since there is no need for it. The correct approach is: First define modes and then quantize them according to a harmonic oscillator. In the early days of the laser theory, that is the early sixties, Lamb handed out licences to physicists for the word "photon". Only those who were lucky enough to obtain such a license were allowed to use the word "photon". These days are long gone by. Today nobody applies for licenses anymore. We have again freedom of speech and photons appear everywhere even when there is no need for them. Often photons are used in a sloppy way like some people use phrases such as "You know what I mean" in conversations when even they themselves do not know what they mean. In these cases photons serve as a Charly Brown security blanket.

Such a sloppy approach is not conducive to unravelling the deeper secrets of the photon that are still waiting to be discovered. We, therefore, welcome this opportunity to readdress the old question "what is a photon?" and argue in favor of the canonical approach to field quantization. At the same time we try to communicate the many fascinating facets of the photon. Needless to say, we do not claim to have understood all sides of the photon. Our position is probably best described by Roy J. Glauber's joke: "I don't know anything about photons, but I know one when I see one". This quote is a paraphrase of the well-known attempt of the American Supreme Court Justice Potter Stewart to define obscenity in the 1964 trial Jacobellis versus Ohio by stating "I know it when I see it". Glauber's application to our dilemma with the photon serves as the motto of our paper. It is worth mentioning that Glauber after his lecture at the Les Houches summer school[4] 1963 was one of the very few people ever given a license for the photon and he had not even applied for one.

Our paper is organized as follows. A brief historical summary of the quantum theory of radiation emphasizes the crucial roles of Max Born, Pascual Jordan and Werner Heisenberg in introducing the quantum mechanics of the field.[5,6] This introductory section also alludes to the problem of a hermitian phase operator[7] that originated from Fritz London[8] and was ignored by Paul Adrian Maurice Dirac's seminal paper[9] on the quantum theory of the emission and absorption of radiation. We then outline the formalism[10] of the quantization of the field in a version well-suited for the description of recent experiments[11,12] in cavity quantum electrodynamics. In this approach we expand the electromagnetic field into a complete set of mode functions. They are determined by the boundary conditions of the resonator containing the radiation. In this language a "photon" is the first excitation of a single mode. The Wigner phase space distribution[13,14] allows us to visualize the quantum state of a system. We present the Wigner functions for a gallery of quantum states, including a single photon number state. Several proposals to measure the Wigner function have been made.[15] Recently experiments[11,12,16] have created and measured the phase space function of a single photon. We conclude by summarizing an approach pioneered by J. A. Wheeler in the context of geometrodynamics.[17] This formalism gives the probability amplitude for a given electric or magnetic field configuration in the vacuum state and does not make use of the notion of mode function. A brief summary and outlook alludes to the question of a wave function of a photon,[18] addressed in more detail in the article by A. Muthukrishnan *et al.* in this issue.

2. History of Field Quantization

It was a desperate situation that Max Planck was facing at the turn of the 19th century. How to explain the energy distribution of black-body radiation measured in the experiments at the Physikalisch-Technische Reichsanstalt by Heinrich Rubens and coworkers with such an unprecedented accuracy? How to bridge the gap between the Rayleigh-Jeans law describing the data correctly for small frequencies and Wien's law valid in the large frequency domain? Planck's revolutionary step is well-known: The oscillators situated in the walls of the black-body resonator can only emit or absorb energy in discrete portions. The smallest energy unit of the oscillator with frequency Ω is $\hbar\Omega$, where in today's notation $\hbar$ is Planck's constant. It is interesting to note that Planck had initially called this new constant Boltzmann's constant — not to be confused with Boltzmann's constant k_B of thermodynamics.

Planck's discovery marks the beginning of quantum mechanics in its early version of Atommechanik à la Bohr-Sommerfeld and the matured wave or matrix mechanics of Erwin Schrödinger and W. Heisenberg. It also constitutes the beginning of the quantum theory of radiation. Although Planck got his pioneering result by quantizing the mechanical oscillators of the wall it was soon realized that it is the light field whose energy appears in discrete portions. This discreteness suggested the notion of a particle which Einstein in 1905 called "light quantum". The concept of a particle was also supported by his insight that this light quantum enjoys a momentum $\hbar k$ where $k = 2\pi/\lambda$ is the wave number of the light of wave length λ. The name "photon" for the light quantum originated much later. It was the chemist[3] Gilbert N. Lewis at Stanford University who in 1926 coined the word "photon" when he suggested a model of chemical bonding. His model did not catch on, however the photon survived him. For more historical and philosophical details we refer to Ref. 3 and the paper by A. Zajonc in this issue.

The rigorous quantum theory of radiation starts in 1925 with the immediate reaction of Born and Jordan[5] on Heisenberg's deep insight[19] into the inner workings of the atom obtained during a lonely night on the island of Helgoland. Indeed, it is in this paper that Born and Jordan show that the non-commuting objects proposed by Heisenberg are matrices. This article[5] also contains the so-called Heisenberg equations of motion. Moreover, it applies for the first time matrix mechanics to electrodynamics. Born and Jordan recall that the electromagnetic field in a resonator is a collection of uncoupled harmonic oscillators and interprete the electromagnetic field as an operator, that is as a matrix. Each harmonic oscillator is then quantized according to matrix mechanics and the commutation relation $[\hat{q},\hat{p}] = i\hbar$ between position and momentum operators $\hat{q}$ and $\hat{p}$, respectively. This work is pushed even further in the famous Drei-Männer-Arbeit[6] where also Heisenberg joined Born and Jordan. This paper elucidates many consequences of the quantum theory of radiation from the matrix mechanics point of view. In particular, it calculates from first principles the energy fluctuations of the black-body radiation. From today's demand for rapid publication in the eprint age, it is quite remarkable to recall the submission and publication dates of these three pioneering papers: July 26, 1925, September 27, 1925, November 16, 1925. All three papers were published in 1925.

A new chapter in the book of the quantized electromagnetic field was opened in 1927 when Dirac[9] considered the interaction of a quantized electromagnetic field with an atom which is also described by quantum theory. In this way he derived the Einstein A- and B-coefficients of spontaneous and induced emission. His paper defines the beginning of quantum electrodynamics leading eventually to the modern gauge theories.

Dirac's paper is also remarkable from a different point of view. He does not quantize the field in terms of non-commuting position and momentum operators but by decomposing the annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$ into action $\hat{n}$ and angle $\hat{\phi}$ operators with $[\hat{n},\hat{\phi}] = i\hbar$. However, such a decomposition is not well-defined, since $\hat{n}$ and $\hat{\phi}$ cannot be conjugate variables. Indeed, they have different type of spectra: The spectrum of $\hat{n}$ is discrete whereas the phase in continuous. The problems arising in the translation of classical action-angle variables which are at the heart of the Bohr-Sommerfeld Atommechanik to action-angle operators had already been pointed out by Fritz London[8] in 1926. He showed that there does not exist a hermitian phase operator $\hat{\phi}$. Since then this problem of finding the quantum mechanical analogue of the classical phase has resurfaced repeatedly whenever there was a substantial improvement in the technical tools of preparing quantum states of the radiation field. These periods are characterized by the development of the maser and laser, the generation of squeezed states, and the amazing one-atom maser. In particular, the generation of squeezed light in the mid-eighties has motivated Stephen Barnett and David Pegg[7] to propose a hermitian phase operator in a truncated Hilbert space.

Enrico Fermi independently developed his own approach[10] towards the quantum theory of radiation. In Ref. 10 Fermi applies the quantum theory of radiation to many physical situations. For example, he treats Lippmann fringes and shows that the radiation emitted by one atom and absorbed by another travels with the speed of light. Notwithstanding Fermi's analysis this problem was discussed later in many papers and it was shown that Fermi's model predicts instantaneous propagation.

3. Mode Functions

After this historical introduction we briefly summarize in the next two sections the essential ingredients of Fermi's approach towards quantizing the electromagnetic field. Here we concentrate on a domain of space that is free of charges and currents.

In the Coulomb gauge with $\vec{\nabla}\cdot\vec{A} = 0$ we find from Maxwell's equations the wave equation

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right)\vec{A}(t,\vec{r}) = 0 \tag{1}$$

for the vector potential $\vec{A} = \vec{A}(t,\vec{r})$ where Δ denotes the three-dimensional Laplace operator.

We shall expand $\vec{A}$ into a complete set of mode functions $\vec{u}_{\vec{k},\sigma} = \vec{u}_{\vec{k},\sigma}(\vec{r})$ defined by the Helmholtz equation

$$\left(\Delta + \vec{k}^2\right)\vec{u}_{\vec{k},\sigma}(\vec{r}) = 0 \tag{2}$$

and the boundary conditions set by the shape of the resonator.

For the example of a resonator shaped like a shoe box the mode functions are products of sine and cosine functions. In order to match the boundary conditions of vanishing transverse electric field on the metallic walls the components of the wave vector $\vec{k}$ have to be integer multiples of π/L_j where L_j denotes the length of the j-th side of the resonator. The vector character of the mode function $\vec{u}_{\vec{k},\sigma}$ is determined by the Coulomb gauge condition which for a rectangular resonator takes the form $\vec{k}\cdot\vec{u}_{\vec{k},\sigma}(\vec{r}) = 0$. Hence, the direction of $\vec{u}$ has to be orthogonal to the wave vector $\vec{k}$. The Coulomb gauge translates into a transverse vector potential which is the reason why this gauge is sometimes referred to as "transverse gauge". Since in general there are two perpendicular directions there are two polarization degrees indicated by the index σ.

At this point it is worthwhile emphasizing that the discreteness of the wave vector is unrelated to quantum mechanics. It is solely determined by the boundary conditions imposed on the Helmholtz equation. Indeed, the variable $\vec{r}$ indicating the position in space is a classical quantity and not a quantum mechanical operator.

For more sophisticated shapes of resonators the mode functions become more complicated. Nevertheless, their basic properties explained above for the elementary example of a box-shaped resonator still hold true. In particular, the mode functions $\vec{u}_\ell(\vec{r})$ are complete and enjoy the orthonormality relation

$$\frac{1}{\sqrt{V_\ell V_{\ell'}}}\int d^3r\,\vec{u}^*_\ell(\vec{r})\cdot\vec{u}_{\ell'}(\vec{r}) = \delta_{\ell,\ell'} \tag{3}$$

where V_ℓ denotes the effective volume of the ℓ-th mode. In order to simplify the notation we have combined the three components of the wave vector $\vec{k}$ and the polarization index σ to one index ℓ.

Due to the completeness of the eigenfunctions we can expand the vector potential

$$\vec{A}(t,\vec{r}) \equiv \sum_\ell \mathscr{A}_\ell\, q_\ell(t)\vec{u}_\ell(\vec{r}) \tag{4}$$

where $\mathscr{A}_\ell$ is a constant that we shall choose later in order to simplify the calculations. The time dependent amplitude q_ℓ of the ℓ-th mode follows from the differential equation

$$\ddot{q}_\ell(t) + \Omega_\ell^2 q_\ell(t) = 0 \tag{5}$$

of a harmonic oscillator of frequency $\Omega_\ell \equiv c|\vec{k}_\ell|$. Here a dot denotes differentiation with respect to time. This equation emerges when we substitute the expansion, Eq. (4), into the wave equation, Eq. (1), and make use of the Helmholtz equation, Eq. (2).

The notion of the field amplitudes in the modes as harmonic oscillators stands out most clearly when we calculate the energy

$$H \equiv \int d^3r\left(\frac{1}{2}\varepsilon_0\vec{E}^2 + \frac{1}{2\mu_0}\vec{B}^2\right) \tag{6}$$

of the electromagnetic field in the resonator. Indeed, when we use the relations

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} = -\sum_\ell \mathscr{A}_\ell\,\dot{q}_\ell\vec{u}(\vec{r}) \tag{7}$$

and

$$\vec{B} = \vec{\nabla}\times\vec{A} = \sum_\ell \mathscr{A}_\ell\, q_\ell\nabla\times\vec{u}(\vec{r}) \tag{8}$$

connecting in Coulomb gauge the electric and magnetic fields $\vec{E}$ and $\vec{B}$ with the vector potential $\vec{A}$ we find after a few lines of calculations[14]

$$H = \sum_\ell H_\ell = \sum_\ell \frac{1}{2}\dot{q}_\ell^2 + \frac{1}{2}\Omega_\ell^2 q_\ell^2. \tag{9}$$

Here we have used the orthonormality relation, Eq. (3), and have chosen the prefactor $\mathscr{A}_\ell \equiv (\varepsilon_0 V_\ell)^{-1/2}$ in the expansion Eq. (4).

4. Field Operators

According to Eq. (9) the electromagnetic field is a collection of harmonic oscillators with conjugate variables q_ℓ and $p_\ell \equiv \dot{q}_\ell$. The natural method to quantize the field is therefore to replace the variables q_ℓ and p_ℓ by operators $\hat{q}_\ell$ and $\hat{p}_\ell$ satisfying the canonical commution relations $[\hat{q}_\ell,\hat{p}_{\ell'}] = i\hbar\delta_{\ell,\ell'}$. In this way we arrive at the operator

$$\hat{\vec{E}}(t,\vec{r}) = -\sum_\ell \mathscr{A}_\ell\,\hat{p}_\ell(t)\vec{u}_\ell(\vec{r}) \tag{10}$$

of the electric field and

$$\hat{\vec{B}}(t,\vec{r}) = \sum_\ell \mathscr{A}_\ell\,\hat{q}_\ell(t)\vec{\nabla}\times\vec{u}_\ell(\vec{r}) \tag{11}$$

of the magnetic field.

From the expressions Eqs. (10) and (11) we recognize that $\hat{\vec{E}}$ and $\hat{\vec{B}}$ must be conjugate variables since $\hat{\vec{B}}$ only contains generalized position operators $\hat{q}_\ell$ whereas $\hat{\vec{E}}$ only involves generalized momentum operators $\hat{p}_\ell$. Therefore, it is not surprising that in general it is not possible to measure the electric and magnetic field simultaneously with arbitrary accuracy. The limits put on the accuracy of field measurements has been the subject of two famous papers by N. Bohr and Leon Rosenfeld.[1]

We conclude by casting the quantum analogue

$$\hat{H} \equiv \sum_\ell \frac{1}{2}\hat{p}_\ell^2 + \frac{1}{2}\Omega_\ell^2\hat{q}_\ell^2 \tag{12}$$

of the Hamiltonian Eq. (9) into a slightly different form. For this purpose it is useful to introduce the annihilation and creation operators $\hat{a}_\ell \equiv [\Omega_\ell/(2\hbar)]^{1/2}(\hat{q}_\ell + i\hat{p}_\ell/\Omega_\ell)$ and $\hat{a}_\ell^\dagger \equiv [\Omega_\ell/(2\hbar)]^{1/2}(\hat{q}_\ell - i\hat{p}_\ell/\Omega_\ell)$, respectively. The commutation relation $[\hat{a}_\ell,\hat{a}_{\ell'}^\dagger] = \delta_{\ell,\ell'}$ follows from the one of $\hat{q}_\ell$ and $\hat{p}_{\ell'}$. The Hamiltonian of the electromagnetic field then takes the form

$$\hat{H} = \sum_\ell \hat{H}_\ell = \sum_\ell \hbar\Omega_\ell\left(\hat{n}_\ell + \frac{1}{2}\right) \tag{13}$$

where $\hat{n}_\ell \equiv \hat{a}_\ell^\dagger\hat{a}_\ell$ denotes the number operator.

The contribution $1/2$ arises from the commutation relations and results in the familiar zero point energy. Since every mode contributes the energy $\hbar\Omega_\ell/2$ and there are infinitely many modes we arrive at an infinite zero point energy of the electromagnetic field. In general we drop this contribution since a constant shift in the energy, that is, in the Hamiltonian, does not influence the dynamics, even if it is infinite. Under certain circumstances this contribution becomes finite and gives rise to a physical effect. For example, we find an attractive force[20] between two neutral conducting metal surfaces. This Casimir force has also been observed experimentally.[14]

5. Quantum States

Operators are only one side of the coin of quantum mechanics. The other one is the description of the quantum system, that is, the electromagnetic field, by a quantum state. In general this state $|\Psi\rangle$ is a multimode state, that is, it involves a quantum state $|\psi_\ell\rangle$ for each mode ℓ. In the most elementary situation the states of the individual modes are independent from each other and the state of the electromagnetic field is a product state

$$|\Psi\rangle \equiv \prod_\ell |\psi_\ell\rangle = \ldots|\psi_{-1}\rangle\otimes|\psi_0\rangle\otimes|\psi_1\rangle\ldots. \tag{14}$$

However, the most interesting states are the ones where two or more modes are correlated with each other. Schrödinger in his famous paper[1] *"On the current situation of quantum mechanics"* triggered by the Einstein-Podolsky-Rosen paper[1] asking the question *"Can quantum-mechanical description of physical reality be considered complete?"* coined the phrase "entangled states". In order to describe entangled states it is useful to first introduce the most elementary quantum states, namely photon number states $|n_\ell\rangle$.

The states $|n_\ell\rangle$ are eigenstates of the operator $\hat{n}_\ell$, that is

$$\hat{n}_\ell|n_\ell\rangle = n_\ell|n_\ell\rangle \tag{15}$$

with integer eigenvalues. Since the states $|n_\ell\rangle$ are eigenstates of the Hamiltonian $\hat{H}_\ell$ of the ℓ-th mode the energy of the field in the state $|n_\ell\rangle$ is then (neglecting the zero-point energy) $n_\ell\hbar\Omega_\ell$, that is n_ℓ times the fundamental unit $\hbar\Omega_\ell$. This feature has led to the notion that n_ℓ quanta of energy $\hbar\Omega_\ell$ are in this mode. But we emphasize that this energy is distributed over the whole resonator. It cannot be localized at a specific position $\vec{r}$. Indeed, recall that we have found the Hamiltonian, Eq. (9), by integrating the energy density, Eq. (6), over the whole resonator. Due to the discreteness in the excitation of the mode in portions of units $\hbar\Omega_\ell$ the expression "photon" for this excitation is appropriate.

We are now in the position to discuss the notion of an entangled state. The state $|\psi\rangle$ of a given mode is in general a superposition of photon number states, that is

$$|\psi\rangle \equiv \sum_n \psi_n |n\rangle. \tag{16}$$

We emphasize that here the subscript n is not a mode index but counts the quanta in a single mode.

Two states $|\psi\rangle$ and $|\tilde{\psi}\rangle$ that are independent of each other are then described by a direct product, that is

$$|\Psi\rangle = |\psi\rangle \otimes |\tilde{\psi}\rangle = \sum_{m,n} \psi_m \tilde{\psi}_n |m\rangle|n\rangle. \tag{17}$$

In case the two states are entangled we find

$$|\Psi\rangle \equiv \sum_{m,n} \Psi_{m,n} |m\rangle|n\rangle. \tag{18}$$

where the expansion coefficients $\Psi_{m,n}$ do not factorize into a product of two contributions solely related to the two individual modes.

Entangled states are the essential ingredients of the newly emerging and rapidly moving field of quantum information processing.[21] They can be created by non-linear optical processes such as parametric down-conversion as discussed in the next section or by beam splitters as outlined in detail by R. Loudon and A. Zajonc in their articles in the present issue.

6. Wigner Functions of Photons

In the following two sections we focus on states of a single mode of the radiation field and for the sake of simplicity suppress the mode index. We introduce the Wigner phase space distribution and discuss experiments measuring the Wigner function of a single photon.

A photon denoted by the quantum state $|1\rangle$ is an excitation of a mode of the electromagnetic field. But how to gain deeper insight into this state?

Here, the Wigner function offers itself as a useful tool to visualize the rather abstract object of a quantum state. It was intoduced in 1932 by Eugene Paul Wigner in a paper[13] concerned with the corrections of quantum mechanics to classical statistical mechanics. It is remarkable that in a footnote Wigner shares the fame as the original proposer of this phase space distribution function. He states: "This expression was found by L. Szilard and the present author some years ago for another purpose".

However, no such paper by Leo Szilard and Wigner exists. Later in life Wigner explained that he had only added this footnote in order to assist Szilard in his search for a research position.[22] It is astonishing that Heisenberg[23] and Dirac,[24] who later was to become Wigner's brother in law, had already earlier introduced this phase space function. In particular, Dirac had also studied many of its properties and amazingly enough Wigner seemed to be unaware of Dirac's work.

We now turn to the definition of the Wigner phase space distribution. For this purpose it is useful to recall that the eigenstates $|E\rangle$ of the single-mode electric field operator $\hat{\vec{E}} = -\mathscr{A}_0 \hat{p}\vec{u}(\vec{r})$ are proportional to the eigenstates $|p\rangle$ of the momentum operator $\hat{p}$. Likewise, the eigenstates $|B\rangle$ of the single mode magnetic field operator $\hat{\vec{B}} = \mathscr{A}_0 \hat{q} \nabla \times \vec{u}(\vec{r})$ are proportional to the eigenstates $|q\rangle$ of the position operator $\hat{q}$.

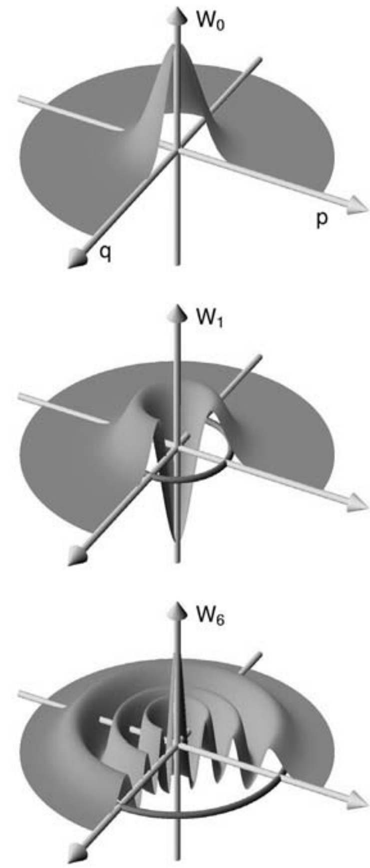

Fig. 1. Gallery of Wigner functions of a single mode of the radiation field. The Wigner function of the vacuum (top) is always positive whereas the ones corresponding to a single photon (center) or six photons (bottom) contain significant domains where the phase space distribution assumes negative values. The circle visible in the quadrant of the foreground indicates where the phase space trajectory corresponding to the energy $\hbar\Omega(n+1/2)$ runs. The scales on the axes are identical in all three cases.

The Wigner function $W = W(q,p)$ of a state $|\psi\rangle$ with wave function $\psi(q) \equiv \langle q|\psi\rangle$ is defined by

$$W(q,p) \equiv \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} d\xi\, e^{-ip\xi/\hbar} \psi^* \left(q - \frac{\xi}{2}\right) \psi \left(q + \frac{\xi}{2}\right) \tag{19}$$

where q and p are conjugate variables. For a massive particle they correspond to position and momentum whereas in the case of the electromagnetic field they represent the amplitude of the magnetic and electric field, respectively.

Hence, the problem of finding the Wigner function of a given wave function amounts to evaluating the integral Eq. (19). For the example of a photon number state $|n\rangle$ of a mode with frequency Ω the wave function $\varphi_n(q;\Omega) \equiv \langle q|n\rangle$ reads[14]

$$\varphi_n(q;\Omega) \equiv N_n(\Omega)\, H_n\left(\sqrt{\frac{\Omega}{\hbar}}q\right) \exp\left(-\frac{1}{2}\frac{\Omega}{\hbar}q^2\right) \tag{20}$$

where $N_n(\Omega) \equiv (\Omega/(\pi\hbar))^{1/4} (2^n n!)^{-1/2}$ and H_n denotes the n-th Hermite polynomial.

When we substitute this expression into the definition, Eq. (19), of the Wigner function and perform the integration we arrive at[14]

$$W_n(q,p) = \frac{(-1)^n}{\pi\hbar} L_n\left[2\eta(q,p)\right]\exp\left[-\eta(q,p)\right] \qquad (21)$$

where $\eta(q,p) \equiv (p^2 + \Omega^2 q^2)/(\hbar\Omega)$ is the scaled phase space trajectory of a classical harmonic oscillator and L_n denotes the n-th Laguerre polynomial.

The two phase space variables q and p enter the Wigner function in a symmetric way. Moreover, the Wigner function is constant along the classical phase space trajectories, that is along circles. Its behavior along the radial direction is determined by the Laguerre polynomial. In order to study these features in more detail we now analyze and display in Fig. 1 the Wigner functions of the ground state, a one-photon and a six-photon state.

We start our discussion with the Wigner function of the ground state, that is $n = 0$ where according to Eq. (20) the wave function

$$\varphi_0(q;\Omega) = N_0(\Omega)\exp\left(-\frac{1}{2}\frac{\Omega}{\hbar}q^2\right) \qquad (22)$$

is a Gaussian. The corresponding Wigner function

$$W_0(q,p) = \frac{1}{\pi\hbar}\exp\left[-\frac{1}{\hbar\Omega}\left(\Omega^2 q^2 + p^2\right)\right], \qquad (23)$$

is then a Gaussian in the generalized position and momentum variables, that is in the electric and magnetic field amplitudes. Thus, the Wigner function of the ground state, that is a mode with no excitation, that is no photons, is everywhere positive.

We now turn to the Wigner function of a single photon, that is of the first excited state $|1\rangle$. Since the first Laguerre polynomial reads $L_1 = 1 - x$ the Wigner function, Eq. (21), takes the form

$$W_1(q,p) = \frac{(-1)}{\pi\hbar}(1-2\eta)e^{-\eta}. \qquad (24)$$

Hence, at the origin of phase space the Wigner function assumes the negative value $W_1(0,0) = (-1)/(\pi\hbar)$. Figure 1 shows that the Wigner function is not only negative at the origin, but also in a substantial part of its neighborhood. It is the existence of negative parts that rules out a probability interpretation of the Wigner function. Nevertheless it can be used to develop a formalism of quantum mechanics in phase space,[14] that is equivalent to the one in Hilbert space.

The negative parts of the Wigner function are a consequence of the wave nature of quantum mechanics. This feature stands out most clearly when we consider the Wigner function of a photon number state with many photons in it. In Fig. 1 we show the Wigner function corresponding to the state $|6\rangle$. We recognize circular wave troughs that alternate with circular wave crests. The Wigner function repeatedly assumes negative values and contains $n = 6$ nodes. The last positive crest is located in the neighborhood of the classical phase space trajectory corresponding to the quantized energy $E = \hbar\Omega(n + \frac{1}{2})$ of this state. Hence, this positive-valued ring represents the classical part of the state $|n\rangle$. The fringes caught inside reflect the quantum nature of the state. In order to gain deeper insight into this separation of wave and particle nature, we recall that a photon number state is an energy eigenstate of a harmonic oscillator. In the limit of large n, that is many quanta of excitation, this state is the superposition of a right- and a left-going wave. Since the Wigner function, Eq. (19), is bilinear in the wave function the interference between these two waves manifests itself in the structures circumnavigated by the classical crest.

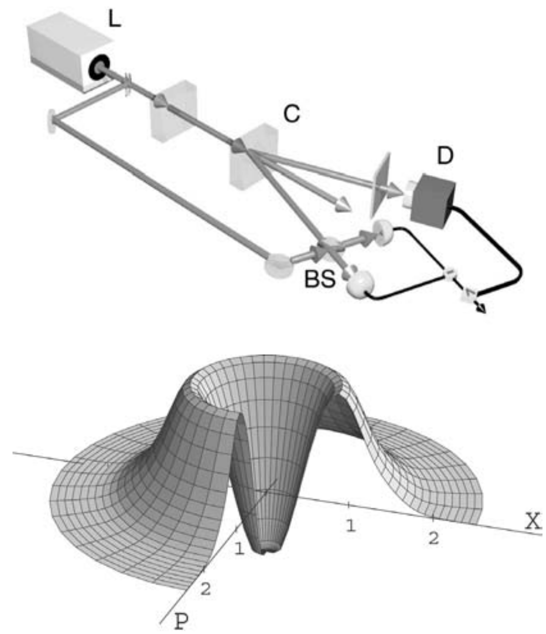

Fig. 2. Quantum state tomography of a single photon. Generation of entangled photons and triggered homodyne detection (top) leads to the reconstruction of the Wigner function (bottom). A laser L creates through a non-linear interaction in a crystal C a pair of photons in two modes. The photon in the upper mode triggers a detector D and the photon in the lower mode gets mixed on a beam splitter BS with a portion of the original laser field which serves as a local oscillator. The difference in the two mixed photo-currents (homodyne detector) is correlated with the detection of the photon in the upper mode. The current distributions for various phases of the laser field together with a mathematical algorithm — the Radon transform — yield the Wigner function of a single photon. After Lvovsky *et al.*, Phys. Rev. Lett. **87**, 050402 (2001)

7. Measured Wigner Functions

Wigner functions of a single photon have recently been observed experimentally. Space does not allow us to present these experiments in every detail, nor can we provide a complete theoretical description. Here we only try to give the flavor of these experiments and refer to the literature[11,12] for more details.

There are essentially two types of experiments. The first approach shown in Fig. 2 uses the method of quantum state tomography to reconstruct the Wigner function, whereas the second technique summarized in Fig. 3 obtains the Wigner function from the output of a Ramsey set-up.

In the tomography approach the quantized light field to be investigated is mixed on a beam splitter with a classical field of rather well-defined phase. The currents emerging from two photodetectors are subtracted. In contrast to many other experiments which only measure the average of the current for the reconstruction of the Wigner function we need the full statistics of the current fluctuations, that is the probability distribution of the current. These measurements are repeated for many different phases of the classical field. A mathematical algorithm, the so-called Radon transform,[14] enables us to obtain from this set of data the Wigner function of the underlying

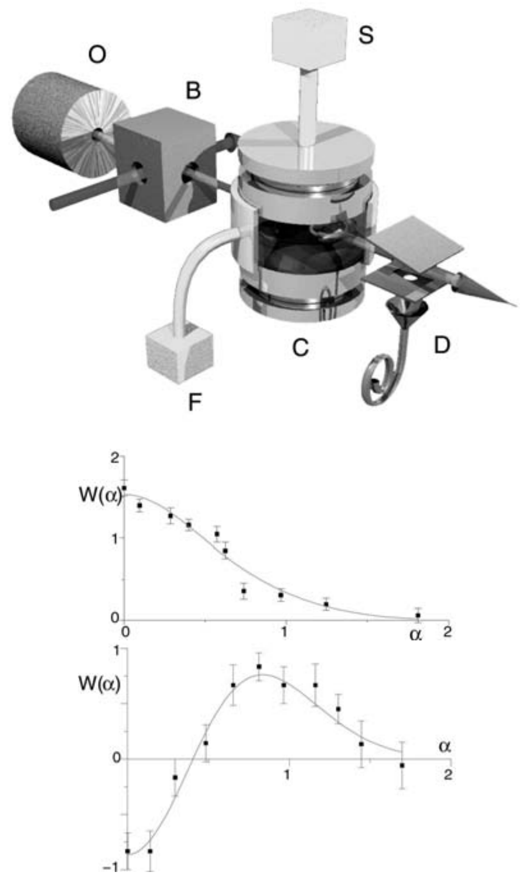

Fig. 3. Ramsey interferometry (top) to reconstruct the Wigner functions (bottom) of the vacuum (bottom upper) and a single photon (bottom lower) in an ideal cavity. An atomic beam of atoms emerging from an oven O and prepared in B in a Rydberg state probes the field in a cavity C. For this purpose two classical light fields F first prepare and then probe two internal levels of the atom: The first field prepares a dipole whereas the second field reads out the change of the dipole due to the interaction with the cavity field. A detector D measures the populations in the two levels as a function of the phase difference between the two classical fields. These Ramsey fringes are recorded for various displacements of a classical field S injected into the cavity. The contrast of the fringes for a given displacement α determines the Wigner function at the phase space point α. After P. Bertet *et al.*, Phys. Rev. Lett. **89**, 200402 (2002)

state. Figure 2 shows the so-reconstructed Wigner function[11] of a single photon state created by a parametric process in a crystal. We recognize the negative parts around the origin.

The second experiment[12] is from the realm of cavity QED. Here an atom probes the quantum state of the field inside a resonator. This field has been prepared earlier by one or more atoms. In this method of state reconstruction the information about the state is stored in the internal states of the atom. In order to be sensitive to interference in the field the atoms enter and are probed in a coherent superposition of their internal states. For the sake of simplicity we have assumed here only two internal states. A detector at the exit of the device measures the populations in the two states as a function of the amplitude of a classical field injected into the resonator. The contrast of the interference structures determines the value of the Wigner function.

In Fig. 3 we show the radial cut of the so-obtained Wigner function of the vacuum and a single photon. Whereas the vacuum enjoys a Gaussian Wigner function, Eq. (23), that is positive everywhere the one corresponding to a single photon, Eq. (24), displays clearly substantial negative parts around the origin.

8. Wave Functional of Vacuum

Find the mode functions appropriate for the problem at hand and quantize every mode oscillator according to the canonical prescription — that is the one-sentence summary of the quantum theory of radiation. The excitations of these modes are the photons. The situation when all mode oscillators are in their ground states defines the vacuum of the electromagnetic field.

This approach relies heavily on the concept of a mode function. We now briefly review a treatment[17] that does not involve mode functions but refers to the complete electromagnetic field given by all modes. This formulation provides us with a probability amplitude $\Psi = \Psi[\vec{B}(\vec{r})]$ for a given magnetic field configuration $\vec{B} = \vec{B}(\vec{r})$ being in the ground state.

In order to motivate this expression we first consider a single mode of frequency Ω_ℓ characterized by the mode index ℓ. We assume that the field in this mode is in the ground state. According to Eq. (22) the corresponding probability amplitude $\psi_\ell(q_\ell) \equiv \varphi_0(q_\ell;\Omega_\ell)$ to find the value q_ℓ determining the magnetic field via Eq. (8) is then the Gaussian distribution

$$\psi_\ell(q_\ell) = \mathcal{N}_\ell \exp\left(-\frac{1}{2}\frac{\Omega_\ell}{\hbar}q_\ell^2\right) \tag{25}$$

where $\mathcal{N}_\ell \equiv N_0(\Omega_\ell)$ denotes the normalization constant.

The probability amplitude Ψ for the vacuum of the complete electromagnetic field, that is all modes in the ground state, with the scaled magnetic field q_{-1} in the mode -1, and the field q_0 in the mode 0, the amplitude q_1 in mode 1 and ... is the product

$$\Psi = \ldots \psi_{-1}(q_{-1}) \cdot \psi_0(q_0) \cdot \psi_1(q_1) \ldots = \prod_\ell \psi_\ell(q_\ell) \tag{26}$$

of the ground state wave functions ψ_ℓ of these modes. This product in wave function space is an example for a multimode state $|\Psi\rangle$ expressed in Eq. (14) in terms of state vectors.

When we recall the Gaussian wave function, Eq. (25) and make use of the property $e^A \cdot e^B = e^{A+B}$ of the exponential function we arrive at

$$\Psi = N \exp\left(-\frac{1}{2\hbar}\sum_\ell \Omega_\ell q_\ell^2\right). \tag{27}$$

Here, we have introduced the normalization constant $N \equiv \prod_\ell \mathcal{N}_\ell$.

In the derivation of the Hamiltonian Eq. (9) we have used the relation

$$\frac{1}{\mu_0}\int d^3r\,\vec{B}^2(\vec{r}) = \sum_\ell \Omega_\ell^2 q_\ell^2. \qquad (28)$$

The integral of the square of the magnetic field translates into a sum of the squares of the mode amplitudes. Hence, we should be able to express the sum in the ground state wave function Ψ, Eq. (27), in terms of a bilinear product of magnetic fields. However, in contrast to Eq. (28) Ψ involves Ω_ℓ only in a linear way. Hence, the connection between the sum in Eq. (27) and the magnetic field must be more complicated. Indeed, Wheeler showed[17] that such a connection exists which finally yields

$$\Psi[\vec{B}(\vec{r})] = N\exp\left[-\frac{1}{16\pi^3\hbar c}\int d^3r_1\int d^3r_2\,\frac{\vec{B}(\vec{r}_1)\cdot\vec{B}(\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|^2}\right]. \qquad (29)$$

The quantity Ψ is the ground state functional. It is not an ordinary function but a functional since it depends not on a point but a whole function $\vec{B}=\vec{B}(\vec{r})$. Indeed, it is the probability amplitude to find the magnetic field distribution $\vec{B}=\vec{B}(\vec{r})$ in the vacuum state. In this approach no explicit mentioning of a mode function is made.

9. Conclusions

The photon has come a long way. From Planck's minimal portion of energy triggering the quantum revolution at the end of the 19th century, via the quantum of excitation of the electromagnetic field dominating the physics of the 20th century, to entangled photons as resources of quantum cryptography and teleportation. In this version photons will surely be central to the quantum technology of the 21st century. At last we have achieved a complete understanding of the photon, we might think.

But is our situation not reminiscent of 1874 when the professor of physics Phillip von Jolly at the University of Munich tried to discourage the young Planck from studying theoretical physics with the words: "Theoretical physics is an alright field ... but I doubt that you can achieve anything fundamentally new in it" (german original: "Theoretische Physik, das ist ja ein ganz schönes Fach ... aber grundsätzlich Neues werden sie darin kaum mehr leisten können"). In hindsight we know how wrong Prof. von Jolly was in his judgement.

Today there exist many hints that the photon might again be ready for suprises. For example, we do not have a generally accepted wave function of the photon. Many candidates[18] offer themselves: Should we use the classical Maxwell field, the energy density, or the Glauber coherence functions.[4] The pros and cons of the various approaches have been nicely argued in the paper by A. Muthukrishnan *et al.* in this volume. But could it be that there is no such wave function at all? Would this exception not point into a new direction?

Closely related to the problem of the proper photon wave function is the question of the position operator of a photon.[25] Might there be a completely new aspect of the photon lurking behind these questions?

D. Finkelstein's article in this volume is even arguing that there is still too much commutativity in quantum mechanics — restricting it further might lead to an even richer land of quantum phenomena.

Make no mistake, we have learned a lot since Einstein's famous admission about his lack of deeper insight into the photon. Nevertheless, we have only started to scratch the surface. Many more exciting discoveries can be expected to appear in the next hundred years of a photon's life.

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For more information, e-mail W. P. Schleich at Schleich@physik.uni-ulm.de or go to http://www.physik.uni-ulm.de/quan/.

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