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APPLICATION NOTE NO. 10 **Revised March 2008**

COMPRESSIBILITY COMPENSATION OF SEA-BIRD CONDUCTIVITY SENSORS

Sea-Bird conductivity sensors provide precise characterization of deep ocean water masses. To achieve the accuracy of which the sensors are capable, an accounting for the effect of hydrostatic loading (pressure) on the conductivity cell is necessary. Conductivity calibration certificates show an equation containing the appropriate pressure-dependent correction term, which has been derived from mechanical principles and confirmed by field observations. The form of the equation varies somewhat, as shown below:

SBE 4, 9, 9*plus*, 16, 19, 21, 25, 26, 26*plus*, and 53 BPR

$$\text{Conductivity (Siemens/meter)} = \text{slope}\ \frac{(g + h f^2 + i f^3 + j f^4)/10}{1 + [\text{CTcor}]\,t + [\text{CPcor}]\,p} + \text{offset} \qquad (\textbf{recommended})$$

or

$$\text{Conductivity (Siemens/meter)} = \text{slope}\ \frac{(a f^m + b f^2 + c + dt)/10}{1 + [\text{CPcor}]\,p} + \text{offset}$$

SBE 16*plus*, 16*plus*-IM, 16*plus* V2, 16*plus*-IM V2, 19*plus*, 19*plus* V2, 37, 45, 49, and 52-MP

$$\text{Conductivity (Siemens/meter)} = \text{slope}\ \frac{g + h f^2 + i f^3 + j f^4}{1 + [\text{CTcor}]\,t + [\text{CPcor}]\,p} + \text{offset}$$

where

- a, b, c, d, m, and CPcor are the calibration coefficients used for older sensors (prior to January 1995). Sea-Bird continues to calculate and print these coefficients on the calibration sheets for use with old software, but recommends use of the g, h, i, j, CTcor, CPcor form of the equation for most accurate results.
- g, h, i, j, CTcor, and CPcor are the calibration coefficients used for newer sensors.
  Note: The SBE 26, 26*plus*, and 53 BPR use the SBE 4 conductivity sensor, so both sets of calibration coefficients are reported on the calibration sheet. *SEASOFT for Waves* **for DOS**, which can be used with the SBE 26 only, only supports use of the a, b, c, d, CTcor, and CPcor coefficients. The current processing software for these instruments, *SEASOFT for Waves* **for Windows**, only supports use of the g, h, i, j, CTcor, CPcor coefficients.
- **CPcor is the correction term for pressure effects on conductivity (see below for discussion)**
- slope and offset are correction coefficients used to make corrections for sensor drift between calibrations; set to 1.0 and 0 respectively on initial calibration by Sea-Bird (see Application Note 31 for details on calculating slope and offset)
- f is the instrument frequency (kHz) for all instruments except the SBE 52-MP.
  For the SBE 52-MP, $f = \text{instrument frequency (kHz)} * (1.0 + \text{WBOTC} * t)^{0.5} / 1000.00$
- t is the water temperature (°C).
- p is the water pressure (decibars).

Sea-Bird CTD data acquisition, display, and post-processing software *SEASOFT for Waves* (for SBE 26, 26*plus*, and 53 only) and *SEASOFT* (for all other instruments) automatically implement these equations.

DISCUSSION OF PRESSURE CORRECTION

Conductivity cells do not measure the specific conductance (the desired property), but rather the conductance of a *specific geometry* of water. The ratio of the cell's length to its cross-sectional area (*cell constant*) is used to relate the measured conductance to specific conductance. Under pressure, the conductivity cell's length and diameter are reduced, leading to a lower indicated conductivity. The magnitude of the effect is not insignificant, reaching 0.0028 S/m at 6800 dbars.

The compressibility of the borosilicate glass used in the conductivity cell (and all other homogeneous, noncrystalline materials) can be characterized by E (Young's modulus) and ν (Poisson's ratio). For the Sea-Bird conductivity cell, $E = 9.1 \times 10^6$ psi, ν = 0.2, and the ratio of indicated conductivity divided by true conductivity is:

$1 + s$
where s = (CPcor) (p)
Typical value for CPcor is $- 9.57 \times 10^{-8}$ for pressure in decibars **or** $- 6.60 \times 10^{-8}$ for pressure in psi

Note: This equation and the mathematical derivations below deal only with the pressure correction term, and do not address the temperature correction term.

MATHEMATICAL DERIVATION OF PRESSURE CORRECTION

For a cube under hydrostatic load:

$\Delta L / L = s = -p (1 - 2 \nu) / E$
where
- p is the hydrostatic pressure
- E is Young's modulus
- ν is Poisson's ratio
- ΔL / L and *s* are strain (change in length per unit length)

Since this relationship is linear in the forces and displacements, the relationship for strain also applies for the length, radius, and wall thickness of a cylinder.

To compute the effect on conductivity, note that $R_0 = \rho L / A$, where R_0 is resistance of the material at 0 pressure, ρ is volume resistivity, L is length, and A is cross-sectional area. For the conductivity cell $A = \pi r^2$, where r is the cell radius. Under pressure, the new length is L (1 + *s*) and the new radius is r (1 + *s*). If R_p is the cell resistance under pressure:

$$R_p = \rho L (1 + s) / (\pi r^2 [1 + s]^2) = \rho L / \pi r^2 (1 + s) = R_0 / (1 + s)$$

Since conductivity is 1/R:

$C_p = C_0 (1 + s)$ and $C_0 = C_p / (1 + s) = C_p / (1 + [Cpcor] [p])$
where
- C_0 is conductivity at 0 pressure
- C_p is conductivity measured at pressure

A less rigorous determination may be made using the material's bulk modulus. For small displacements in a cube:

$\Delta V / V = 3 \Delta L / L = -3p (1 - 2 \nu) / E$ or $\Delta V/V = -p / K$
where
- ΔV / V is the change in volume per volume or volume strain
- K is the bulk modulus. K is related to E and ν by $K = E / 3 (1 - 2 \nu)$.

In this case, $\Delta L / L = -p / 3K$.