Empirical equation for the index of refraction of seawater

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We have determined an empirical equation for the index of refraction of water as a function of temperature, salinity, and wavelength at atmospheric pressure. The experimental data selected by Austin and Halikas ["The index of refraction of seawater," SIO Ref. 76-1 (Scripps Institution of Oceanography, La Jolla, Calif., 1976)] were fitted to power series in the variables. A ten-parameter empirical equation that reproduces the original data to within its experimental errors was obtained.

The optical index of refraction is one of the important inherent optical properties of seawater. It is used, for example, to obtain the specific volume for seawater,[1] to analyze underwater radiation propagation, and to design underwater lenses and viewing ports.

For studies of the upper mixed layer of the ocean, we can focus our attention on the index of refraction of seawater at atmospheric pressure. Specifically, pressure effects down to ≈ 100 m are small. Based on an analysis of the pressure-dependent expression for the index of refraction given by McNeil,[2] we estimate that water pressure at a depth of 100 m (≈ 10 kg/cm^2) increases the index of refraction by $\approx 1.37 \times 10^{-4}$ from its value at atmospheric pressure. Furthermore, the maximum variation of this increase is 1.2×10^{-5} for temperatures in the range $0 < T < 30$ °C and salinities in the range $0 < S < 35$‰. This maximum variation is well within the errors of both the experimental database and our fit to it (as discussed below). In summary, one can treat the effect of pressure on the index of refraction in the top 100 m by simply adding $1.37 \times 10^{-6} D$ to the index of refraction at atmospheric pressure (where D is the depth in meters).

Sager[3] gives a table of data for the index of refraction at temperature T (0 °C $\leq T \leq$ 30 °C in steps of 5 °C), salinity S (0‰ $\leq S \leq$ 40‰ in steps of 2.5‰),

and fixed wavelength $\lambda_0 = 589.3$ nm. This set of data has been frequently referenced.[4,5] Another set of data has been provided by Austin and Halikas (A&H).[6] They have reviewed previous research on the refractive index of seawater and have presented an extensive summary of experimental data as well as interpolations and extrapolations. Their selected reference data for $n(S, T, \lambda)$ at atmospheric pressure are reproduced here as Table 1; the experimental data were originally measured by Mehu and Johannin-Gilles,[7] and the stated accuracy is 3×10^{-5}. The additional values at $\lambda = 700$ nm in Table 1 are extrapolations from the experimental data that were made by A&H.

McNeil (1977)[2] used the A&H data to obtain an empirical equation for the refractive index of seawater as a function of wavelength, temperature, salinity, and pressure. Setting $P = 0$ (i.e., atmospheric pressure) in his expression yields

$$n(S, T, \lambda) = 1.3247 - 2.5 \times 10^{-6} T^2$$
$$+ S(2 \times 10^{-4} - 8 \times 10^{-7} T)$$
$$+ \frac{3300}{\lambda^2} - \frac{3.2 \times 10^7}{\lambda^4}, \tag{1}$$

where n is the index of refraction, S is the salinity in parts per thousand (‰), T is the temperature in degrees Celsius, and λ is the wavelength in nanometers.

We define δn as the difference between the value of $n(S, T, \lambda)$ given by Eq. (1) and that listed in Table 1. Figure 1 shows a plot of δn for all the entries in Table 1. Although Eq. (1) may be sufficient for some purposes, it has clear deficiencies, and the apparent

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Table 1. Data for Index of Refraction as a Function of Wavelength, Salinity, and Temperature at Atmospheric Pressure as Given in Ref. 6

λ (nm)	S (‰)	Temperature (°C)						
		1.0	5.0	10.0	15.0	20.0	25.0	30.0
404.7	0.0	1.34375	1.34368	1.34348	1.34316	1.34274	1.34224	1.34166
	34.998	1.35093	1.35072	1.3504	1.34999	1.3495	1.34894	1.34831
435.8	0.0	1.34121	1.34114	1.34094	1.34062	1.34021	1.33971	1.33913
	34.998	1.34831	1.34811	1.34778	1.34736	1.34688	1.34632	1.34569
467.8	0.0	1.33913	1.33906	1.33886	1.33854	1.33813	1.33764	1.33706
	34.998	1.34615	1.34596	1.34563	1.3452	1.34471	1.34416	1.34355
480.0	0.0	1.33844	1.33837	1.33817	1.33786	1.33745	1.33695	1.33638
	34.998	1.34544	1.34525	1.34492	1.3445	1.34401	1.34345	1.34284
508.5	0.0	1.33701	1.33694	1.33674	1.33644	1.33603	1.33554	1.33497
	34.998	1.34397	1.34378	1.34344	1.34302	1.34253	1.34199	1.34138
546.1	0.0	1.33544	1.33537	1.33518	1.33487	1.33447	1.33398	1.33341
	34.998	1.34235	1.34215	1.34183	1.3414	1.34092	1.34037	1.33977
577.0	0.0	1.33434	1.33428	1.33408	1.33378	1.33338	1.33289	1.33233
	34.998	1.34122	1.34104	1.3407	1.34028	1.33979	1.33924	1.33865
579.1	0.0	1.33427	1.33421	1.33402	1.33371	1.33331	1.33282	1.33226
	34.998	1.34115	1.34097	1.34064	1.34021	1.33972	1.33917	1.33858
589.3	0.0	1.33395	1.33388	1.33369	1.33339	1.33299	1.3325	1.33194
	34.998	1.34081	1.34063	1.3403	1.33987	1.33938	1.33883	1.33824
643.8	0.0	1.33241	1.33234	1.33215	1.33186	1.33146	1.33098	1.33042
	34.998	1.33924	1.33905	1.33872	1.3383	1.33781	1.33726	1.33666
700.0	0.0	1.33109	1.33103	1.33084	1.33055	1.33016	1.32968	1.32913
	34.998	1.33788	1.33771	1.33738	1.33695	1.33644	1.33591	1.33532

systematic error dependencies in Fig. 1 suggest possible systematic problems with the functional dependencies of Eq. (1) on S, T, and λ.

Matthaus[4] also used the data of Mehu and Johannin-Gilles to obtain an empirical equation for the index of refraction as a function of S, T, and λ,

$$n(S, T, \lambda) = 1.447824 + 3.0110 \times 10^{-4}S - 1.8029$$
$$\times 10^{-5}T - 1.6916 \times 10^{-6}T^2 - 4.89040$$
$$\times 10^{-1}\lambda + 7.28364 \times 10^{-1}\lambda^2 - 3.83745$$
$$\times 10^{-1}\lambda^3 - S(7.9362 \times 10^{-7}T$$
$$- 8.0597 \times 10^{-9}T^2 + 4.249 \times 10^{-4}\lambda$$
$$- 5.847 \times 10^{-4}\lambda^2 + 2.812 \times 10^{-4}\lambda^3), \qquad (2)$$

where S is salinity in ‰, T is temperature in degrees Celsius, and λ is wavelength in micrometers.

Equation (2) has 12 parameters, and the λ dependence only contains positive powers of λ. This is surprising because physically we would expect the index of refraction to have inverse dependencies on wavelength. The maximum deviation of this empirical equation is stated to be within $\pm 4 \times 10^{-5}$. We define $\delta n'$ as the difference between the value of $n(S, T, \lambda)$ given by Eq. (2) and that listed in Table 1. Figure 2 shows a compilation of all the $\delta n'$ with the same terminology as in Fig. 1. One can see that there is also an obvious pattern in Fig. 2, indicating possible systematic problems with the functional dependence on S, T, λ. Discrepancies in the data at 700 nm are so large that they are shown on a different scale at the right. This is not surprising because

these are not experimental data, and Matthaus did not use them in his fit.

Shifrin[8] also gives an empirical equation for the refractive index of light in ocean water. With 40 empirical coefficients, the absolute errors in his equation (with respect to the original data) do not exceed 2×10^{-4} with a probability of 0.95. Although this accuracy was sufficient for his requirements (i.e.,
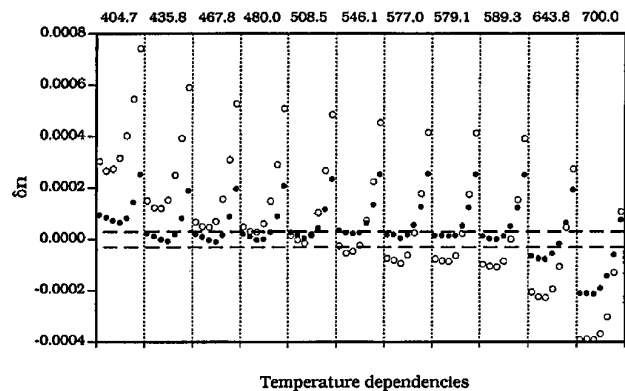

Fig. 1. Differences between the A&H data and results calculated from Eq. (1), which was obtained by McNeil. The sets of data points at each wavelength are separated by vertical dashed lines; the wavelength of each of these sets is given in units of nanometers at the top of the plot. There are 11 sets corresponding to each of the 11 wavelengths in Table 1. At a given wavelength, the δn values for $S = 0$‰ are the filled circles, and those for $S = 35$‰ are open circles. Within a wavelength range, the points from left to right are for temperatures of 1, 5, 10, 15, 20, 25, and 30 °C. The horizontal dashed lines correspond to the quoted errors in the experimental data.
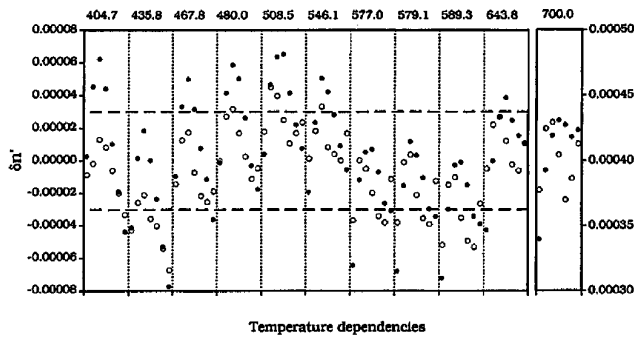
Fig. 2. Differences between the A&H data and the results calculated from Eq. (2), which was obtained by Matthaus. The terminology is the same as in Fig. 1. Deviations at 700 nm are so large they are plotted separately with an expanded scale.
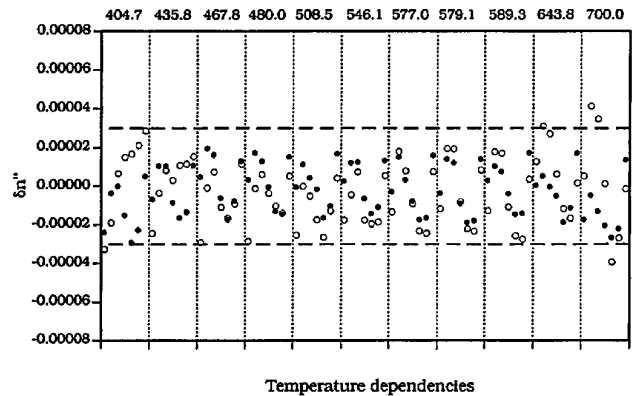

Fig. 3. Differences between the A&H data and results calculated from our new equation, Eq. (3). The terminology is the same as in Fig. 1. Most of the data points lie between the horizontal lines that correspond to the quoted errors in the experimental data.

calculations of the coefficient of molecular scattering of light by seawater), it is relatively low.

We have obtained an empirical equation that significantly improves the fit to the data of Table 1. Our approach was to fit the data to a power series that contains various powers of temperature, salinity, and wavelength and all their cross terms. The choice of powers was based on the following considerations:

(1) According to both A&H and Seaver,[9] the dependence of the index of refraction on salinity is linear, and this linearity is accurate at least to the fifth decimal digit. Because this is less than the accuracy of the experimental data, we assumed a simple linear dependence on salinity S.

(2) Positive powers of temperature up to T^3 were included.

(3) Because the index of refraction is expected to have an inverse dependence on wavelength, negative powers of λ up to λ^{-4} were included.

The resulting polynomial, which contains 40 terms, was fitted to the data from Table 1 to determine each coefficient and its standard deviation with a least-squares procedure. We then eliminated all those terms for which the standard deviation of the coefficient was larger than the coefficient itself. The remaining terms were again fitted to the data from Table 1; the result is

$$n(S, T, \lambda) = n_0 + (n_1 + n_2 T + n_3 T^2)S + n_4 T^2$$
$$+ \frac{n_5 + n_6 S + n_7 T}{\lambda} + \frac{n_8}{\lambda^2} + \frac{n_9}{\lambda^3}, \quad (3)$$

where, as before, S is the salinity in ‰, T is the temperature in degrees Celsius and λ is the wavelength in nanometers. The coefficients have the following values:

The ranges of validity are $0° < T < 30$ °C, $0‰ < S < 35‰$, and 400 nm $< \lambda < 700$ nm.

We now define $\delta n''$ as the difference between the value of $n(S, T, \lambda)$ given by Eq. (3) and that listed in Table 1. Figure 3 shows a compilation of all the $\delta n''$; the terminology is the same as in Fig. 1, but the vertical axis is expanded. The rms deviation of the $\delta n''$ is 1.5×10^{-5}.

Figure 3 clearly shows that the systematic patterns that appear in Fig. 1 have disappeared. In fact, for almost every data point, $\delta n''$ is less than the errors specified for the experimental data, which are again indicated by the horizontal dashed lines. The exceptions are a couple of points near the end of the axis. These correspond to data at 700 nm that are, in fact, not experimental data but are the result of extrapolations by A&H.

We also compared our analytic result, Eq. (3), with Sager's data at a fixed wavelength $\lambda_0 = 589.3$ nm. We define $\delta n'''$ as the difference between the value of $n(S, T, \lambda_0)$ given by Eq. (3) and Sager's index of refraction data at the same values of S and T. Figure 4 shows a compilation of all these $\delta n'''$; the vertical dashed lines separate the different salinity sections shown at the top of the graph. The maximum deviation is less than 5×10^{-5} and is generally significantly less. Again, the distributions of $\delta n'''$ are indicative of systematic effects but are generally within experimental errors. For reference, the deviations $\delta n''$ for the A&H data at 589.3 nm are also shown as open circles. Sager's data is stated to be accurate to within several digits (einige Einheiten) in the fifth digit; consequently, our new equation for the index of refraction also reproduces Sager's data (depending on the definition of several digits) to within experimental accuracy.

$n_0 = 1.31405,$ $n_1 = 1.779 \times 10^{-4},$ $n_2 = -1.05 \times 10^{-6},$ $n_3 = 1.6 \times 10^{-8},$

$n_4 = -2.02 \times 10^{-6},$ $n_5 = 15.868,$ $n_6 = 0.01155,$ $n_7 = -0.00423,$

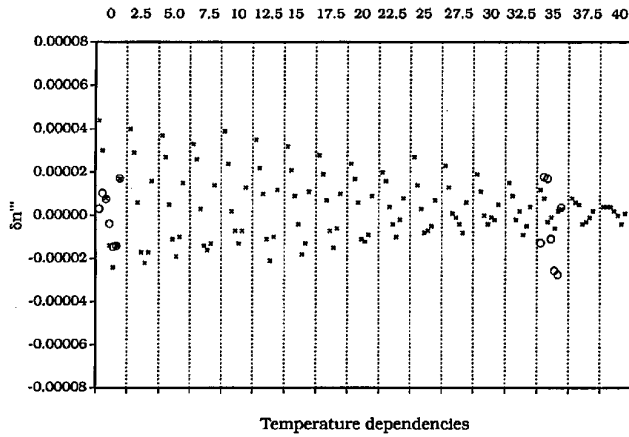$n_8 = -4382,$ $n_9 = 1.1455 \times 10^6.$

Fig. 4. Differences between Sager's data at a wavelength of 589.3 nm and results calculated from our Eq. (3) are shown as crosses. The sets of data points at each salinity are separated by vertical dashed lines and are labeled in units of ‰ at the top of the plot. Within each salinity section the data points from left to right are from 0 to 30 °C in steps of 5 °C. For reference, the $\delta n''$ data at 589.3 nm from Fig. 3 are reproduced here and shown as open circles.

In conclusion, we have provided a simple analytic expression for the index of refraction of seawater, $n(S, T, \lambda)$, that reproduces all the experimental data to within experimental errors.

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